Student Name

Mathematics Stage 4

AREA

Book 2 Develop and use the formula to find the area of circles and sectors to solve problems

Version: 260207

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CONTENTS

OVERVIEW

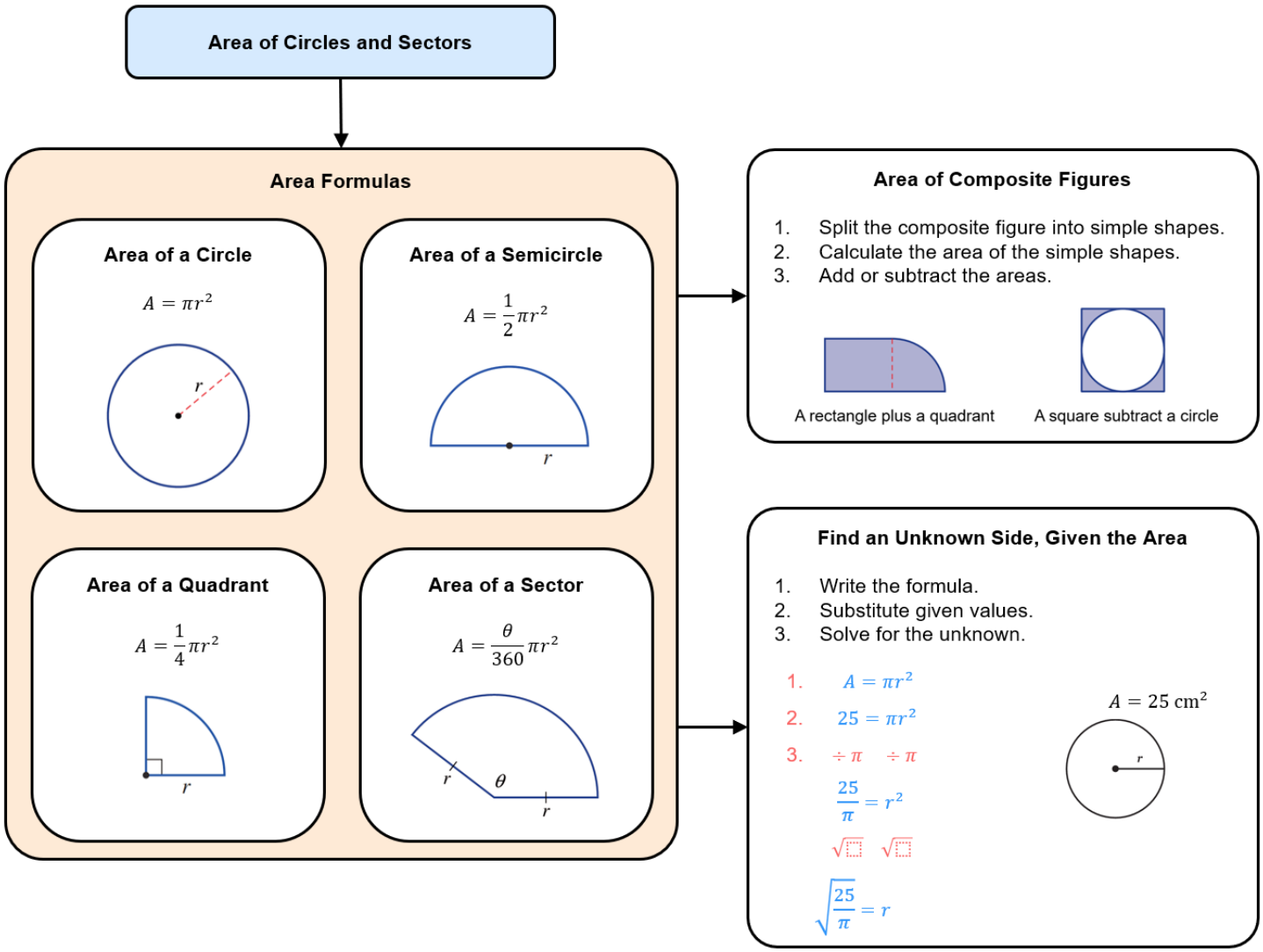
SYLLABUS CONTENT

MA4-ARE-C-01 applies knowledge of area and composite area involving triangles, quadrilaterals and circles to solve problems

Develop and use the formula to find the area of circles and sectors to solve problems

- Develop and apply the formula to find the area of a circle: $A = \pi r^2$, where r is the length of the radius
- Explain how the area of a sector can be developed from the area of a circle $A = \frac{\theta}{360} \times \pi r^2$
- Find the area of quadrants, semicircles and sectors, and apply these formulas in the context of real-life problems
- Calculate the areas of composite shapes involving quadrants, semicircles and sectors to solve problems

SUMMARY



REVIEW OF PRIOR KNOWLEDGE

1 Square these numbers.

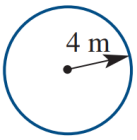
a 4^2 **b** 8^2 **c** 11^2 **d** 1.3^2

2 Round these numbers to two decimal places.

a 9.424777 **b** 191.63715 **c** 15.707963 **d** 0.125663

3 Identify the radius and diameter of these circles.

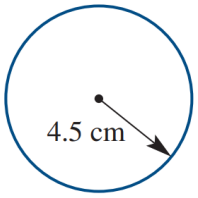
a



Radius: ……

Diameter: ……

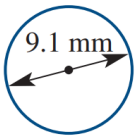
b



Radius: ……

Diameter: ……

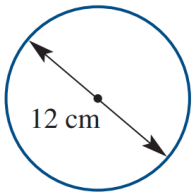
c



Radius: ……

Diameter: ……

d



Radius: ……

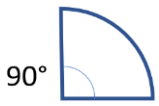
Diameter: ……

4 Simplify these, leaving your answer in exact form.

a $2 \times \pi \times 3$ **b** $\pi \times 7 \times 4$ **c** $\frac{1}{2} \times 8 \times \pi$ **d** $\pi \times 8^2$

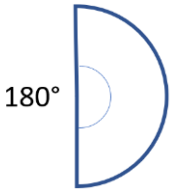
5 What fraction of a whole circle is shown? All angles show the interior angle.
Answer as a simplified fraction.

a

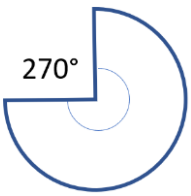


$\frac{\square}{360} = \frac{\square}{\square}$

b

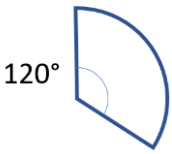


c



d

e



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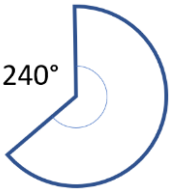

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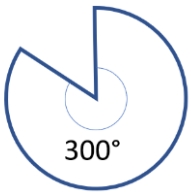

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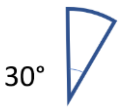
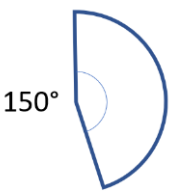


i



j

k

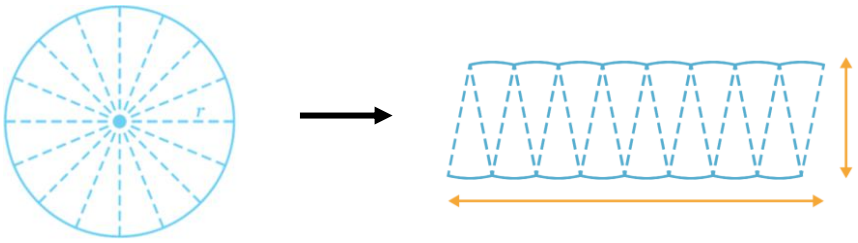
l

AREA OF A CIRCLE

Investigation Area of a circle.

Area of a Circle: https://www.geogebra.org/m/fuwvtu7w

Imagine a circle divided into sectors, then placed together into a shape that resembles a parallelogram.



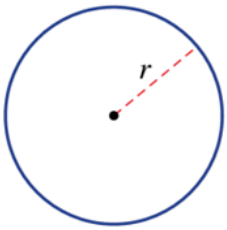
Label the base and height of the parallelogram, then calculate the area.

$$A = bh$$
$$= \ldots\ldots\ldots \times \ldots\ldots\ldots$$
$$= \ldots\ldots\ldots\ldots\ldots\ldots$$

Area of a Circle

$A = \pi r^2$

Radius



Example Calculate the area of a circle.

Calculate the area of these circles to 2 decimal places.

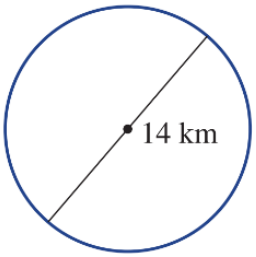
$A = \pi r^2$

$= \pi(2)^2$

≈ 12.57 cm^2

What does it mean to square a number?

Squaring a number means to multiply it by .....



$A = \pi r^2$

$= \pi(7)^2$

≈ 153.94 km^2

Explain why we multiplied by 7^2 in this example.

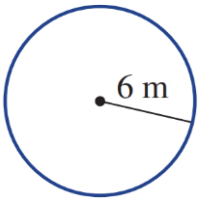
In this example, the ..... is 14.

The area of a circle uses the .....

The radius is half the diameter: .....

Calculate the area of these circles to 2 decimal places.

a



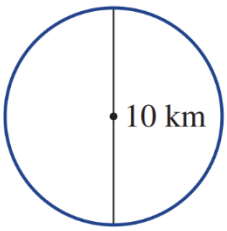
$A = \pi r^2$

$= \pi(\quad)^2$

$= .....$

113.10 m^2

b



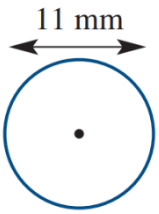
$A = \pi r^2$

$= .....$

$= .....$

78.54 km^2

c



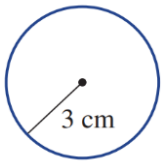
95.03 mm^2

Key ideas

1. We calculate the area of a circle using the formula .....
2. The formula always uses the ..... of a circle
3. If the diameter is given, we need to change it to ..... by ..... the diameter before doing any calculations.

1 Calculate the area of these circles correct to two decimal places.

a



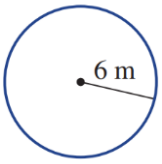
$A = \pi r^2$

$= \pi(\quad)^2$

$= \ldots\ldots\ldots\ldots$

28.27 cm²

b



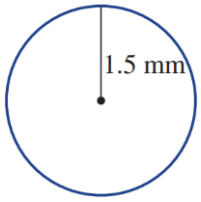
$A = \pi r^2$

$= \ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots$

113.10 m²

c



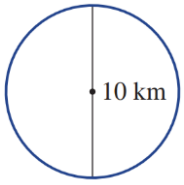
$A = \pi r^2$

$= \ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots$

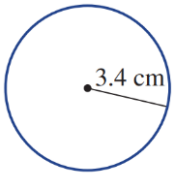
7.07 mm²

d



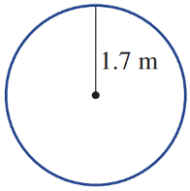
78.54 km²

e



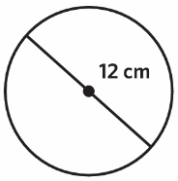
36.32 cm²

f



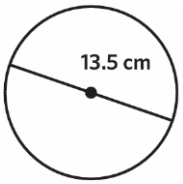
9.08 m²

g



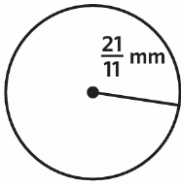
113.10 cm²

h



143.14 cm²

i



2.86 mm²

2 A pizza tray has a diameter of 30 cm. Calculate its area to the nearest whole number of cm^2

707 cm²

3 Two circular plates have radii 12 cm and 13 cm.

Find the difference in their area, correct to two decimal places.

78.54 cm²

4 A tanker ship spills a circular oil slick 1 km in diameter.

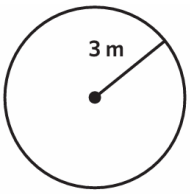
The news reports that the area covered by the oil slick is over a square kilometre.

Are they correct?

No, they are incorrect. The area is only 0.785 km²

5 Calculate the area of these circles. Leave your answer in terms of π.
The first question has been done for you.

a

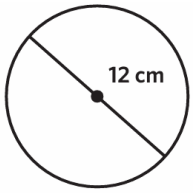


$A = \pi(3)^2$
$= 9\pi \text{ m}^2$

b

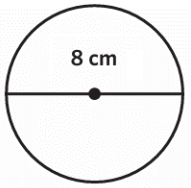
25π km²

c



36π cm²

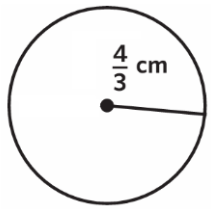
d



16π cm²

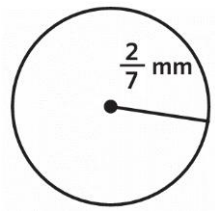
6 Find the area of the circle in terms of π as a fraction.

a



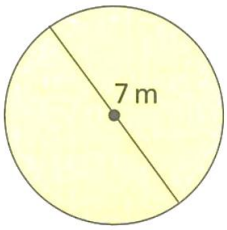
$\frac{16}{9}\pi$ cm²

b



$\frac{4}{49}\pi$ mm²

7 Ed calculated the area of this circle as 49π m².



a Explain why he is incorrect.

The radius is $\frac{7}{2}$, not 7

b Write the correct area in terms of π as a fraction

$\frac{49}{4}\pi$ cm²

8 A circle has radius of 2 cm. For these questions, answer in terms of π.

a What happens to the area if you double the radius?

Original: $\pi(2)^2 = 4\pi$, New: $\pi(4)^2 = 16\pi$, so the area quadruples

b What happens to the area if you triple the radius?

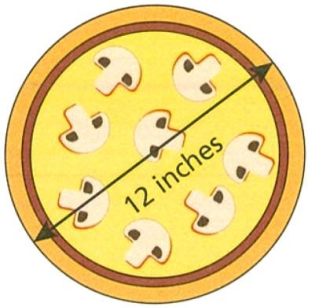
It will increase by a factor of 9

c What happens to the area if you quadruple the radius?

It will increase by a factor of 16

9 Pizza sizes are given using the diameter, measured in inches.
The 12-inch pizza is cut into 8 equal slices. Rayyan eats 3 slices.

What is the area of the pizza that Rayyan ate? Answer in square inches to 1 d.p.



Whole pizza: $\pi(6)^2 = 36\pi$ in^2

$\frac{3}{8} \times 36\pi = \frac{27\pi}{2}$ in^2 ≈ 42.4 in^2

10 You order a 9-inch pizza from a restaurant.

After a while, the waiter tells you that they no longer have 9-inch pizza bases and will instead give you two 5-inch pizzas instead.

He tries to convince you, saying that you will get more since $2 \times 5 = 10$.

Explain to him why this is not a good deal.

Note: pizza sizes are measured according to the diameter.

9-inch pizza: $\pi(4.5)^2 \approx 63.62$ in^2

2 5-inch pizzas: $\pi(2.5)^2 \times 2 \approx 39.27$ in^2

This is not a good deal as two 5-inch pizzas are significantly smaller than a single 9-inch pizza

(This happens because the radius gets squared; if you halve the radius, the area is a quarter, so giving two pizzas won't make up the difference)

11 Anastasia sells pizzas for \$24. Her pizzas have a radius of 18 cm.

She is creating a new pizza size that has a radius of 21 cm.

How much should she sell the new size so her customers pay the same amount per cm²?

Current \$/cm²: $\frac{\$24}{\pi(18)^2} = \$0.02357851009/\text{cm}^2$

New area: $\pi(21)^2 = 1385.44236$ cm²

New price = $1385.44236 \text{ cm}^2 \times \$0.02357851009/\text{cm}^2$

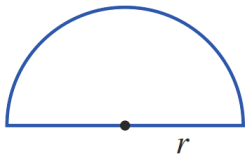
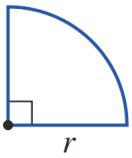
$= \$32.67$

Alternate method

new pizza is $\left(\frac{21}{18}\right)^2$ as large.

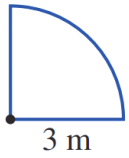
$\$24 \times \left(\frac{21}{18}\right)^2 = \$32.67$

AREA OF QUADRANTS AND SEMICIRCLES

Semicircle	Quadrant
A semicircle is half a circle.  $A = \frac{1}{2}\pi r^2$	A quadrant is a quarter of a circle.  $A = \frac{1}{4}\pi r^2$

Example Calculate the area of a semicircle and quadrant.

Calculate the area of these parts of circles to 2 decimal places.



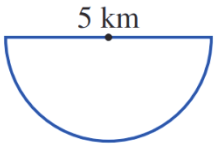
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4}\pi(3)^2$

$= 7.07 \text{ m}^2$

Why do we multiply the area of the whole circle by $\frac{1}{4}$ when calculating the area of a quadrant?

A quadrant is a ……of a circle.



$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(2.5)^2$

$= 9.82 \text{ km}^2$

Why did we multiply by 2.5^2 in this example?

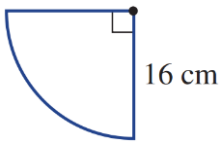
In this example, the …… is 5.

The area of a circle uses the ……

The radius is half the diameter: ……

Calculate the area of these parts of circles to 2 decimal places.

a



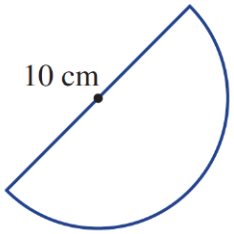
$A = \frac{1}{4}\pi r^2$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

201.06 cm²

b



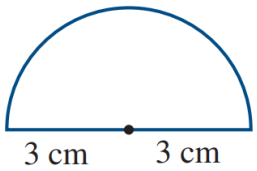
$A = \frac{1}{2}\pi r^2$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

39.27 cm²

c



14.14 cm²

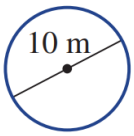
Key ideas

1. The area of a circle is calculated using the formula ……
2. A half circle is called a ……, so the area of a semicircle is ……
3. A quadrant is called a ……, so the area of a quadrant is ……

FOUNDATION

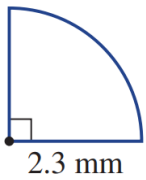
1 Identify the radius of these shapes.

a



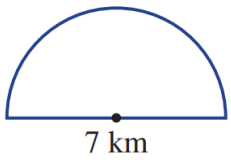
5 m

b



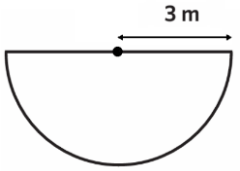
2.3 mm

c



3.5 km

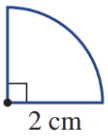
d



3 m

2 Calculate the area of these parts of circles. Round to 2 d.p.

a



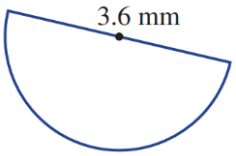
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4}\pi(\quad)^2$

$= \ldots\ldots\ldots\ldots\ldots$

3.14 cm²

b



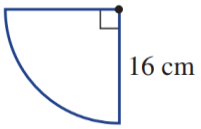
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(\quad)^2$

$= \ldots\ldots\ldots\ldots\ldots$

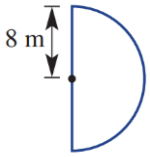
5.09 mm²

c



201.06 cm²

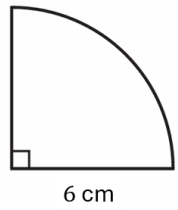
d



100.53 m²

3 Calculate the area of each shape. Give your answers in terms of π.
The first has been done for you.

a

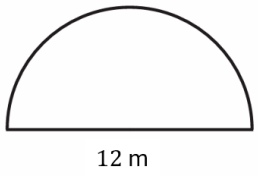


$A = \frac{\pi(6)^2}{4}$

$= \frac{36\pi}{4}$

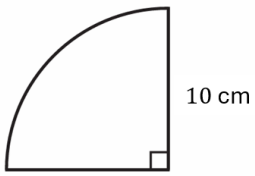
$= 8\pi$ cm²

b



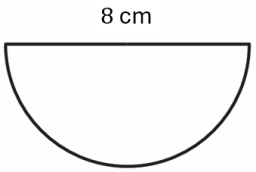
18π m²

c



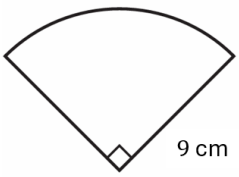
25π cm²

d



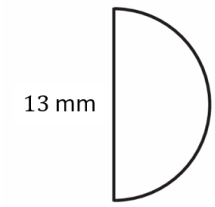
8π cm²

e



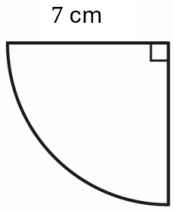
$\frac{81}{4}\pi$ cm²

f



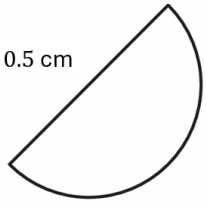
$\frac{169}{8}\pi$ mm²

g



$\frac{49}{4}\pi$ cm²

h



$\frac{\pi}{32}$ cm²

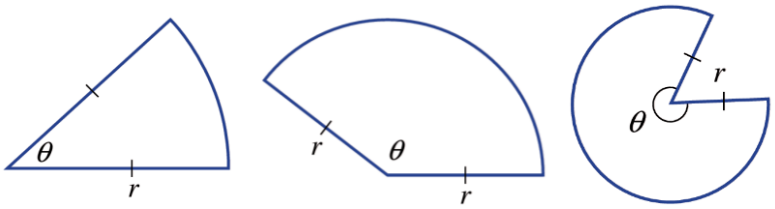
AREA OF A SECTOR

Area of a Sector: https://www.geogebra.org/m/h75rvzn8

Area of a Sector

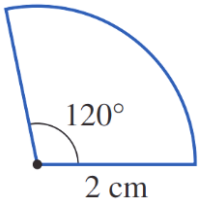
$$A = \frac{\theta}{360} \times \pi r^2$$

Fraction of circle ↑ Area of whole circle ↑



Example Calculate the area of a sector.

Calculate the area of these sectors to 2 decimal places.



$$A = \frac{\theta}{360}\pi r^2$$

$$= \frac{120}{360}\pi(2)^2$$

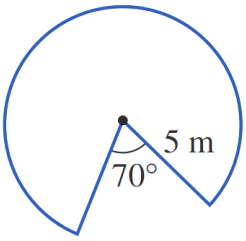
$$= 4.19 \text{ cm}^2$$

What does $\frac{\theta}{360}$ represent?

$\frac{\theta}{360}$ *represents the ..... of the circle.*

Why do we multiply $\frac{\theta}{360}$ by the area of the whole circle?

To find a fraction of a whole, we ..... the fraction by the whole.



$$A = \frac{\theta}{360}\pi r^2$$

$$= \frac{290}{360}\pi(5)^2$$

$$= 63.27 \text{ mm}$$

Why is $\theta = 290°$ in this example?

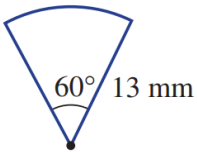
θ represents the angle ..... the sector.

$70 + \theta =$ .....°,

so $\theta = 290°$

Calculate the area of these sectors to 2 decimal places.

a



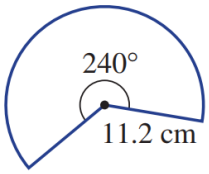
$$A = \frac{\theta}{360}\pi r^2$$

$=$ .....

$=$ .....

88.49 mm²

b



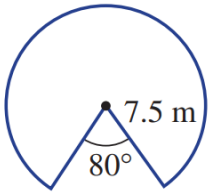
$$A = \frac{\theta}{360}\pi r^2$$

$=$ .....

$=$ .....

262.72 cm²

c



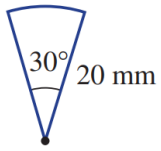
137.44 m²

Key ideas

1. To find the area of a sector, first find the ..... of the circle the sector represents by using the ..... inside the sector.

1 Calculate the area of these sectors. Round your answer to 1 d.p.

a



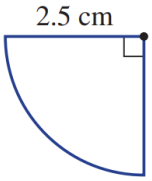
$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\quad}{360} \times \pi(\quad)^2$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

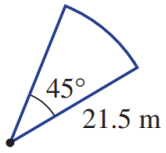
104.7 mm²

b



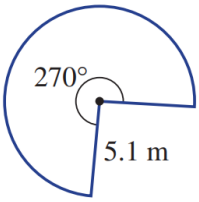
4.91 cm²

c



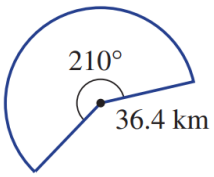
181.5 m²

d



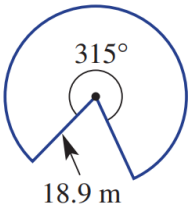
61.3 m²

e



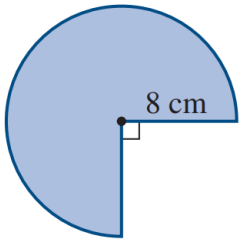
2428.1 km²

f



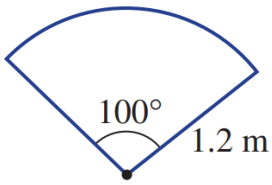
981.9 m²

4 Calculate the area of the sector shown, answer in terms of π.



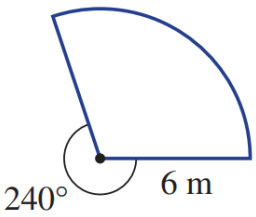
48π cm²

2 A simple bus wiper blade wipes an area over 100°, as shown.
Find the area wiped by the blade, correct to two decimal places.



1.26 m²

3 For the sector shown, Jerry calculated the fraction of the circle as $\frac{240}{360} = \frac{2}{3}$.



a Explain why he is wrong.

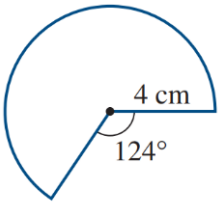
The angle inside the sector is not 240°

b Calculate the area of the sector, round to 1 d.p.

37.7 m²

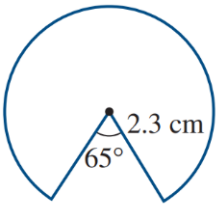
4 Calculate the area of these sectors. The angle shown is the exterior angle.
Round your answers to 1 d.p.

a



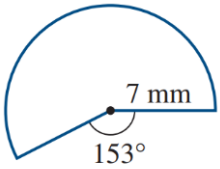
33.0 cm²

b



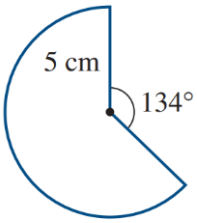
13.6 cm²

c



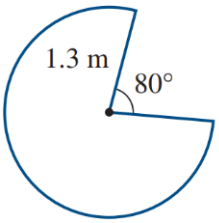
88.5 mm²

d



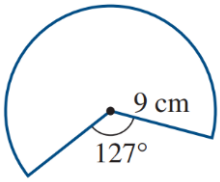
49.3 cm²

e



4.1 m²

f



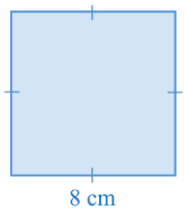
164.7 cm²

MIXED PRACTICE

FOUNDATION

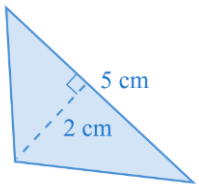
1 Calculate the area of these shapes. Answer to 2 d.p. where appropriate.

a



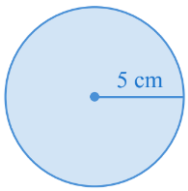
64 cm²

b



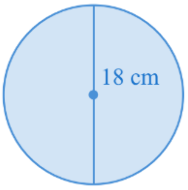
5 cm²

c



78.54 cm²

d

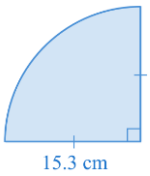


254.47 cm²

e

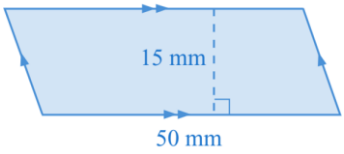
38.11 cm²

f



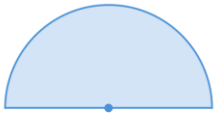
183.85 cm²

g



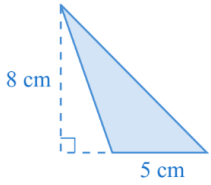
750 cm²

h



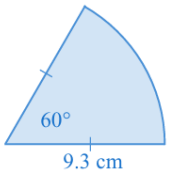
47.52 cm²

i



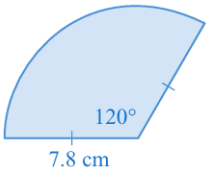
20 cm²

j



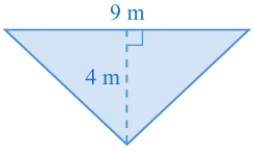
45.29 cm²

k



63.71 cm²

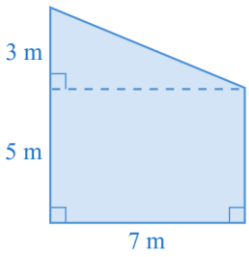
l



18 cm²

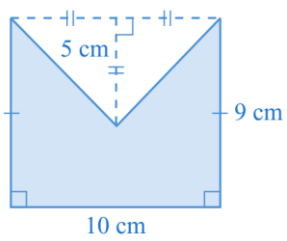
2 Calculate the area of these composite shapes.

a



45.5 m²

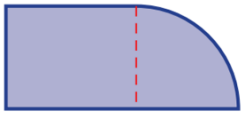
b



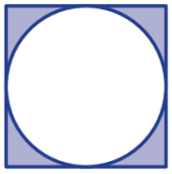
65 cm²

COMPOSITE FIGURES INVOLVING CIRCLES

- A **composite figure** is made up of multiple simple shapes.



A rectangle plus a quadrant

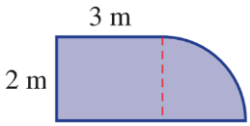


A square subtract a circle

- To find the area of composite figures, add or subtract the areas of simple shapes.
  1. Split the composite figure into squares, rectangles, and triangles.
  2. Calculate the area of each simple shape.
  3. Add or subtract the areas.

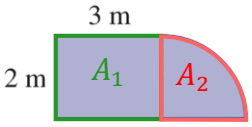
Example Calculate the area of a composite shape involving circles.

Calculate the area of the composite shape to two decimal places.



$A_1 = 2 \times 3$

$= 6 \text{ m}^2$



$A_2 = \frac{1}{4}\pi(2)^2$

$= 3.14 \text{ m}^{2s}$

$A_{\text{Total}} = A_1 + A_2$

$= 6 + 3.14$

$= 9.14 \text{ m}^2$

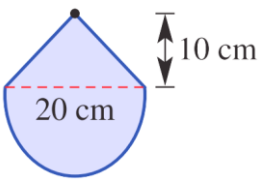
What two shapes did we split the composite shape into?

We split the composite shape into a ..... and a .....

What is the formula for the area of a quadrant?

Calculate the shaded area. Round your answer to 2 decimal places.

a



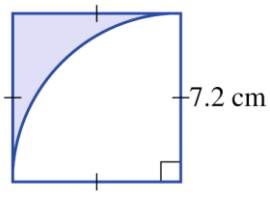
$A_1 =$

$A_2 =$

$A_{\text{Total}} =$

257.08 cm²

b



$A_1 =$

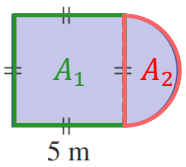
$A_2 =$

$A_{\text{Total}} =$

11.12 cm²

1 Calculate the areas of these composite shapes using addition. Answer to 2 d.p.

a



$A_1 =$ .....

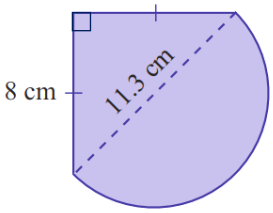
$A_2 =$ .....

$A_{\text{Total}} = A_1 + A_2$

$=$ .....

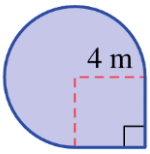
34.82 m²

b



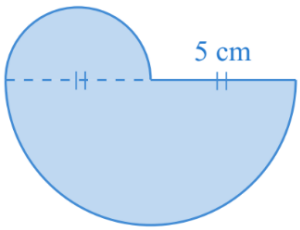
82.14 cm²

c



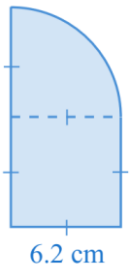
53.70 m²

d



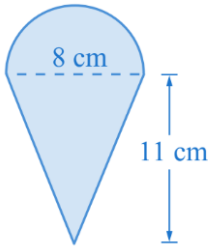
49.09 cm²

e



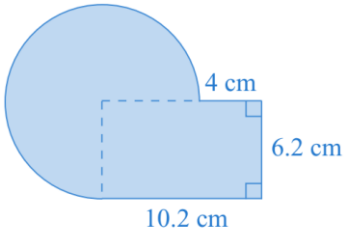
68.63 cm²

f



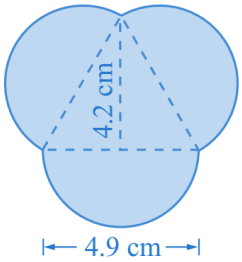
69.13 cm²

g



153.81 cm²

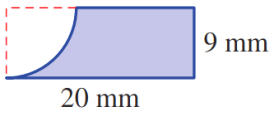
h



38.58 cm²

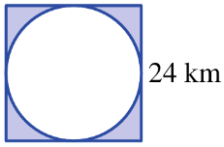
2 Calculate the areas of these composite shapes using subtraction.

a



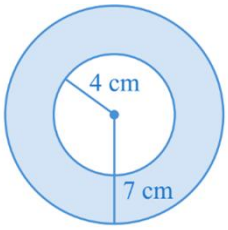
116.38 mm²

b



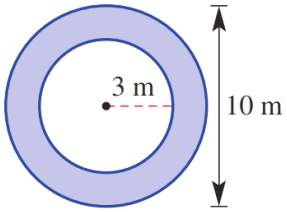
123.61 km²

c



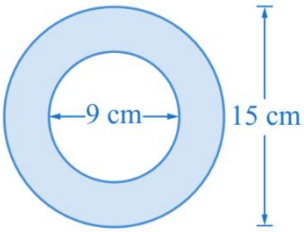
103.67 cm²

d



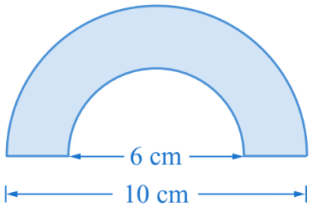
50.27 m²

e



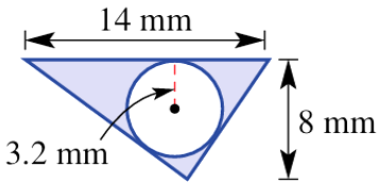
113.09 cm²

f



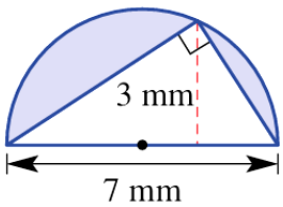
25.13 cm²

g



23.83 mm²

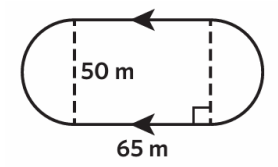
h



8.74 mm²

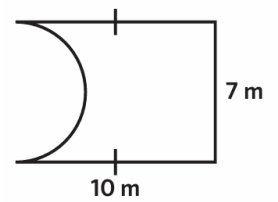
3 Calculate the area of these shapes. Answer to 2 decimal places.

a



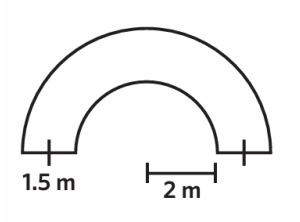
5213.50 m²

b



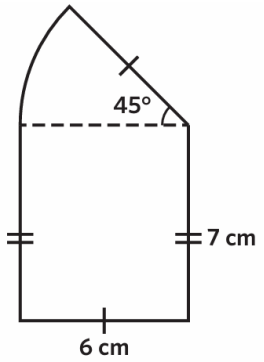
50.76 m²

c



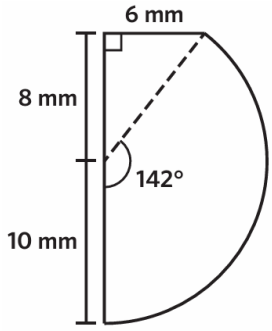
12.96 m²

d



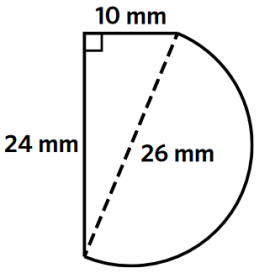
56.14 cm²

e



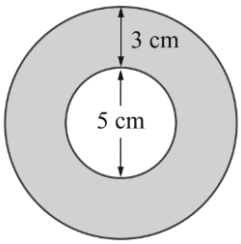
147.92 mm²

f



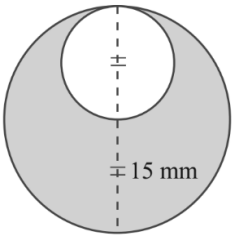
385.46 mm²

g



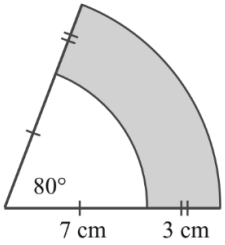
75.40 cm²

h



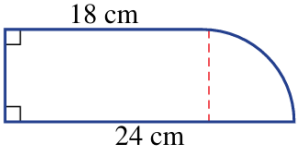
530.14 mm²

i



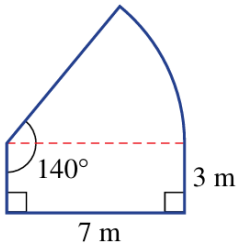
35.60 cm²

j



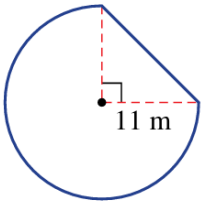
136.27 cm²

k



42.38 m²

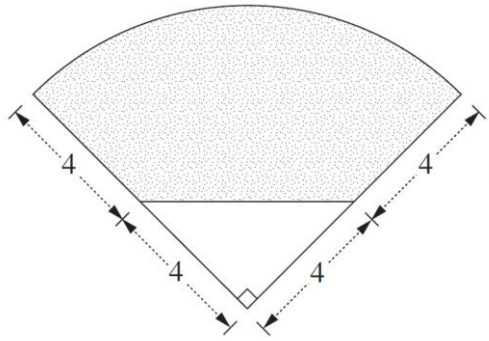
l



345.60 m²

4 **2009 HSC Standard 2 Band 4**

What is the area of the shaded part of this shape, correct to the nearest square centimetre?



NOT TO SCALE

All measurements are in centimetres.

$\frac{1}{4}\pi(8)^2 - \frac{16}{2}$

= 42.27 cm²

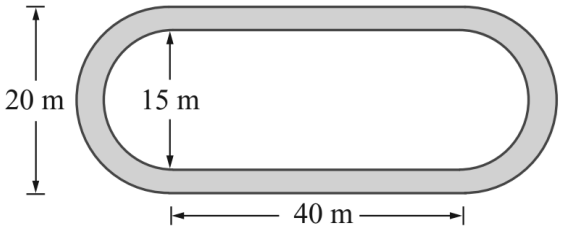
5 A shop sells pizza sectors. The two options are:

15 cm radius pizza with an angle of 45° or 26 cm diameter pizza with an angle of 60°.

Which piece gives the bigger area and by how much? Round the answer to two decimal places.

The 60° sector, by around 0.13 cm²

6 The racetrack shown needs to be resurfaced.
Calculate the area to be resurfaced to 2 decimal places.



The racetrack is an oval made up of a rectangle and two semicircles.

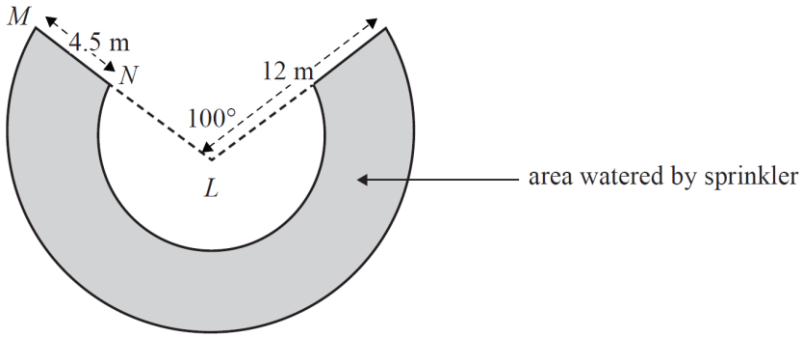
Area is outer oval – inner oval.

$A = (40 \times 20 + \pi(10)^2) - (40 \times 15 + \pi(7.5)^2)$

337.44 m²

7 **NAPLAN A+**

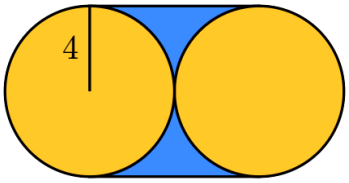
A golf course has a sprinkler system that waters the grass in the shape of a sector, as shown.
A sprinkler is positioned at point L and can turn through an angle of 100°.
The section of grass that is watered is 4.5 m wide at all points.
Water can reach a maximum of 12 m from the sprinkler at L.
What is the area of grass that this sprinkler will water?
Round your answer to the nearest square metre.



$A = \frac{260}{360}\pi(12)^2 - \frac{260}{360}\pi(7.5)^2$

= 199 m²

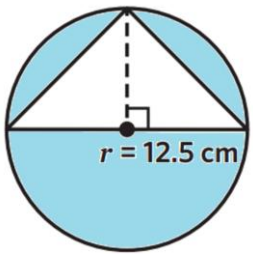
8 Find the area of the dark region to 2 d.p.



Area of dark region = Square – circle

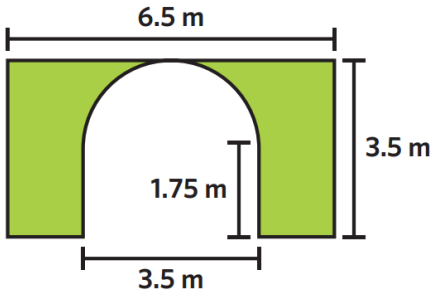
$8^2 - \pi(4)^2 = 13.73 \text{ units}^2$

9 A triangle is inscribed in a circle, as shown. Calculate the shaded area to 2 d.p.



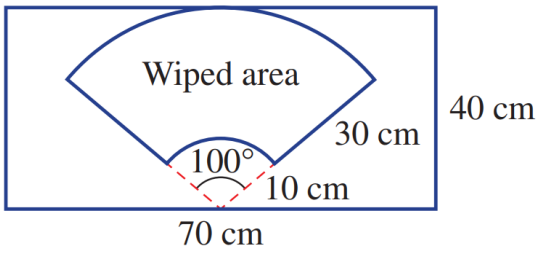
$\pi(12.5)^2 - \frac{1}{2}(25)(12.5) = 334.62 \text{ cm}^2$

10 Tui has a wall that has a height of 3.5 m and width of 6.5 m.
She has cut an arch into the wall that consists of a semicircle with a diameter of 3.5 m and a rectangle that is 1.75 m high.
Calculate the area of the wall she has left, correct to 2 decimal places.



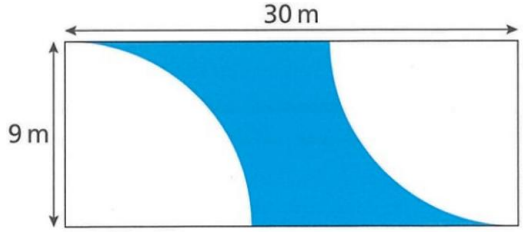
$3.5 \times 6.5 - (3.5 \times 1.75) - \frac{1}{2}\pi(1.75)^2 = 11.81 \text{ m}$

11 Determine the percentage of the windscreen covered by the wiper to 2 d.p.



$\frac{\frac{100}{360}\pi(40^2) - \frac{100}{360}\pi(10)^2}{40 \times 70} \times 100 = 46.75\%$

12 The floor in an art gallery is to be painted. The diagram shows the region to be painted.
A tin of paint covers 6.5 m^2.
Each tin of paint costs $12.95.
Determine the cost of painting the floor.

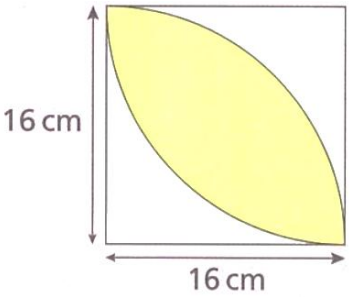


$\text{Area} = 30 \times 9 - 2 \times \left(\frac{1}{4}\pi(9)^2\right) = 142.765 \text{ m}^2$

Tins of paint needed = $142.765 \div 6.5 \approx 22$ tins of paint needed

$22 \times 12.95 = \$284.90$

13 Work out the area of the shaded region.
(Hint: use the area of a triangle!)



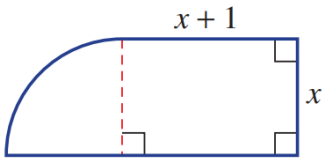
Consider a quadrant. The shaded segment is:

Shaded segment $= \frac{1}{4}\pi(16)^2 - \frac{1}{2}(16)(16) = 73.06\ \text{cm}^2$

There are two identical segments, so $146.12\ \text{cm}^2$

14 Find expressions, in simplest form, for the perimeter (P) and area (A) of these shapes.
Answer in terms of π ; for example, 4π. Do not use decimals.

a

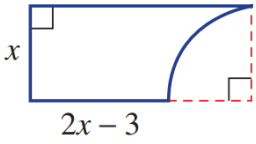


$P =$

$A =$ $4x + 2 + \frac{\pi x}{2}$

$x^2 + x + \frac{\pi x^2}{4}$

b

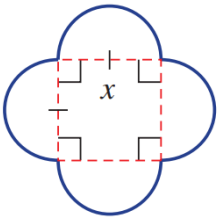


$P =$

$A =$ $6x - 6 + \frac{\pi x}{2}$

$2x^2 - 3x - \frac{\pi x^2}{4}$

c

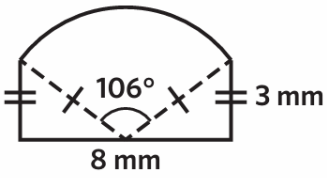


$P =$ $4\pi x$

$A =$

$x^2 + 2\pi x^2$

15 Find the area of the composite shape to 2 d.p.

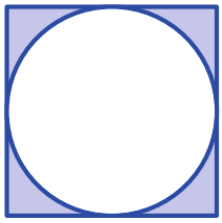


Using right-angled triangle, $r = \sqrt{4^2 + 3^2} = 5$

$A = 2\left(\frac{1}{2} \times 3 \times 4\right) + \frac{106}{360}\pi(5)^2$

$= 35.13\ \text{mm}^2$

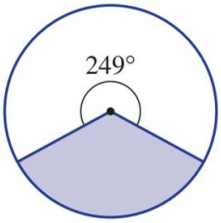
16 What fraction of the square is shaded?



$\frac{(2r)^2 - \pi r^2}{(2r)^2} = \frac{4r^2 - \pi r^2}{4r^2} = \frac{4 - \pi}{4}$

17 What percentage of the total area is occupied by the shaded region in these diagrams? Round the answer to one decimal place.

a



$\frac{111}{360} \times 100 = 30.8\%$

b

$\frac{\frac{1}{2}\pi r^2}{2r^2} \times 100$

$= \frac{\frac{1}{2}\pi}{2} \times 100$

$= 78.5\%$

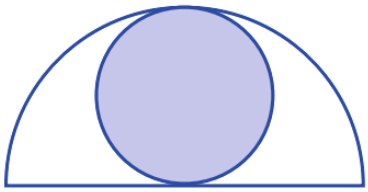
c

$\frac{r^2 - \frac{1}{4}\pi r^2}{r^2} \times 100$

$= \frac{r^2\left(1 - \frac{1}{4}\pi\right)}{r^2} \times 100$

$= 21.5\%$

18 What percentage of the large semicircle is shaded?



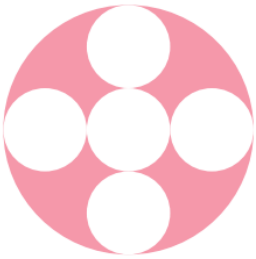
Let radius of small circle be r

$\frac{\pi r^2}{\frac{1}{2}\pi(2r)^2} \times 100$

$= \frac{\pi r^2}{2\pi r^2} \times 100$

$= 50\%$

19 What percentage of the circle is shaded? Answer to 2 d.p.



Let radius of small circle be r

$\frac{\pi(3r)^2 - 4\pi r^2}{\pi(3r)^2} \times 100$

$= \frac{5\pi r^2}{9\pi r^2} \times 100$

$= 55.56\%$

20 The diagram shows two circles with the same centre.
The radius of the outer circle is equal to the diameter of the inner circle.
If the region between the two circles is divided into six equal segments, what fraction of the circle is shaded in the diagram?

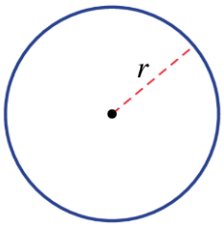
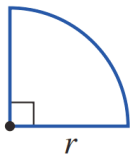
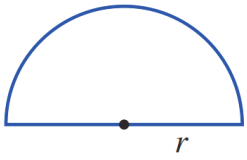
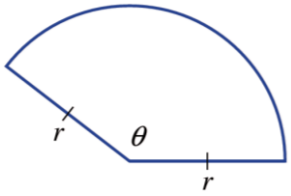
Let inner circle have radius r

Area of ring: $\pi(2r)^2 - \pi(r)^2 = 3\pi r^2$

Half the ring is shaded, so shaded area = $1.5\pi r^2$

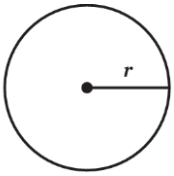
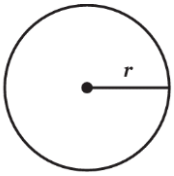
$\frac{1.5\pi r^2}{4\pi r^2} = \frac{3}{8}$

FINDING THE RADIUS GIVEN AREA

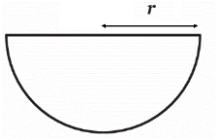
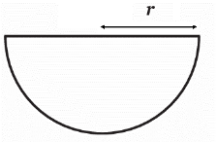
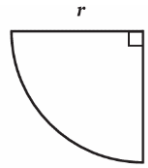
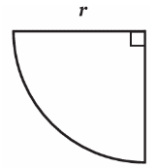
Circle	Quadrant	Semicircle	Sector
$A = \pi r^2$ 	$A = \frac{\pi r^2}{4}$ 	$A = \frac{\pi r^2}{2}$ 	$A = \frac{\theta}{360}\pi r^2$ 

- If you are given the area and asked to find the radius:
  1. Write the formula.
  2. Substitute given values.
  3. Solve for the unknown value using inverse operations.
- To avoid rounding errors, do not enter anything into the calculator until the last step.

Example Calculate the radius given the area.

Worked Example	Your Turn
The area of a circle is $A = 25\ \text{m}^2$ Find the radius to 3 d.p. 	The area of a circle is $A = 18\ \text{m}^2$ Find the radius. 
1. Write the formula $A = \pi r^2$ 2. Substitute $25 = \pi r^2$ 3. Solve $\div \pi \quad \div \pi$ $\frac{25}{\pi} = r^2$ $\sqrt{\ } \quad \sqrt{\ }$ $\sqrt{\frac{25}{\pi}} = r$ $2.821\ \text{cm} = r$	1. 2. 3. 2.394 m

Example Calculate the radius given the area.

Worked Example	Your Turn
The area of a semicircle is $A = 25\ \text{m}^2$ Find the radius to 3 d.p.  1. Write the formula $A = \frac{\pi r^2}{2}$ 2. Substitute $25 = \frac{\pi r^2}{2}$ 3. Solve $\times 2 \quad \times 2$ $50 = \pi r^2$ $\div \pi \quad \div \pi$ $\frac{50}{\pi} = r^2$ $\sqrt{\ } \quad \sqrt{\ }$ $\sqrt{\frac{50}{\pi}} = r$ $3.989\ \text{cm} = r$	The area of a semicircle is $A = 20\ \text{m}^2$ Find the radius to 3 d.p.  1. 2. 3. 3.568 m
The area of a quadrant is $A = 13.5\ \text{m}^2$ Find the radius to 3 d.p.  1. Write the formula $A = \frac{\pi r^2}{4}$ 2. Substitute $13.5 = \frac{\pi r^2}{4}$ 3. Solve $\times 4 \quad \times 4$ $54 = \pi r^2$ $\div \pi \quad \div \pi$ $\frac{54}{\pi} = r^2$ $\sqrt{\ } \quad \sqrt{\ }$ $\sqrt{\frac{54}{\pi}} = r$ $4.146\ \text{cm} = r$	The area of a quadrant is $A = 21.25\ \text{m}^2$ Find the radius to 3 d.p.  1. 2. 3. 5.202 m

1 Calculate the radius of each circle, rounding your answer to 1 d.p.

a $A = 70$ cm^2

$A = \pi r^2$

$70 = \pi r^2$

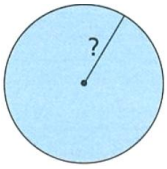
$\frac{70}{\pi} = r^2$

$\sqrt{\frac{70}{\pi}} = r$

..... $= r$

4.7 cm

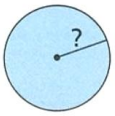
b $A = 98$ cm^2



$A = \pi r^2$

5.6 cm

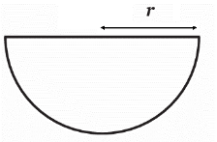
c $A = 0.45$ m^2



0.4 cm

2 Calculate the radius of each semicircle or quadrant, rounding your answer to 2 d.p.

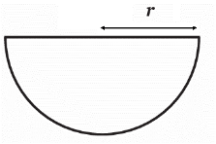
a $A = 15.5$ mm^2



$A = \frac{\pi r^2}{2}$

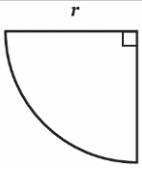
3.14 mm

b $A = 14.8$ cm^2



3.07 cm

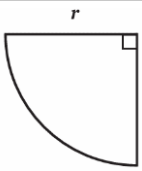
c $A = 6$ mm^2



$A = \frac{\pi r^2}{4}$

2.76 mm

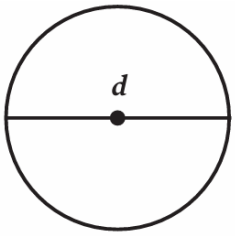
d $A = 3.8$ cm^2



2.20 cm

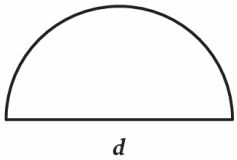
3 Find the *diameter* for these areas, rounding your answer to 1 d.p.
(Find the radius, then multiply by 2)

a $A = 12$ cm^2



3.9 cm

b $A = 14$ mm^2



6.0 mm

4 Read the example for when the area is given in terms of π and answer the questions.

a The area of a circle is 25π cm^2.
Calculate the radius.

$A = \pi r^2$

$25\pi = \pi r^2$

$\div \pi \quad \div \pi$

$25 = r^2$

$\sqrt{} \quad \sqrt{}$

$5 = r$

The radius is 5 cm.

b The area of a circle is 9π cm^2
Calculate the radius.

$A = \pi r^2$

$9\pi = \pi r^2$

$9 = r^2$

$\ldots = r$

3 cm

c The area of a circle is 100π cm^2
Calculate the radius.

10 cm

d The area of a semicircle is 32π cm^2
Calculate the radius.

8 cm

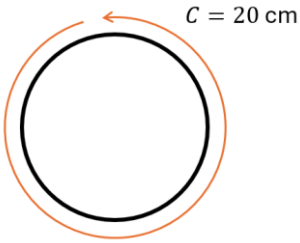
e The area of a quadrant is 9π cm^2
Calculate the radius

6 cm

f The area of a quadrant is 23π cm^2
Calculate the radius to 1 d.p.

9.6 cm

5 Ferdinand measures the circumference of his circular plate to be 20 cm.



a What is the radius of the plate?
(circumference is $C = 2\pi r$)

3.183 cm

b What is the area that his plate covers?

31.83 cm²

6 Complete the table.

	Radius	Diameter	Area $A = \pi r^2$ (in terms of π)	Circumference $C = 2\pi r$ (in terms of π)
a	3	6	9π	6π
b	6	12	36π	12π
c	6	12	36π	12π
d	2	4	4π	4π
e	1	2	π	2π
f	4	8	16π	8π
g	8	16	64π	16π
h	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{4\pi}{9}$	$\frac{4\pi}{3}$
i	$\frac{3}{10}$	$\frac{3}{5}$	$\frac{9\pi}{100}$	$\frac{3\pi}{5}$
j	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{4\pi}{25}$	$\frac{4\pi}{5}$
k	$\frac{2}{25}$	$\frac{4}{25}$	$\frac{4\pi}{625}$	$\frac{4\pi}{25}$

7 **NAPLAN A**

The radius of a circle is 6.5 centimetres.

A square has the same area as this circle.

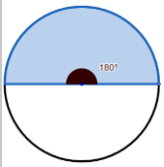
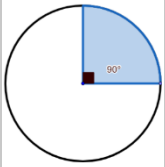
Calculate the side length of the square, in centimetres correct to one decimal place.

$A = \pi(6.5)^2$

$l = \sqrt{\pi(6.5)^2}$

11.52 cm

8 Fill in the table.

	Sketch	Radius	Angle	Fraction of circle	Circum-ference	Arc Length	Area of circle	Area of sector
a		6	180°	$\frac{1}{2}$	12π	6π	36π	18π
b		6	$90°$	$\frac{1}{4}$	12π	3π	36π	9π
c		6	$45°$	$\frac{1}{8}$	12π	1.5π	36π	4.5π
d		6	$270°$	$\frac{3}{4}$	12π	9π	36π	27π
e		6	$60°$	$\frac{1}{6}$	12π	2π	36π	6π
f		6	$120°$	$\frac{1}{3}$	12π	4π	36π	12π
g		6	$330°$	$\frac{11}{12}$	12π	11π	36π	33π
h		12	$330°$	$\frac{11}{12}$	24π	22π	144π	132π
i		12	$30°$	$\frac{1}{12}$	24π	2π	144π	12π
j		12	$90°$	$\frac{1}{4}$	24π	6π	144π	36π

9

a Calculate the exact circumference of a circle with diameter 5 cm.

5π cm

b Calculate the exact area of a circle with diameter 5 cm.

25π cm

c The area of a circle is 48 cm^2. Calculate the radius of the circle to 1 d.p.

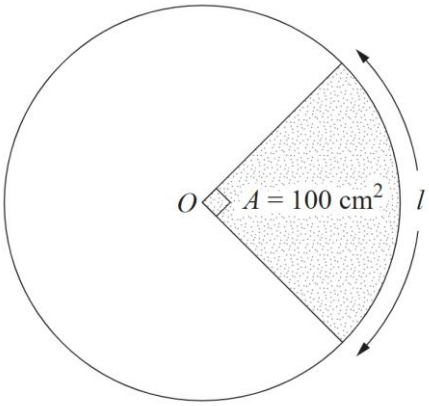
3.9 cm

d The circumference of a circle is 48 cm. Calculate the area to 1 d.p.

183.4 cm

10 **2017 HSC Standard 2 Band 5**

In the circle, the area of the quadrant is 100 cm^2.
What is the arc length l, correct to one decimal place?



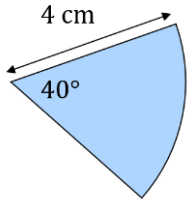
$100 = \frac{1}{4}\pi r^2$

$r = 11.283$ cm

$l = \frac{1}{4}(2\pi r) = 17.7$ cm

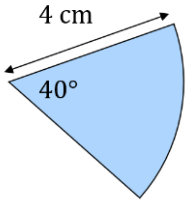
11 Consider the sector below. It is not drawn to scale.

a Calculate the area of the sector to 2 d.p.



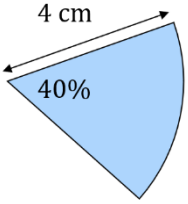
5.59 cm²

b Calculate the perimeter to 2 d.p.



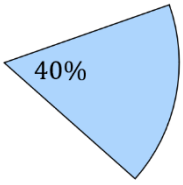
10.79 cm

c The sector is 40% of the circle. Calculate the area of the sector.



20.11 cm²

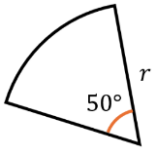
d The sector is 40% of the circle and has an area of 200 cm^2. Calculate the area of the whole circle.



$0.4 \times A = 200$
$\therefore A = 500$ cm²

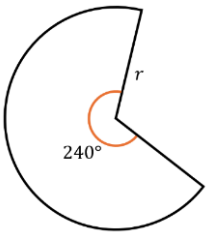
12 Find the unknown value in these sectors.

a The area is 16 cm^2.
Find the radius to 2 d.p.



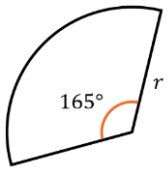
$$A = \frac{\theta}{360}\pi r^2$$
$$16 = \frac{50}{360}\pi r^2$$
$$16 = \frac{5}{36}\pi r^2$$
$$\times 36 \quad \times 36$$
$$576 = 5\pi r^2$$
$$\div 5\pi \quad \div 5\pi$$
$$\frac{576}{5\pi} = r^2$$
$$\sqrt{} \quad \sqrt{}$$
$$6.06 \text{ cm} = r$$

b The area is 105.6 cm^2.
Find the radius to 1 d.p.



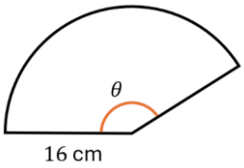
7.1 cm

c $A = 323$ cm^2
Find diameter to 2 d.p.



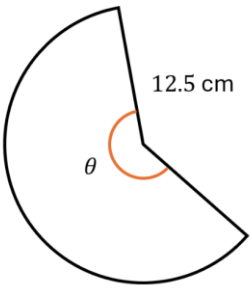
29.95 cm

d The area is 335.1 cm^2.
Find the angle.



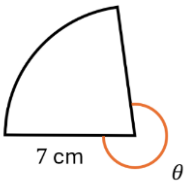
150°

e The area is 272.7 cm^2
Find the angle.



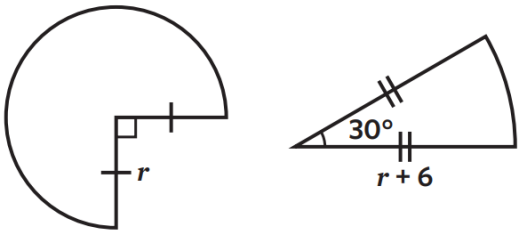
200°

f The area is 32.07 cm^2
Find the outside angle.



285°

13 The shapes in the diagram below have equal areas. Determine the value of r.



$\frac{3}{4}\pi r^2 = \frac{1}{12}\pi(r+6)^2$

$\frac{3}{4}r^2 = \frac{1}{12}(r+6)^2$

$\frac{r^2}{(r+6)^2} = \frac{1}{9}$

By trial and error, the radius is 3.

Alternate method using quadratic equations:

$\frac{3}{4}r^2 = \frac{1}{12}(r^2+12r+36)$

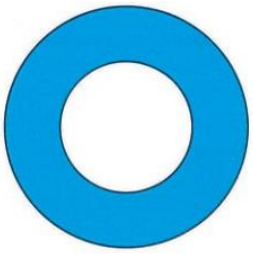
This simplifies to $-2r^2+3r+9=0$

$(-2r-3)(r-3)=0$

$r = 3$ or $-\frac{3}{2}$, but r must be positive.

$r = 3$

14 A shape is made up of two circles with the same centre.
The diameter of the large circle is 12 cm.
The area of the shaded region is 50 cm^2.
Find the diameter of the smaller circle.



Area of the shaded region is $A = \pi R^2 - \pi r^2$

$36\pi - \pi r^2 = 50$

$-\pi r^2 = 50 - 36\pi$

$r^2 = \frac{50-36\pi}{-\pi}$

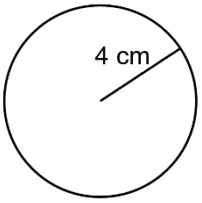
$\therefore r = 4.48$ cm

$\therefore$ diameter of smaller circle is 8.96 cm

CHECK YOUR UNDERSTANDING

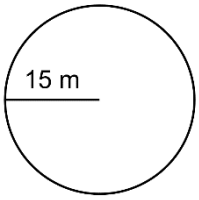
Area of a circle

1 What is the area of this circle?



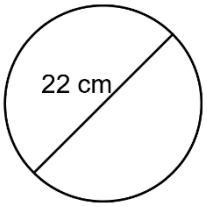
a 25.14 cm^2 **b** 50.27 cm^2 **c** 201.09 cm^2 **d** 12.57 cm^2

2 What is the area of this circle?



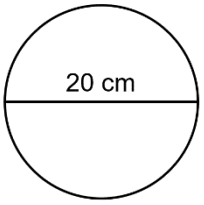
a 2827.4 m^2 **b** 47.1 cm^2 **c** 706.9 m^2 **d** 94.2 cm^2

3 What is the area of this circle?



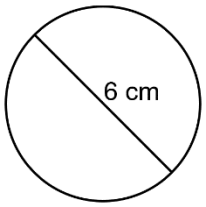
a 69.12 cm^2 **b** 380.13 cm^2 **c** 1520.53 cm^2 **d** 6082.12 cm^2

4 What is the area of this circle?



a 314.16 cm^2 **b** 1256.64 cm^2 **c** 62.83 cm^2 **d** 5026.55 cm^2

5 What is the area of this circle?



a 36π cm^2 **b** 12π cm^2 **c** 9π cm^2 **d** 6π cm^2

6 The radius of a circle is 9 cm. What is the area?

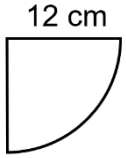
a 9π cm^2 **b** 18π cm^2 **c** 81π cm^2 **d** 20.25π cm^2

7 The diameter of a circle is 16 cm. What is the area?

a 64π cm^2 **b** 16π cm^2 **c** 128π cm^2 **d** 256π cm^2

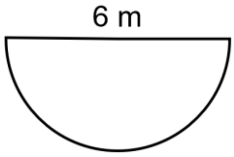
Area of semicircles and quadrants

1 What is the area of the quadrant?
Round your answer to 1 d.p.



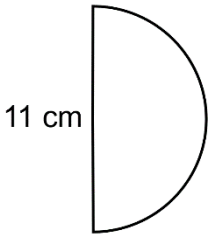
a 28.3 cm^2 **b** 18.8 cm^2 **c** 452.4 cm^2 **d** 113.1 cm^2

2 What is the area of the semi-circle?
Round your answer to 1 d.p.



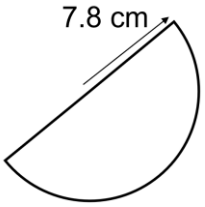
a 28.3 m^2 **b** 56.5 m^2 **c** 14.1 m^2 **d** 9.4 m^2

3 What is the area of the semi-circle?
Round your answer to the nearest whole.



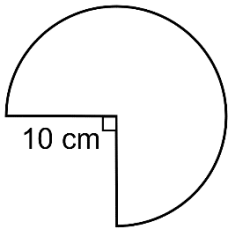
a 48 m^2 **b** 28 m^2 **c** 95 m^2 **d** 380 m^2

4 What is the area of the semi-circle?
Round your answer to 2 d.p.



a 40.10 cm^2 **b** 191.13 cm^2 **c** 95.57 cm^2 **d** 49.01 cm^2

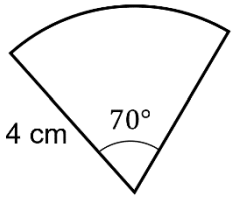
5 What is the area of this shape?
Round your answer to 1 d.p.



a 78.5 cm^2 **b** 235.6 cm^2 **c** 47.1 cm^2 **d** 942.5 cm^2

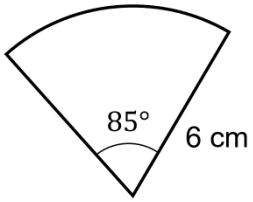
Area of a sector

1 What is the area of the sector?



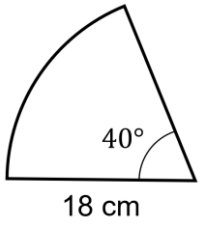
a $\frac{70}{360} \times \pi \times 4$ **b** $\frac{70}{360} \times \pi \times 4^2$ **c** $\frac{70}{360} \times \pi \times 8$ **d** $\frac{70}{360} \times \pi \times 8^2$

2 What is the area of the sector?



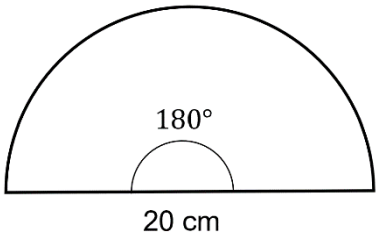
a $\frac{85}{360} \times \pi \times 12$ **b** $\frac{85}{360} \times \pi \times 6$ **c** $\frac{85}{360} \times \pi \times 6^2$ **d** $\frac{85}{360} \times \pi \times 12^2$

3 What is the area of the sector?



a 1020 cm^2 **b** 28.3 cm^2 **c** 113 cm^2 **d** 12.6 cm^2

4 What is the area of the sector?



a 628.3 cm^2 **b** 312.4 cm^2 **c** 157.1 cm^2 **d** 70.5 cm^2