

Student Name

Mathematics Stage 4

INDICES

Book 2 Examine cube roots and square roots

Version: 260207

Report mistakes & suggest improvements:
<https://MrDingMaths.com>

CONTENTS

Overview	2
Square Roots	4
Cube Roots	10
Estimating Square Roots and Cube Roots.....	13
Evaluate Expressions with Roots	15
Check Your Understanding	20

OVERVIEW

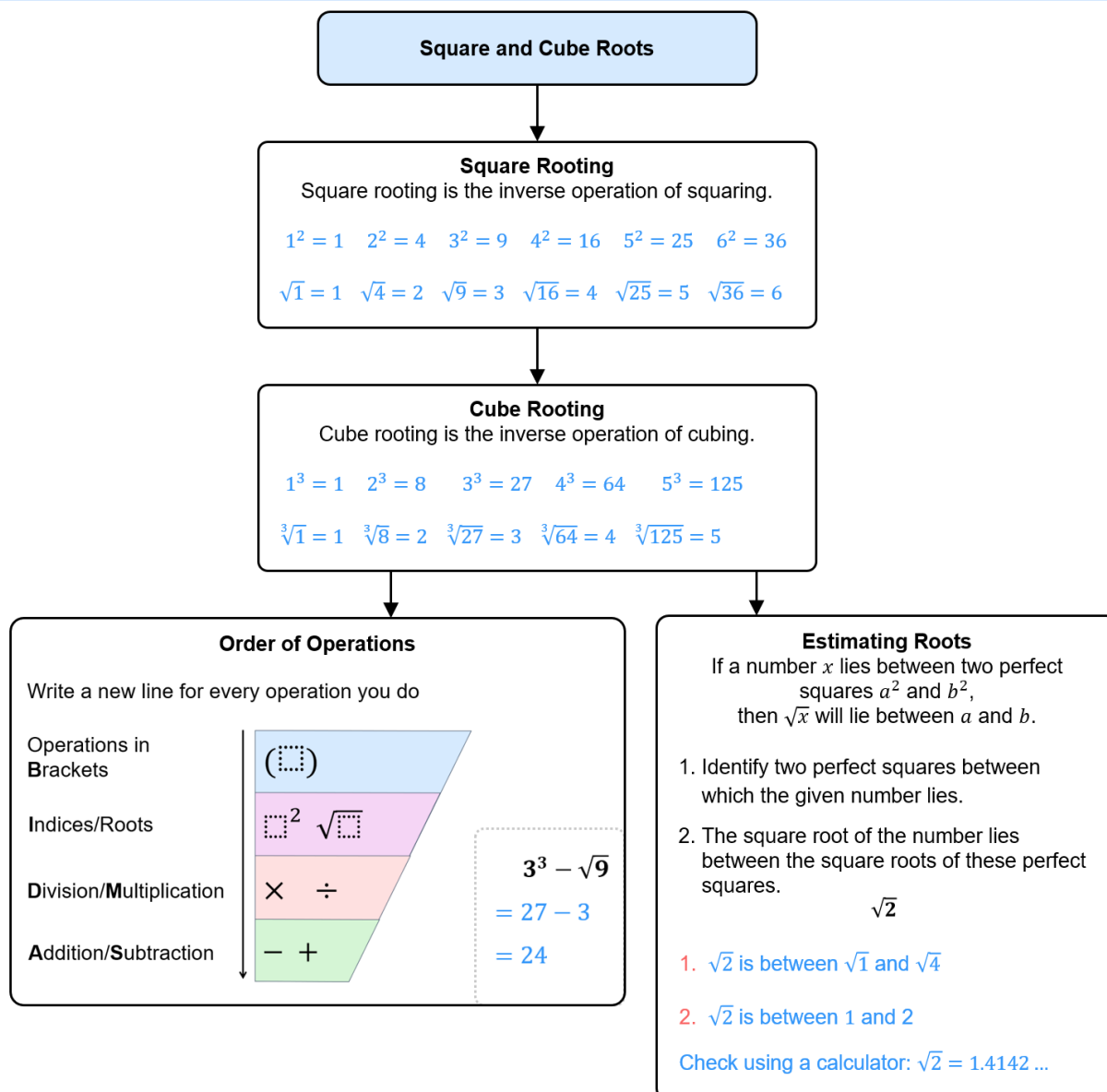
SYLLABUS CONTENT

MA4-INT-C-01 operates with primes and roots, positive-integer and zero indices involving numerical bases and establishes the relevant index laws

Examine cube roots and square roots

- Use the notations for square root ($\sqrt{\quad}$) and cube root ($\sqrt[3]{\quad}$)
- Recognise and describe the relationship between squares and square roots, and cubes and cube roots for positive numbers
- Verify, through numerical examples, that $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- Estimate the square root of any non-square whole number and the cube root of any non-cube whole number, then check using a calculator
- Identify and describe exact and approximate solutions in the context of square roots and cube roots
- Apply the order of operations to evaluate expressions involving square roots, cube roots, square numbers and cube numbers

SUMMARY



REVIEW OF PRIOR KNOWLEDGE

1 Complete the following using the order of operations.

a $3 \times 5 + 4 - 2$

b $3 \times 5 - 2 + 4$

c $3 - 5 + 4 \times 2$

d $3 - 5 \times 4 + 2$

e $(4 + 5)^2$

f $4 + 5^2$

g $2 + (3 \times 4^2)$

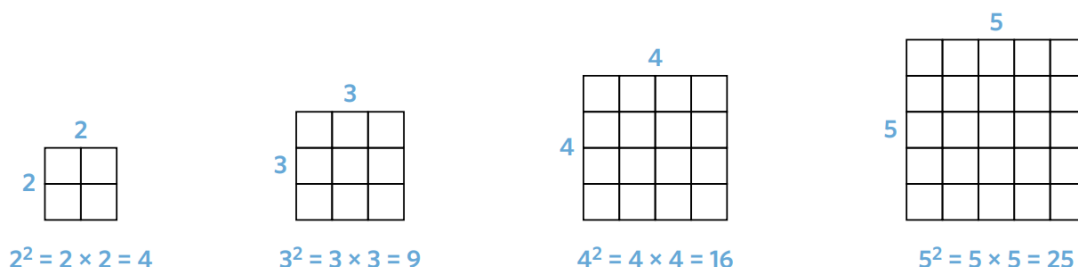
h $6 \times (2^3 + 8)$

i $4 + (6 \div (2 \times 5)^2)$

SQUARE ROOTS

SQUARING A NUMBER

- Squaring a number means to multiply the number by itself.
- This is the same as raising to the power of 2.
- By squaring a number, we make a square with that side length.
- A **perfect square** the square of a whole number.



Example Identify perfect squares.

$1^2 = 1 \times 1 =$	$9^2 =$
$2^2 =$	$10^2 =$
$3^2 =$	$11^2 =$
$4^2 =$	$12^2 =$
$5^2 =$	$13^2 =$
$6^2 =$	$14^2 =$
$7^2 =$	$15^2 =$
$8^2 =$	

Square these numbers.

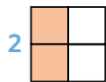
a 8^2 $=$	b 13^2 $=$
64	169
c 100^2 $=$	d 0.4^2 $=$
10 000	0.16
e 1.1^2 $=$	f 15^2 $=$
1.21	225

SQUARE ROOTS

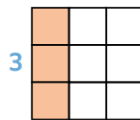
- **Square rooting** $\sqrt{\quad}$ a number is the inverse operation of squaring a number.

$$4^2 = 16$$

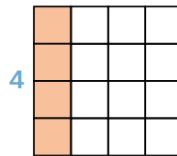
$$\sqrt{16} = 4$$



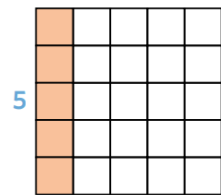
$$\sqrt{4} = 2$$



$$\sqrt{9} = 3$$



$$\sqrt{16} = 4$$



$$\sqrt{25} = 5$$

🔗 Square Roots: <https://www.geogebra.org/m/wxvue3sb>

Example Identify square roots of perfect squares.

$\sqrt{1} =$	$\sqrt{4} =$	$\sqrt{25} =$	$\sqrt{121} =$
$\sqrt{16} =$	$\sqrt{49} =$	$\sqrt{36} =$	$\sqrt{100} =$
$\sqrt{9} =$	$\sqrt{81} =$	$\sqrt{64} =$	$\sqrt{144} =$

Key ideas

1. Squaring a number means to a number by itself.
2. Square rooting is the operation of squaring. It will “undo” a square.

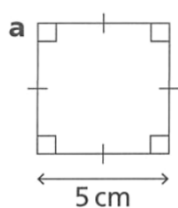
1 Evaluate:

a 6^2	b 6 squared	c 6 to the power of 2
36	36	36
d 6×6	e 5^2	f 11^2
36	25	121
g 13^2	h 12^2	i 9^2
169	144	81
j 100^2	k 0^2	l 1^2
10 000	0	1

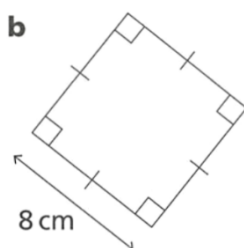
2 Evaluate:

a $\sqrt{36}$	b Square root of 36	c $\sqrt{25}$
6	6	5
d $\sqrt{169}$	e $\sqrt{4}$	f $\sqrt{0}$
13	2	0
g $\sqrt{49}$	h $\sqrt{100}$	i $\sqrt{400}$
7	10	20
j $\sqrt{1}$	k $\sqrt{49}$	l $\sqrt{144}$
1	7	12
m $\sqrt{169}$	n $\sqrt{196}$	o $\sqrt{225}$
13	14	15

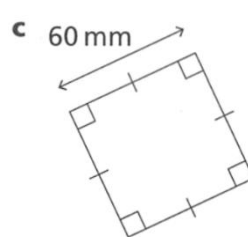
3 Calculate the area of each square.



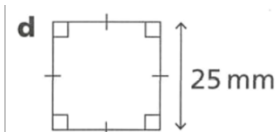
25 cm²



64 cm²



3600 mm²



625 mm²

4 Find the side lengths of these squares.

a



7 cm

b



8 cm

c



6 cm

d



11 cm

5 Find the missing values.

a $\underline{\quad}^2 = 9$

3

b $\underline{\quad}^2 = 25$

5

c $\underline{\quad}^2 = 16$

4

d $\underline{\quad}^2 = 49$

7

6 List all the square numbers between 101 and 200. Hint: There are only four.

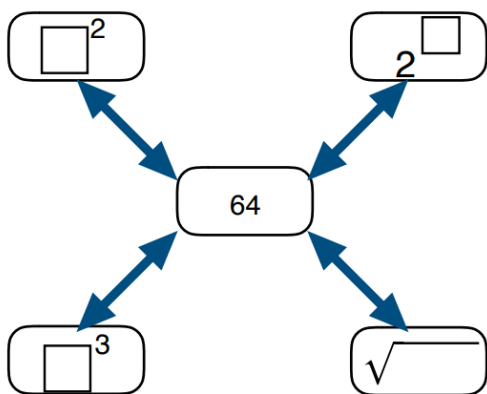
121, 144, 169, 196

7 Type $\sqrt{-1}$ into the calculator. Try this with square roots of other negative numbers. Explain.

You cannot square root a negative number, since the square of a number must be positive.

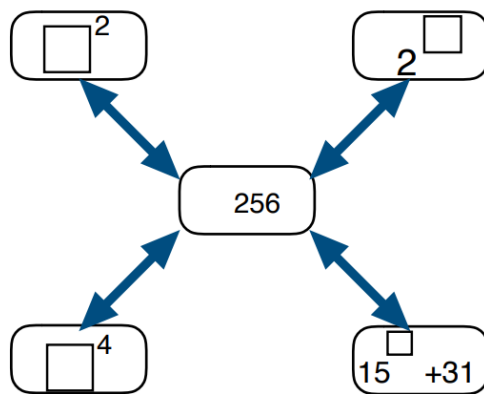
8 Fill in the missing numbers to make each box equal to each other.

a



Clockwise from top-left: 8^2 , 2^6 , $\sqrt{4096}$, 4^3

b



Clockwise from top-left: 16^2 , 2^8 , $15^2 + 31$, 4^4

9 First determine the prime factorisation of the quantity, then use it to find the square root.

a $196 = 2 \times 2 \times 7 \times 7$
 $= 2^2 \times 7^2$
 $= 14^2$
 $\sqrt{196} = \dots\dots\dots$

14

b $100 = 2 \times 2 \times 5 \times 5$
 $= \dots\dots\dots$
 $= \dots\dots\dots$
 $\sqrt{100} = \dots\dots\dots$

10

c $225 = 3 \times 3 \times \dots\dots\dots \times \dots\dots\dots$
 $= 3^2 \times \dots\dots\dots$
 $= \dots\dots\dots$
 $\sqrt{225} = \dots\dots\dots$

15

d $900 =$
 $\sqrt{900} =$

30

10 Find the square root of each of the following numbers using its prime factorisation.

a $441 = 3 \times 3 \times 7 \times 7$

21

b $5625 = 3 \times 3 \times 5 \times 5 \times 5 \times 5$

75

c $9801 = 3 \times 3 \times 3 \times 3 \times 11 \times 11$

99

d $1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

36

11

a Draw the factor tree of 784

b Draw the factor tree of 4356

Use the prime factorisation of 784 to find $\sqrt{784}$

28

Hence find $\sqrt{4356}$

66

12

a List the factors of the number 20

1, 20 2, 10 4, 5

b List the factors of the number 49

1, 7, 49

c The number of factors of a square number will always be odd. Why is this true?

Factors usually come in "pairs," so most numbers have an even number of factors.

Square numbers have a "pair" that is a number and itself, so it only counts as 1. An even number of factors plus 1 is always odd.

13

a Evaluate 12^2

144

b Evaluate $3^2 \times 4^2$

144

c The rule $(ab)^2 = a^2b^2$ can be used to link 12^2 and $3^2 \times 4^2$. Check this rule is always true using three pairs of other numbers (you may use a calculator).

14 Use the rule above to evaluate mentally:

a $2^2 \times 4^2$

64

b $6^2 \times 2^2$

144

c $10^2 \times 5^2$

2500

15 Use a calculator to:

a Evaluate $\sqrt{36} + \sqrt{64}$

14

b Evaluate $\sqrt{36 + 64}$

10

c Is $\sqrt{36} + \sqrt{64} = \sqrt{36 + 64}$?

no

16 Use a calculator to:

a Evaluate $\sqrt{36} \times \sqrt{64}$

48

b Evaluate $\sqrt{36 \times 64}$

48

c Is $\sqrt{36} \times \sqrt{64} = \sqrt{36 \times 64}$?

yes

17 Find the smallest whole number n such that $\sqrt{2016n}$ is a whole number.

$$2016n = 2^5 \times 3^2 \times 7 \times n$$

For a square root to be a whole number, it must be a perfect square.

So all the powers must be multiples of 2.

Smallest number for n is $2 \times 7 = 14$

CUBE ROOTS

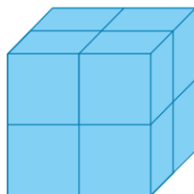
CUBING A NUMBER

- **Cubing** a number means to use a number in a multiplication 3 times.
It is shown by the index number ³
- By cubing a number, we have made a cube with that side length.

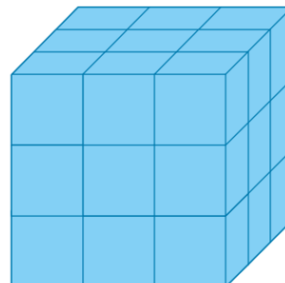
$$1^3 = 1 \times 1 \times 1 = 1$$



$$2^3 = 2 \times 2 \times 2 = 8$$



$$3^3 = 3 \times 3 \times 3 = 27$$



CUBE ROOTS

- Cube rooting a number is the inverse operation of cubing a number.
- The cube root symbol is written as $\sqrt[3]{}$
- Cube Roots: <https://www.geogebra.org/m/hmmv2sud>

Example Perfect cubes and cube roots

$1^3 =$	$4^3 =$	$\sqrt[3]{1} =$	$\sqrt[3]{216} =$
$2^3 =$	$5^3 =$	$\sqrt[3]{27} =$	$\sqrt[3]{64} =$
$3^3 =$	$6^3 =$	$\sqrt[3]{8} =$	$\sqrt[3]{125} =$

Evaluate the following numbers.

a 4^3

b 5^3

c 3^3

64

125

27

d $\sqrt[3]{1000}$

e $\sqrt[3]{64}$

f $\sqrt[3]{8}$

10

4

2

Key ideas

1. Cubing a number means to use a number in a multiplication times.
2. Cube rooting is the operation of cubing. It will “undo” a cube.

1 Evaluate the following.

a	2^3	b	2 cubed	c	2 to the power of 3
d	4^3	e	3^3	f	5^3
g	10^3	h	$\sqrt[3]{64}$	i	$\sqrt[3]{125}$
j	$\sqrt[3]{1000}$	k	$\sqrt[3]{1}$	l	$\sqrt[3]{0}$

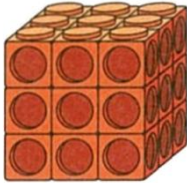
2 How many small cubes are used to make each bigger cube?

a



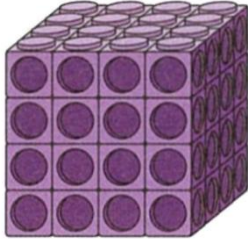
8

b



27

c



64

3 Evaluate each expression.

a	2^4		f	4^3	
b	8^2		g	3^4	
c	$\sqrt[3]{27}$		h	$\sqrt{9}$	
d	3^4		i	2^6	
e	$\sqrt[3]{1000}$		j	$\sqrt{256}$	

Therefore, match the left and right columns in the table above.

4 Using a calculator, or otherwise, evaluate the following:

a $\sqrt[3]{27}$

3

b $\sqrt[4]{16}$

2

c $\sqrt[5]{100\,000}$

10

d $\sqrt[3]{125}$

5

e $\sqrt[3]{343}$

7

5

a Use the fact that $5^7 = 78\,125$ to work out $\sqrt[7]{78\,125}$

5

b Use the fact that $\sqrt[4]{1296} = 6$ to evaluate 6^4

1296

c Given that $2^{10} = 1024$, work out $(\sqrt[10]{1024})^2$

4

6

a Evaluate 3^3

27

b Evaluate $(-3)^3$

-27

c Complete the sentence:

A positive number cubed will always be

positive

A negative number cubed will always be

negative

7 $x^3 = 71$. What is $(-x)^3$? Explain how you know.

-71, as $(-x)^3 = -x^3$

8 $\sqrt[3]{p} = 43$. What is $\sqrt[3]{-p}$?

-43

ESTIMATING SQUARE ROOTS AND CUBE ROOTS

- If $a^2 < x < b^2$, then $a < \sqrt{x} < b$
- To estimate square roots:
 1. Identify two perfect squares between which the given number lies.
 2. The square root of the number lies between the square roots of these perfect squares.
- To get the exact value, you'll usually need to use a calculator.

Example Estimate square roots

$\sqrt{2}$ 1. $\sqrt{2}$ is between $\sqrt{1}$ and $\sqrt{4}$ 2. $\sqrt{2}$ is between 1 and 2 Check using a calculator: $\sqrt{2} = 1.4142 \dots$	Why did we choose $\sqrt{1}$ and $\sqrt{4}$, rather than $\sqrt{3}$? $\sqrt{1}$ and $\sqrt{4}$ make numbers.
---	---

Estimate the value of these square roots.

a $\sqrt{13}$

$\sqrt{13}$ is between $\sqrt{9}$ and $\sqrt{16}$

$\sqrt{13}$ is between and

$\sqrt{13}$ is between 3 and 4

b $\sqrt{17}$

$\sqrt{17}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$

$\sqrt{17}$ is between and

$\sqrt{17}$ is between 4 and 5

c $\sqrt{68}$

$\sqrt{68}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$

$\sqrt{68}$ is between and

$\sqrt{68}$ is between 8 and 9

Key ideas

1. We can estimate square roots if we know which two numbers it lies between.

1 Complete the number line.



Using the number line, between which two integers does each value lie?

i $\sqrt{50}$	ii $\sqrt{29}$	iii $\sqrt{45}$	iv $\sqrt{63}$
7 and 8	5 and 6	6 and 7	7 and 8

2 Estimate the value of these square roots and plot their approximate location on a number line. $\sqrt{3}$ has been done for you.

a $\sqrt{1}$	b $\sqrt{2}$	c $\sqrt{3}$	d $\sqrt{7}$
$\sqrt{1} = 1$	$1 < \sqrt{2} < 2$	$\sqrt{1} < \sqrt{3} < \sqrt{4}$ $1 < \sqrt{3} < 2$	$2 < \sqrt{7} < 3$
e $\sqrt{16}$	f $\sqrt{35}$	g $\sqrt{50}$	h $\sqrt{64}$
$\sqrt{16} = 4$	$6 < \sqrt{35} < 7$	$7 < \sqrt{50} < 8$	$\sqrt{64} = 8$



Did you place $\sqrt{3}$ before or after $\sqrt{2}$? Why?

After, since $\sqrt{3}$ must be larger than $\sqrt{2}$

Check the location of your estimates using a calculator.

3 Estimate the value of these cube roots and plot their approximate location on a number line.

a $\sqrt[3]{8}$	b $\sqrt[3]{16}$	c $\sqrt[3]{80}$	d $\sqrt[3]{310}$
$\sqrt[3]{8} = 2$	$\sqrt[3]{8} < \sqrt[3]{16} < \sqrt[3]{27}$	$4 < \sqrt[3]{80} < 5$	$6 < \sqrt[3]{310} < 7$

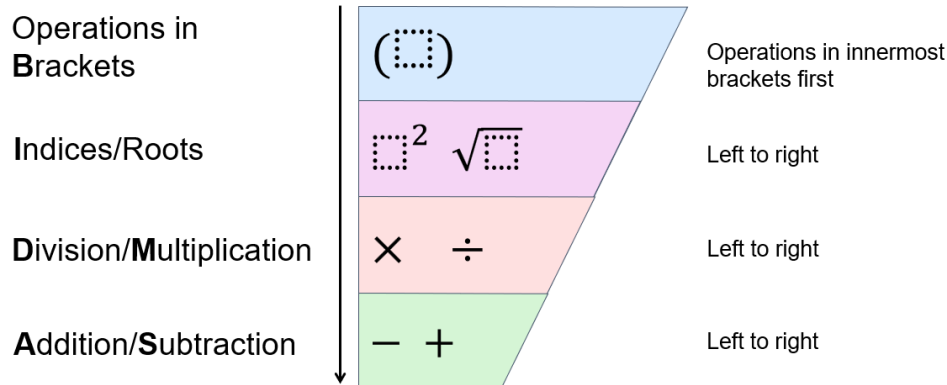


EVALUATE EXPRESSIONS WITH ROOTS

ORDER OF OPERATIONS

- Evaluate roots after operations in brackets and before multiplication/division.
- If both indices and roots are present, evaluate from left to right.
- **Note:** There are invisible brackets inside square roots. $\sqrt{a + b} = \sqrt{(a + b)}$

Write a new line for every operation you do



Example Evaluate expressions with roots.

$3^3 - \sqrt{9}$ $= 27 - 3$ $= 24$	Which operations did we do first? <i>We did and first</i>
$\sqrt{8^2 + 6^2}$ $= \sqrt{64 + 36}$ $= \sqrt{100}$ $= 10$	Add brackets to the expression. Which operations did we do first? <i>We did and because they were inside</i>

Evaluate these expressions.

a $\sqrt{81} - 3$	b $\sqrt{121} - 2^4$	c $\sqrt{3^2 + 4^2}$
6	-5	5
d $3^2 - \sqrt{9}$	e $\sqrt[3]{10^2 + 5^2}$	f $\sqrt{\frac{20 \times 3 + 15}{3}}$
6	5	5
g $(\sqrt{9})^3 + 5 \times \sqrt{100}$	h $(\sqrt{7^2} - \sqrt{16}) \div \sqrt{144}$	i $3(\sqrt{4} + \sqrt{25})^2$
77	$\frac{1}{4}$	147

1 Evaluate these expressions according to the order of operations.

a $\sqrt{11+5} \times 2$	b $6 \times \sqrt{3+1}$	c $45 \div \sqrt{17+8}$
8	12	9
d $50 \div 5^2 + 8$	e $100 \div (4+1)^2$	f $100 \div \sqrt{20+5}$
10	4	20
g $\sqrt{3^2+4^2}$	h $\sqrt{5^2-3^2}$	i $\sqrt{9} \times \sqrt{64} \div \sqrt{36}$
5	4	2
j $4^3 + 2^2$	k $3^3 + 2 \times 4^2$	l $5^2 \div \sqrt{9+16}$
68	59	5
m $\frac{5 \times (3+1)^2}{17+3}$	n $\sqrt{\frac{100-64}{9}}$	o $\sqrt{\frac{10^2-8^2}{3^2}}$
4	2	2
p $\sqrt[3]{\frac{5^2-3^2}{2}} \times \sqrt[3]{8}$	q $\sqrt{\frac{9-5}{9}}$	r $\sqrt{\frac{28+4}{28+22}} + 5$
4	$\frac{2}{3}$	$\frac{29}{5}$

2 Evaluate the following.

a $12 + 4 \times \sqrt{9}$

b $(12 + 4) \times \sqrt{9}$

c $12 + \sqrt{4 \times 9}$

d $\sqrt{12 + 4} \times 9$

e $12 + \sqrt{4} \times \sqrt{9}$

f $12 \times \sqrt{4} + 9$

g $12 \times \sqrt{4} - 9$

h $12 \times (\sqrt{4} - 9)$

i $12 - 9 \times \sqrt{4}$

j $12^2 - 9 \times \sqrt{4}$

k $(12^2 - 9) \times \sqrt{4}$

l $12^2 - \sqrt{9 \times 4}$

24

15

48

-84

18

-6

36

126

30

270

33

138

3 Explain what is wrong with this solution and provide the correct solution.

$$\begin{aligned} (\sqrt{81} - \sqrt{36})^2 &= (\sqrt{81})^2 - (\sqrt{36})^2 \\ &= 81 - 36 \\ &= 45 \end{aligned}$$


Didn't follow order of operations.

$$(9 - 6)^2 = 3^2 = 9$$

4 Evaluate the following:

a $\sqrt{9} + \sqrt{16} \times \sqrt{25}$

b $(\sqrt{9} + \sqrt{16}) \times \sqrt{25}$

c $(\sqrt{9} + \sqrt{16})^2 \times \sqrt{25}$

d $2(\sqrt{9} + \sqrt{16})^2 \times \sqrt{25}$

23

35

245

490

5 Evaluate the following.

a $\sqrt[3]{59 + 5}$

b $\sqrt[3]{64 \times 1000}$

4

40

c $\sqrt[3]{27} + \sqrt[3]{8}$

d $\sqrt[3]{64} - 4 + \sqrt{81}$

5

9

e $\sqrt[3]{125} + \sqrt{100}$

f $(\sqrt[3]{216} - \sqrt[3]{64})^2 \times 25$

15

100

g $(\sqrt[3]{5})^3 + \sqrt{25} + 11$

h $\frac{\sqrt[3]{64}}{8}$

21

$\frac{1}{2}$

6 Show the working out to evaluate the following. Check your answer using a calculator.

a $(6 \times 6) \div 3^2 - 1 + 11^2$

b $10 + ((10 - 4) \times (2^3 - 6))^2 \div 2 + 2$

124

84

c $\sqrt{(12 \times 2^2 - (4^2 - 4)) \times 3}$

d $\sqrt{\frac{2+4^2}{8-2 \times 3}}$

18

3

e $\sqrt{((5^2 - 17) + (6^2 - 7) + 2) \div 3}$

f $\sqrt{\frac{(45 \div (30 - 5 \times 3))^3 \times \sqrt{4+5}}{(-3)^2}}$

$\sqrt{13}$

3

7 Evaluate the following expression.

$$\frac{\sqrt{49} - 1}{\sqrt[3]{-64}} \times \frac{\sqrt{64}}{\sqrt{25}} \div \frac{(-2)^2}{2^3} \times \frac{\sqrt[3]{125}}{3^2}$$

$$-\frac{8}{3}$$

8 Evaluate the following.

a $\left(\frac{1}{5}\right)^3$

$$\frac{1}{125}$$

b $\left(\frac{4}{7}\right)^2$

$$\frac{16}{49}$$

c $\left(2\frac{1}{4}\right)^2$

$$\frac{81}{16}$$

d $\sqrt{\frac{1}{4}}$

$$\frac{1}{2}$$

e $\sqrt[3]{-\frac{8}{27}}$

$$-\frac{2}{3}$$

f $\sqrt{1\frac{24}{25}}$

$$\frac{7}{5}$$

g 0.5^3

$$\frac{1}{8}$$

h $\sqrt{0.04}$

$$0.2$$

i $\sqrt[3]{0.343}$

$$0.7$$

j $\left(\frac{1}{2} + \frac{1}{3}\right)^2$

$$\frac{25}{36}$$

k $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^3$

$$\frac{31}{108}$$

l $\left(\sqrt{49} \times \frac{1}{5}\right)^3$

$$\frac{343}{125}$$

m $\sqrt{\frac{2}{5} \times 1\frac{3}{5}}$

$$\frac{4}{5}$$

n $(1.8 - 0.9)^2$

$$0.81$$

o $\sqrt{1.2 - 1 + 0.05}$

$$0.5$$

p $\sqrt{6\frac{19}{25}}$

$$\frac{13}{5}$$

q $\sqrt[3]{1.728}$

$$\frac{6}{5}$$

r $\sqrt{3\frac{1}{9} \times 1\frac{1}{6} \times \frac{2}{3}}$

$$\frac{14}{9}$$

CHECK YOUR UNDERSTANDING

Square roots

1	A square has an area 64 cm^2 . What is the length of one side of the square?			
	a 4 xm	b 6 cm	c 8 xm	d 32 xm
2	What is the square root of 9?			
	a 3	b 4.5	c 81	d 18
3	$\sqrt{36}$			
	a 6	b 18	c 40	d 9
4	$\sqrt{64}$			
	a 4	b 6	c 8	d 32
5	$\sqrt{144}$			
	a 72	b 12	c 36	d 20 736
6	Which statement is true?			
	a $\sqrt{16} = 8$	b $\sqrt{4} = 16$	c $\sqrt{16} = 4$	d $\sqrt{40} = 4$
7	Between what two numbers does $\sqrt{71}$ lie between?			
	a 35 and 36	b 8 and 9	c 7 and 8	d 9 and 10

Cube roots

1	$\sqrt[3]{27}$	a 3	b 9	c 81	d 327
2	$\sqrt[3]{125}$	a 5	b 25	c $\frac{125}{3}$	d 375
3	What two whole numbers does $\sqrt[3]{65}$ lie between?	a 8 and 9	b 7 and 8	c 5 and 6	d 4 and 5
4	What two whole numbers does $\sqrt[3]{32}$ lie between?	a 2 and 3	b 3 and 4	c 9 and 10	d 10 and 11

Evaluating expressions with roots

1	$5^3 + \sqrt{9}$	a 18	b 24	c 128	d 134
2	Which of these expressions does not evaluate to an integer?	a $\sqrt{1} + \sqrt{4}$	b $\sqrt{81} + \sqrt[3]{2}$	c $\sqrt[3]{27} + \sqrt[3]{1}$	d $\sqrt{8} + \sqrt{8}$
3	Which of these expressions does not evaluate to an integer?	a $\sqrt{25} + \sqrt{16}$	b $\sqrt{49} + \sqrt[3]{16}$	c $\sqrt[3]{64} + \sqrt[3]{1}$	d $\sqrt{100} + \sqrt{4}$
4	$\sqrt{8^2}$	a 8	b 64	c 4	d 16
5	Tom says $\sqrt{16} + \sqrt{9} = 7$ Katie says $\sqrt{16 + 9} = 7$ Who is correct?	a Only Tom	b Only Katie	c Both	d Neither