Student Name

Mathematics Stage 5 Paths

PROPERTIES OF GEOMETRICAL FIGURES

Book 4 Establish and apply properties of similar shapes and solids

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-GEO-P-01 establishes conditions for congruent triangles and similar triangles and solves problems relating to properties of similar figures and plane shapes

Establish and apply properties of similar shapes and solids

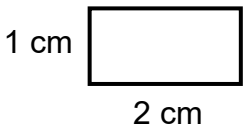
- Establish for 2 similar figures with similarity ratio $a:b$ that their areas are in the ratio $a^2:b^2$ and their volumes are in the ratio $a^3:b^3$
- Solve problems involving areas and volumes of similar shapes and solids

AREAS OF SIMILAR FIGURES

Investigation: Area and volumes of similar figures.

The diagrams below show two similar rectangles, with the second being an enlargement.

Original



$A_1 =$ .....

Image

$A_2 =$ .....

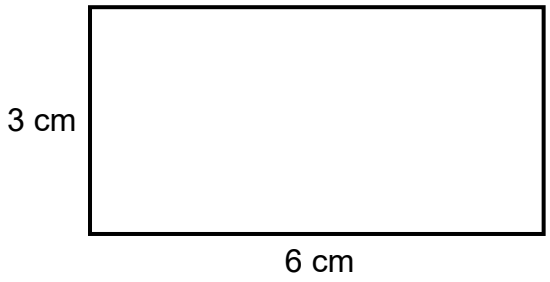
The length ratio of original : image is: .....

The area ratio of original : image is: .....

Original

$A_1 =$ .....

Image



$A_3 =$ .....

The length ratio of original : image is: .....

The area ratio of original : image is: .....

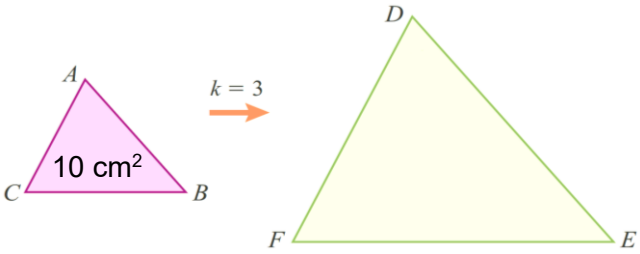
AREAS OF SIMILAR SHAPES

Similar Areas	
If two figures are in the ratio	$a : b$
their areas are in the ratio	$a^2 : b^2$

1. Determine ratio of lengths.
2. Determine ratio of areas.
3. Use a ratio table to find area of unknown.

Example Calculate area and volume of similar figures.

A triangle has been enlarged with scale factor of $k = 3$. The original area is 10 cm^2, calculate the area of the enlarged triangle.



1. Length ratio: $1 : 3$
2. Areas in ratio: $1^2 : 3^2$
   $= 1 : 9$
3. Ratio table:

Original	:	Image
1	:	9
10	:	90

The area of the enlarged figure is 90 cm^2.

How did we know the areas will be in the ratio $1 : 9$?

The lengths are in the ratio $1 : 3$,
So, areas are in the ratio $1 : \ldots\ldots^2 = 1 : 9$

What are two ways to get 90 using the ratio table?

We can use the rows:

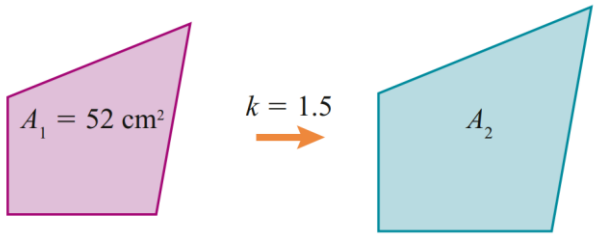
Multiply 10 by ……

We can use the columns:

Multiply 9 by ……

Calculate the areas of these similar figures.

a

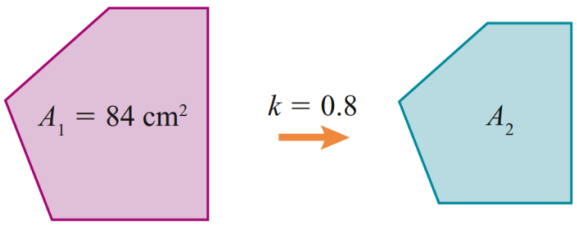


1. Length ratio:
2. Areas in ratio:
3. Ratio table:

Original	:	Image
	:	
	:	

117 cm^2

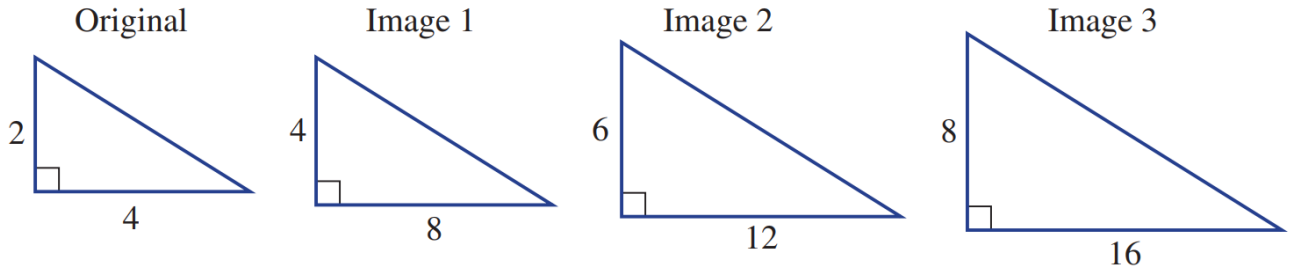
b



Original	:	Image
	:	
	:	

53.76 cm^2

1 Consider these three similar triangles (not drawn to scale). All units are in cm.



a Complete this table, comparing each image to the original.

Triangle	Original	Image 1	Image 2	Image 3
Length ratio	1	1:2	1:3	1:4
Area	4 cm^2	16	36	64
Area ratio	1	1: 4	1:9	1:16

b What would be the area ratio if the length ratio is $1:n$?

$1:n^2$

c What would be the area ratio if the length ratio is:

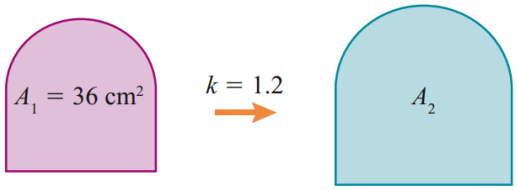
i 1:10? ..... 1:100 **ii** 1:20? ..... 1:400 **iii** 1:100? 1:10 000

d What would be the area ratio if the length ratio is $1:\frac{1}{2}$? .....

$1:\frac{1}{4}$

2 Use the information given to find the unknown area for each pair of similar figures.

a

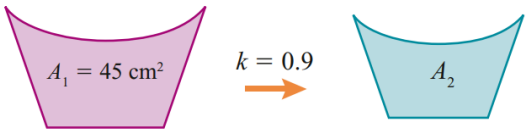


1. Length ratio:
2. Areas in ratio:
3. Ratio table:

Original	:	Image
1	:	1.2^2
36	:	51.84

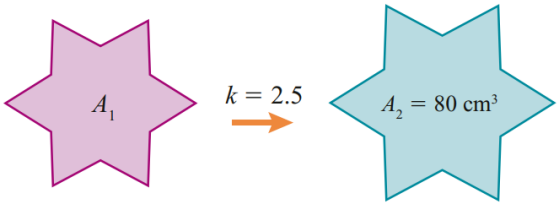
51.84 cm²

b



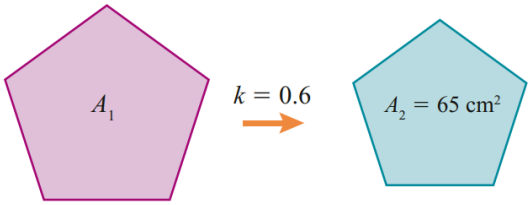
36.45 cm²

c



12.8 cm²

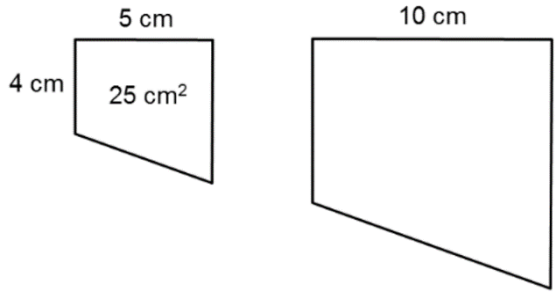
d



180.$\dot{5}$ cm²

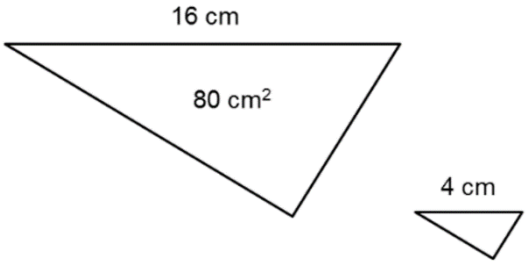
3 These shapes are similar.

a What is the area of the larger shape?



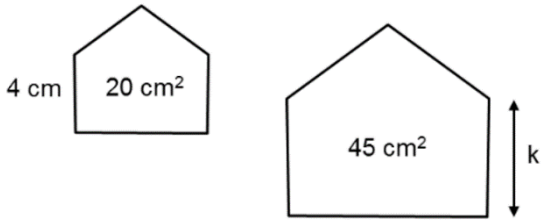
$\left(\frac{10}{5}\right)^2 = \frac{x}{25}$
$x = 100$ cm²

b What is the area of the smaller shape?



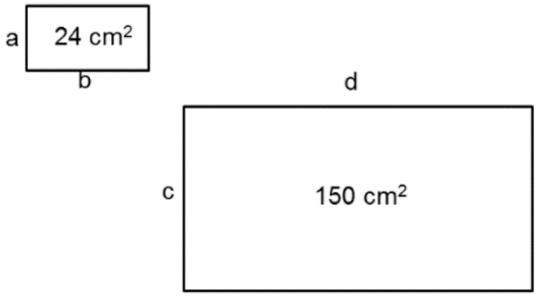
$\frac{x}{80} = \left(\frac{4}{16}\right)^2$
$x = 5$ cm²

c What is the length k?



$\left(\frac{k}{4}\right)^2 = \frac{45}{20}$
$k = 6$ cm

d Find some values of a, b, c, d if they are all integers.



$k^2 = \frac{150}{24}$
$k = 2.5$
a, b are factor pairs of 24 and c, d must be factors pairs of 150:
Possible 4-pairs of (a, b, c, d):
$(2, 12, 5, 18)$
$(4, 6, 10, 15)$
$(6, 4, 15, 10)$
$(12, 2, 18, 5)$

4 A scale diagram of a park uses a scale of 1 cm = 10 m.

a Write this as a ratio; remember that ratios always have the same units for both quantities.

$1:1000$

b A park is 15 cm across on the diagram. Work out the equivalent distance in real life in m.

$15 \text{ cm} \times 1000 = 15000 \text{ cm} = 150 \text{ m}$

c A playground in the park as an area of 4.3 cm² on the diagram.
Work out the area the playground takes up in real life in m²

$4.3 \text{ cm}^2 \times 1000^2 = 4\,300\,000 \text{ cm}^2 = 430 \text{ m}^2$

5 A map is drawn to a scale of 1:1000. That means the area is in the ratio $1:1000^2$.

a The area of a nature reserve is 3.6 ha (1 ha = 10 000 m^2).
What will be its area on the map, in square metres?

$36\,000\ m^2 \div 1000^2 = 0.036\ m^2$

b What is this is square centimetres?

$0.036 \times 100^2 = 360\ cm^2$

c The area on the map of a school and its playgrounds is 96 cm^2.

What is its actual area in ha?

$96 \times 1000^2 = 96\,000\,000\ cm^2 = 9600\ m^2 = 0.96\ ha$

6 The ratio of the area of a field on a scale drawing to its actual area is 1 : 2500.
What is the ratio of the dimensions of the field on the diagram to its actual dimensions?

$1:\sqrt{2500} = 1:50$

7 The matching sides of two similar figures are in the ratio 4 : 5.

a Write the area ratio of the smaller to the larger prism.

$16:25$

b If the area of the larger figure is 600 cm^2, what is the area of the smaller solid?

Areas in ratio 16 : 25

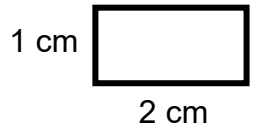
$600 \times \frac{16}{25} = 384\ cm^2$

VOLUMES OF SIMILAR SOLIDS

Investigation: Area and volumes of similar figures.

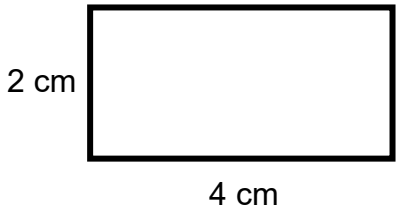
The diagrams below show two similar rectangles, with the second being an enlargement.

Original



$A_1 =$ .....

Image

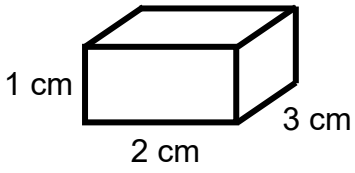


$A_2 =$ .....

The length ratio of original : image is: .....

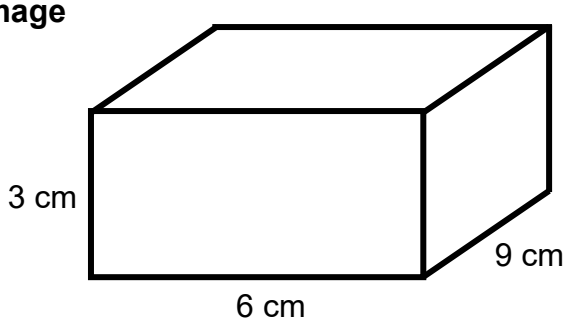
The volume ratio of original : image is: .....

Original



$V_1 =$ .....

Image



$V_2 =$ .....

The length ratio of original : image is: .....

The volume ratio of original : image is: .....

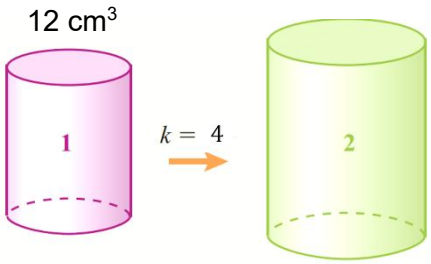
VOLUMES OF SIMILAR SOLIDS

Similar Areas and Volumes	
If two figures are in the ratio	$a:b$
their areas are in the ratio	$a^2:b^2$
their volumes are in the ratio	$a^3:b^3$

1. Determine ratio of lengths.
2. Determine ratio of volumes.
3. Use a ratio table to find area of unknown.

Example Calculate area and volume of similar figures.

A cylinder has been enlarged with scale factor of $k = 4$. The original volume is 12 cm^3, calculate the volume of the enlarged cylinder.



1. Length ratio: $1 : 4$
2. Volumes in ratio: $1^3 : 4^3$
   $= 1 : 64$
3. Ratio table:

Original	:	Image
1	:	64
12	:	768

The volume of the enlarged figure is 768 cm^3.

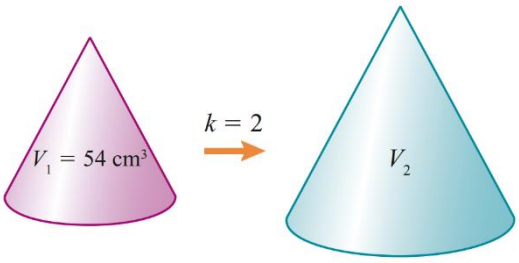
How did we know the volumes will be in the ratio $1:64$?

The lengths are in the ratio $1:4$,
So, volumes are in the ratio $1:\ldots\ldots^3 = 1:64$

How did we go from $1:64$ to $12:768$?

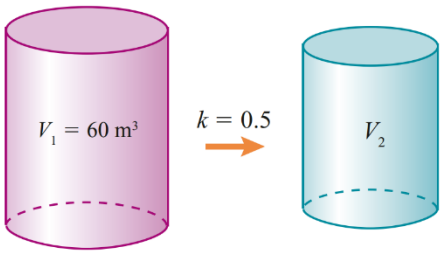
Calculate the volumes of these similar solids.

a



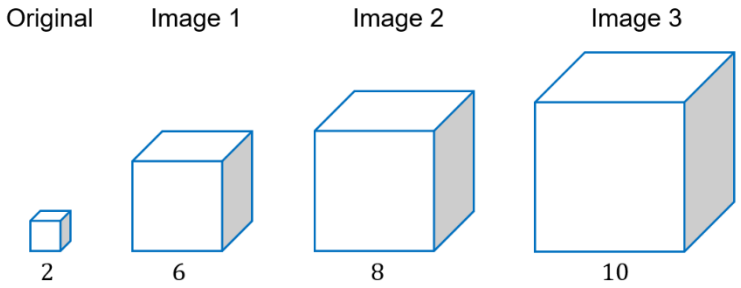
432 cm^3

b



7.5 m^3

1 Shown here is a cube of side length 2 and its images after enlargement. All units are in cm.



a Complete this table, comparing each image to the original.

Cube	Original	Image 1	Image 2	Image 3
Length ratio	1	1:3	1:4	1:5
Volume	8 cm^3	216 cm³	512	1000
Volume ratio	1	1: 27	1:64	1:125

b What would be the volume ratio if the length ratio is $1:n$?

$1:n^2$

c What would be the volume ratio if the length ratio is:

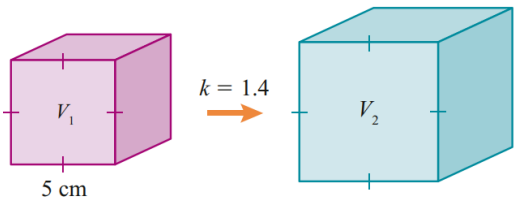
i 1:10? ..... 1:1000 **ii** 1:20? ..... 1:8000 **iii** 1:100? 1:1 000 000

d What would be the volume ratio if the length ratio is $1:\frac{1}{2}$?

$1:\frac{1}{8}$

2 Given that the following pairs of solids are similar, find the unknown volume using the information given. Round to 2 d.p. where appropriate.

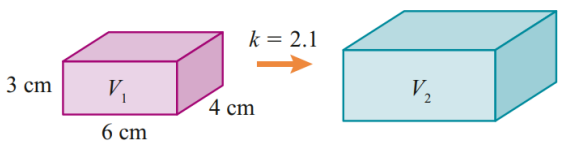
a



1. Length ratio:
2. Volumes in ratio:
3. Ratio table:

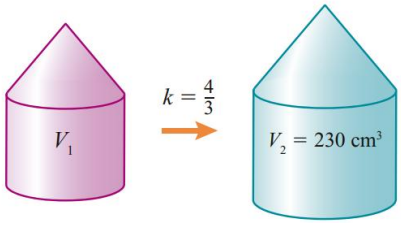
Original	:	Image
1	:	1.4^3
125	:	343

b



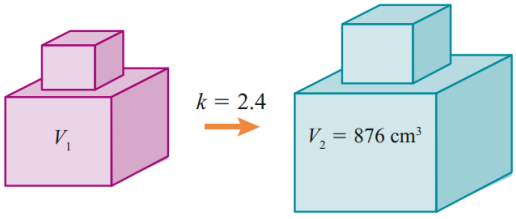
666.79 cm³

c



97.01 cm³

d



63.37 cm³

8 The matching lengths of two similar prisms are 2 cm and 5 cm.

a Write the length ratio of the smaller to the larger prism.

$2:5$

b Write the volume ratio of the smaller to the larger prism

$8:125$

c If the volume of the smaller prism is 64 cm^3, what is the volume of the larger prism?

$\frac{x}{64} = \frac{125}{8}$

1000 cm^3

9 The matching sides of two similar solids are in the ratio 3 : 4.

a Write the volume ratio of the smaller to the larger prism.

$27:64$

b If the volume of the larger solid is 384 cm^3, what is the volume of the smaller solid?

$\frac{x}{384} = \frac{27}{64}$

162 cm^3

10 The surface area and volume of a solid are 350 cm^2 and 480 cm^3 respectively.

The solid is enlarged using a scale factor $k = 1.8$.

Calculate the surface area and volume of the larger solid.

$360 \times 1.8^2 = 1134$ cm^2

$380 \times 1.8^3 = 2799.36$ cm^3

11 A box holds exactly 360 ball bearings.
If all the dimensions of the box are doubled, how many ball bearings would it hold?

New volume is $360 \times 2^3 = 2880$ ball bearings

12 Complete the table.

Ratio of Lengths	Ratio of Areas	Ratio of Volumes
$2:5$	$4:25$	$8:125$
$5:3$	$25:9$	$125:27$
$1:\_\_\_$ 10	$1:10$	$\_\_\_:1000$
$4:3$	$16:\_\_\_$ 9	$\_\_\_:27$ 64
$0.5:0.3$	$0.25:0.09$	$0.125:0.027$
$1:0.1$	$1:0.01$	$\_\_\_:0.001$
$0.2:0.001$	$0.04:0.00001$	$0.008:0.000001$
$a:\_\_\_:\_\_\_$ $a:b:c$	$\_\_\_:b^2:\_\_\_$ $a^2:b^2:c^2$	$\_\_\_:\_\_\_:c^3$ $a^3:b^3:c^3$
$2a:\_\_\_:\_\_\_$ $2a:3b:4c$	$\_\_\_:9b^2:\_\_\_$ $4a^2:9b^2:16c^2$	$\_\_\_:\_\_\_:64c^3$ $8a^3:27b^3:64c^3$
$a^2:b^3:c^4$	$a^4:b^6:c^8$	$a^6:b^9:c^{12}$
$\sqrt{a}:b:c^{\frac{3}{2}}$	$a:b^2:c^3$	$a^{\frac{3}{2}}:b^3:c^{\frac{9}{2}}$

13 A model is made of a statue and the volumes of the statue and its model are in the ratio 27 : 1.

a What is the ratio of the height of the statue to the height of the model?

3:1

b What is the ratio of the surface areas?

9:1

14 When it is hot, a dog cannot sweat to cool down but uses moisture evaporation from its tongue to regulate its body temperature. This is not effective for large animals.
Consider what would happen if a dog's dimensions were increased by a factor of 10.

a By what factor would its volume be increased?

1000

b By what factor would the surface area of its tongue be increased?

100

c What effect would this have on the animal?

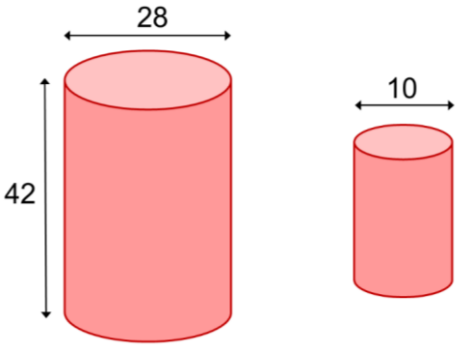
The dog would produce far more heat but have relatively less surface area to dissipate it.
The doggy would overheat!

15 The two jars are similar. What is the volume of the larger jar?

$\frac{x}{0.6} = \left(\frac{24}{16}\right)^3$

$x = 2.025$ L

16 These cylinders are similar. What is the volume of the smaller cylinder? Answer in terms of π.



$\frac{x}{\pi(14)^2(42)} = \left(\frac{10}{28}\right)^3$

$x = 375\pi$ units^3

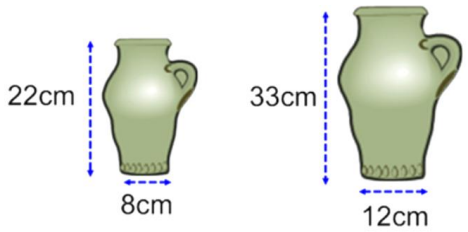
17 A solid metal ball is melted down and made into similar balls with $\frac{1}{5}$ the radius.

How many smaller solid balls can be made?

Each ball is $\left(\frac{1}{5}\right)^3 = \frac{1}{125}$ the original volume

$\therefore$ 125

18 These are two similar vases of difference sizes.



a If the surface area of the smaller vase is 550 cm^2,

Calculate the surface area of the larger vase.

$\frac{x}{550} = \left(\frac{33}{22}\right)^2$

$x = 1237.5$ cm^2

b If the volume of the larger vase is 3800 cm^3,

Calculate the surface area of the smaller vase.

$\frac{x}{3800} = \left(\frac{22}{33}\right)^2$

$x = 1688.\dot{8}$ cm^2