

Student Name

Mathematics Stage 5 Paths

PROPERTIES OF GEOMETRICAL FIGURES

Book 2 Develop and use the conditions for congruent triangles.

Version: 260207

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CONTENTS

Conditions for Congruent Triangles2

OVERVIEW

SYLLABUS CONTENT

MA5-GEO-P-01 establishes conditions for congruent triangles and similar triangles and solves problems relating to properties of similar figures and plane shapes

Develop and use the conditions for congruent triangles

- Establish the 4 congruence tests for triangles (SSS, SAS, AAS and RHS)
- Use the congruence tests to identify a pair of congruent triangles from a selection of 2 or more triangles

REVIEW OF PRIOR KNOWLEDGE

1 These two triangles are congruent.

a Name the vertex on XYZ that matches:

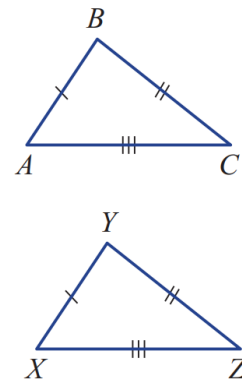
i B ii A iii C

b Name the side on XYZ that matches:

i AB ii AC iii BC

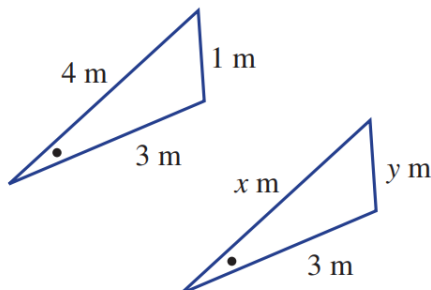
c Name the angle in ABC that matches:

i $\angle X$ ii $\angle Y$ iii $\angle Z$

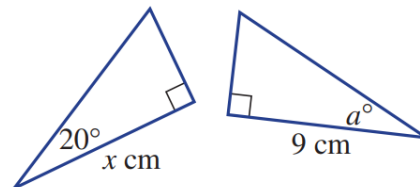


2 These pairs of triangles are congruent. Find the values of the pronumerals.

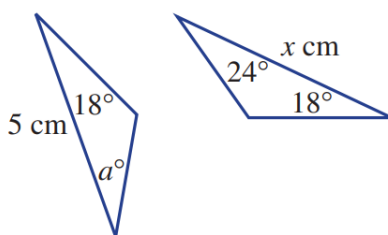
a



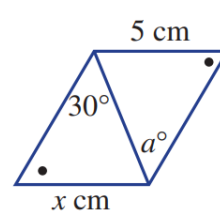
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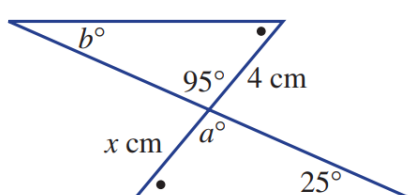
c



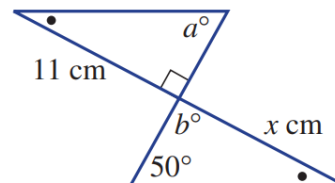
d



e

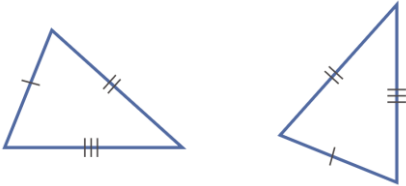
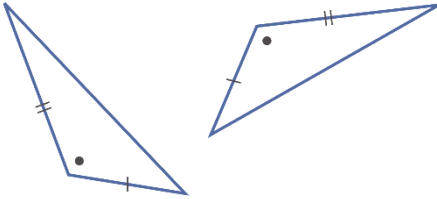
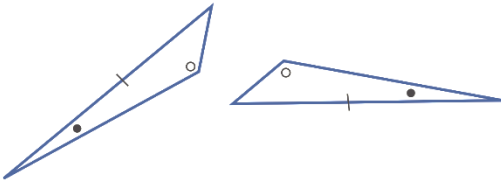
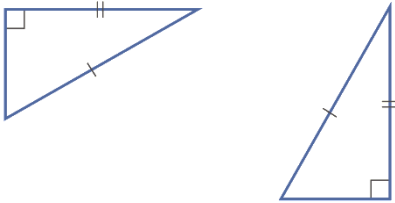


f



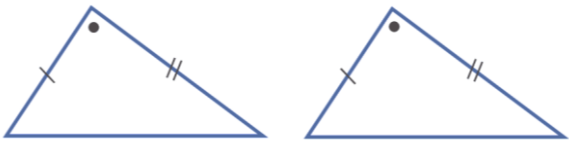
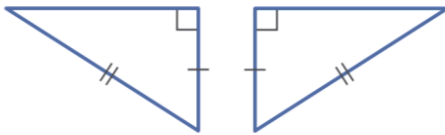
CONDITIONS FOR CONGRUENT TRIANGLES

- If two triangles satisfy one of these four tests, then the two triangles are congruent.

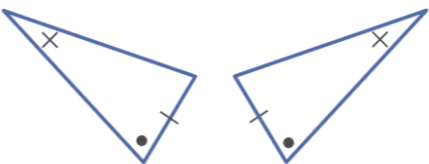
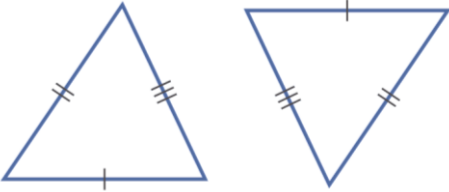
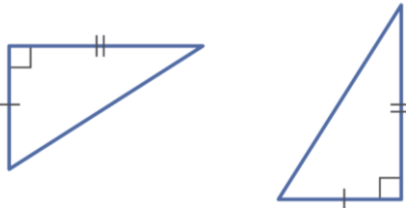
SSS	SAS
 Three pairs of corresponding sides.	 Two pairs of corresponding sides and the included angle equal.
AAS	RHS
 Two pairs of corresponding angles and one pair of corresponding sides equal.	 For a right-angled triangle, the hypotenuse and another corresponding side equal.

Example Identify congruent triangles using the four congruence tests.

Which of the tests (SSS, SAS, AAS or RHS) prove the congruence of these pairs of triangles?

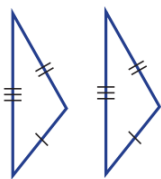
 SAS	 RHS
--	---

Which congruence test (SSS, SAS, AAS or RHS) proves these pairs of triangles are congruent?

a  AAS	b  SSS
c  AAS	d  SAS

1 Pick the congruence test (SSS, SAS, AAS or RHS) for each pair of congruent triangles.

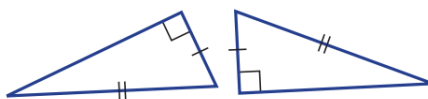
a



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

SSS

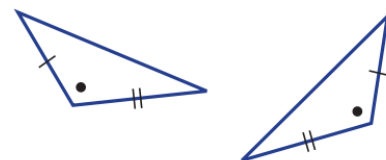
b



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

RHS

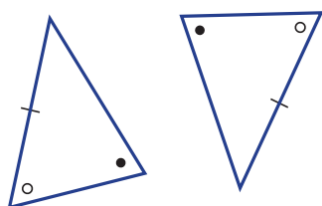
c



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

SAS

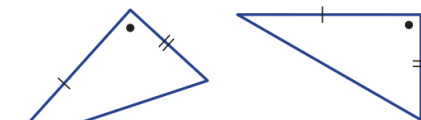
d



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

AAS

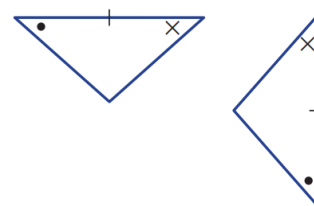
e



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

SAS

f



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

AAS

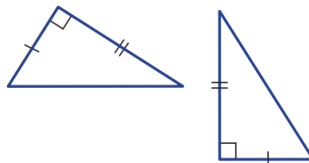
g



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

RHS

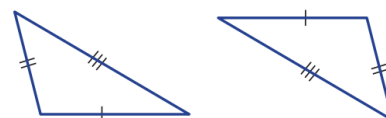
h



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

SAS

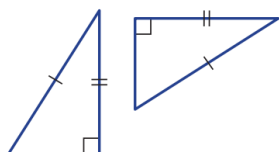
i



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

SSS

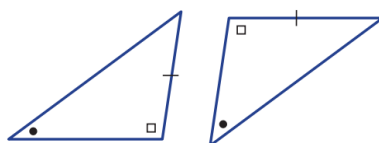
j



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

RHS

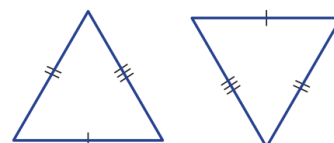
k



- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

AAS

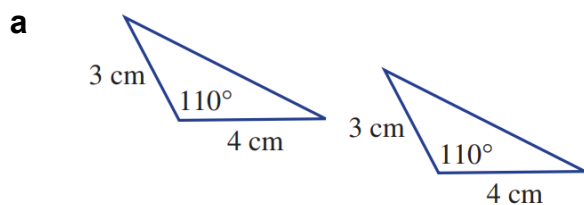
l



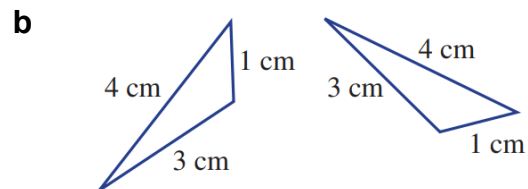
- ☐ SSS ☐ SAS
☐ AAS ☐ RHS

SSS

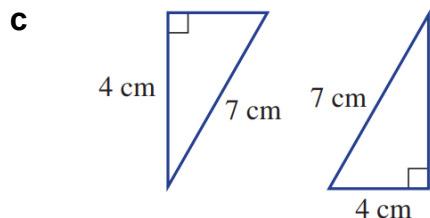
2 Which test (SSS, SAS, AAS or RHS) proves the congruence of these triangles?



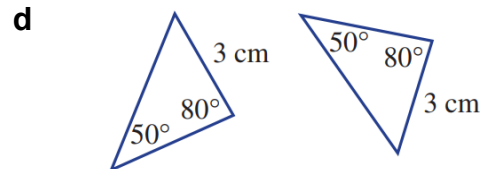
SAS



SSS



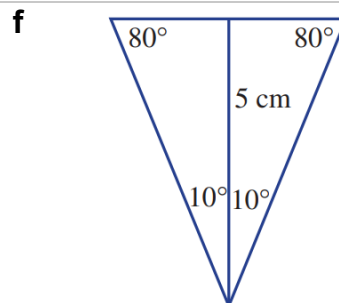
RHS



AAS

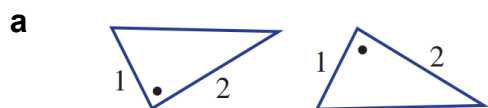


RHS

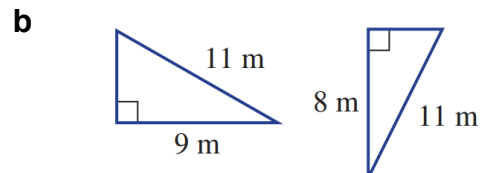


AAS

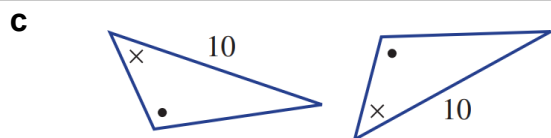
3 Decide if these pairs of triangles are congruent. If they are, write the test (SSS, SAS, AAS or RHS).



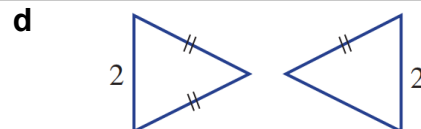
SAS



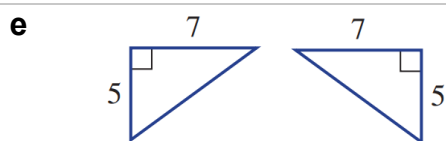
Not congruent



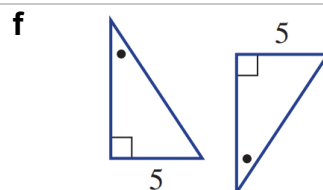
AAS



Not congruent



SAS

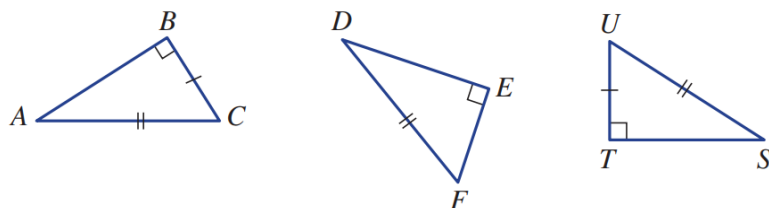


AAS

DEVELOPMENT

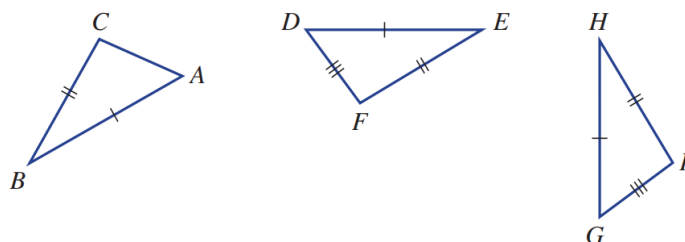
- 4 For each set of three triangles, choose the two that are congruent. Choose the appropriate test (SSS, SAS, AAS or RHS) and write a congruence statement (e.g. $\triangle ABC \equiv \triangle FGH$).

a



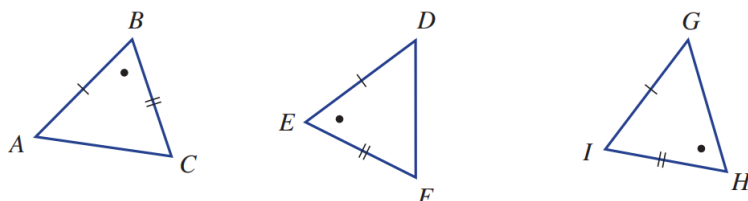
$\triangle ABC \equiv \triangle STU$ (RHS)

b



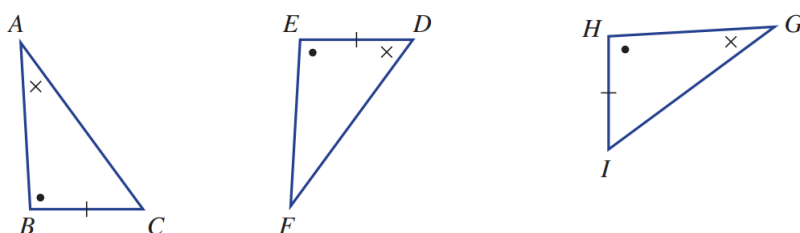
$\triangle DEF \equiv \triangle GHI$ (SSS)

c



$\triangle ABC \equiv \triangle DEF$ (SAS)

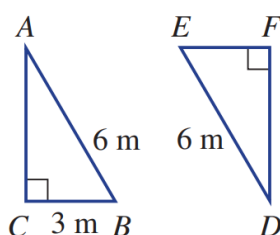
d



$\triangle ABC \equiv \triangle GHI$ (AAS)

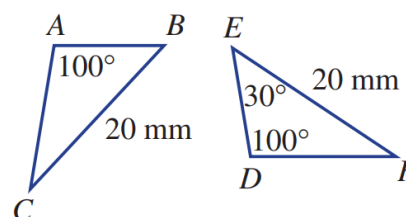
- 5 Add 1 piece of information to each diagram to make the two triangles congruent, and state the congruence test you are using.

a



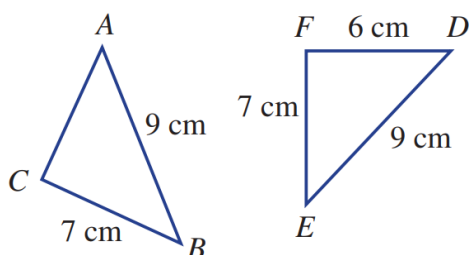
$EF = 3 \text{ m}$, RHS

b



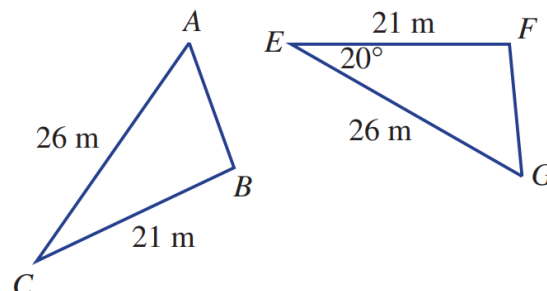
$\angle ABC = 30^\circ$, AAS

c



$AC = 6 \text{ cm}$, SSS

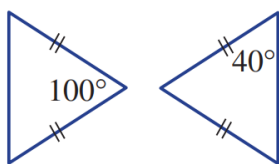
d



$\angle ACB = 20^\circ$, SAS

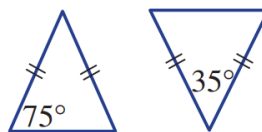
- 6 Decide if each pair of triangles is congruent.
You first need to use the angle sum of a triangle to help calculate some of the angles.

a



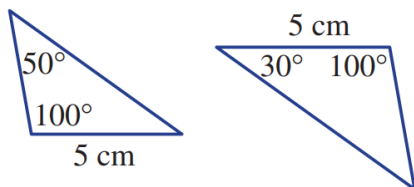
yes

b



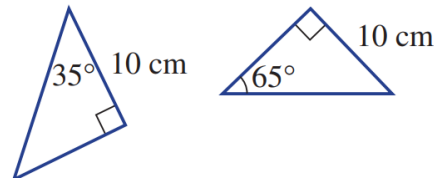
no

c



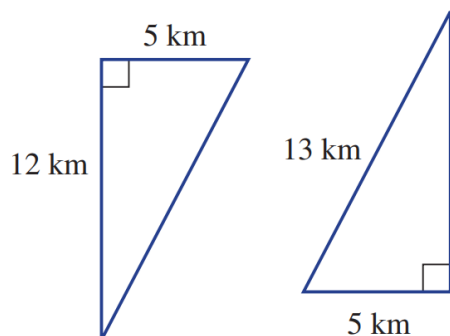
yes

d



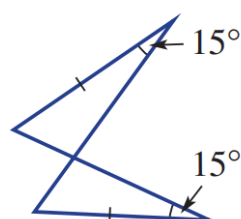
no

- 7 Two adjoining triangular areas of land have the dimensions shown.
Use Pythagoras' theorem to help decide if the two areas of land are congruent.



yes

- 8 Consider this diagram including two triangles.



- a Explain why there are two pairs of equal matching angles.

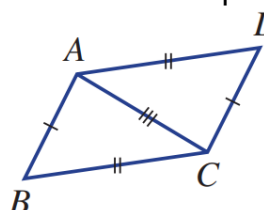
There is a pair of equal 15° angles (given)

And there is a pair of equal vertically opposite angles

- b Give the reason (SSS, SAS, AAS or RHS) why there are two congruent triangles.

AAS

- 9 ABCD is a parallelogram.



- a Give the reason why $\triangle ABC \equiv \triangle CDA$.

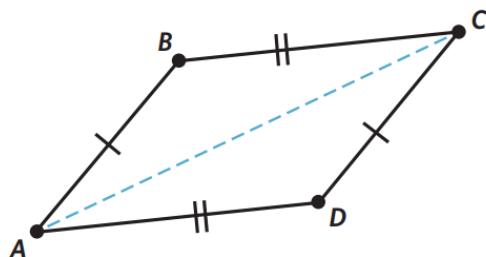
SSS

- b What does this say about $\angle B$ and $\angle D$?

They are the same as corresponding angles in congruent triangles are equal.
This is also a property of a parallelogram (opposite angles equal)

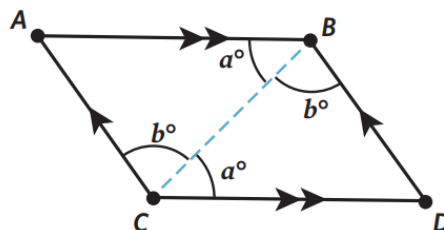
10 Determine a test that could be used to prove these triangles are congruent.

a $\triangle ADC$ and $\triangle ABC$



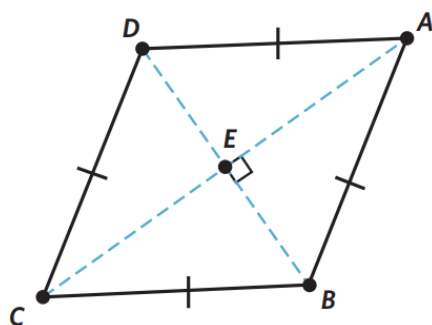
SSS

b $\triangle ABC$ and $\triangle DCB$



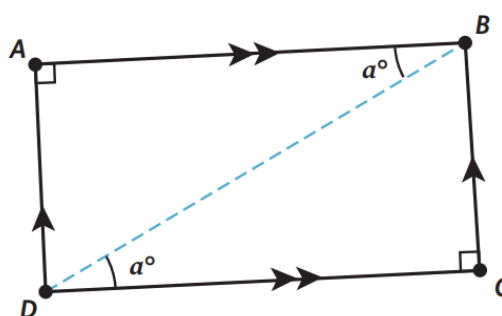
AAS

c $\triangle BEA$ and $\triangle DEA$



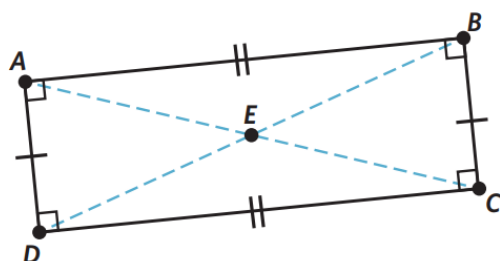
RHS

d $\triangle ABD$ and $\triangle DCB$



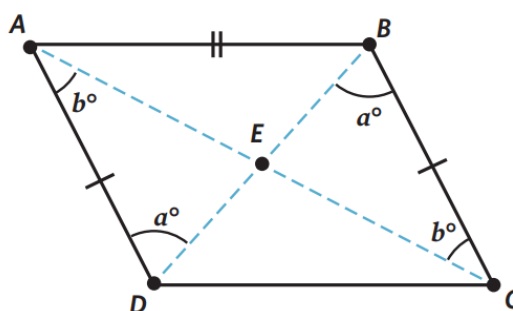
AAS

e $\triangle ABC$ and $\triangle DCB$



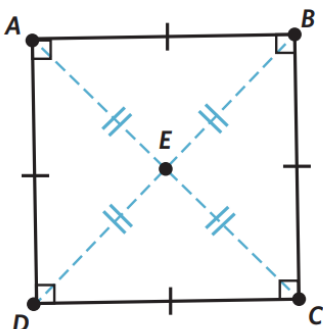
SSS

f $\triangle AED$ and $\triangle BEC$



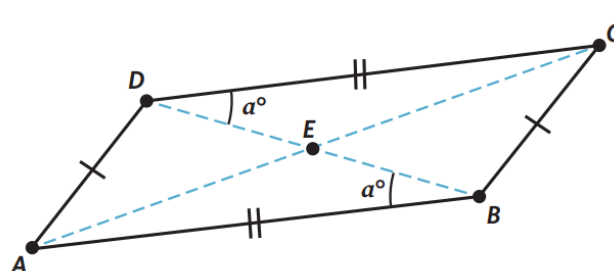
AAS

g $\triangle AEB$, $\triangle AED$, $\triangle DEC$, and $\triangle CEB$



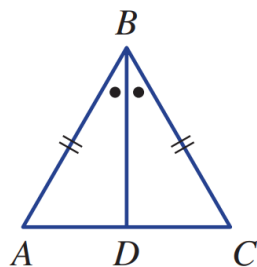
SSS

h $\triangle AEB$ and $\triangle DEC$



AAS

11 Consider this diagram.



- a Explain why there are two pairs of matching sides of equal length for the two triangles $\triangle ABD$ and $\triangle CBD$

$AB = BC$ (given)

BD is a common side

- b Give the reason (SSS, SAS, AAS or RHS) why there are two congruent triangles.

SAS

- c Write a congruence statement.

$\triangle ABD \equiv \triangle CBD$ (SAS)

- d Explain why AC is perpendicular (90°) to DB .

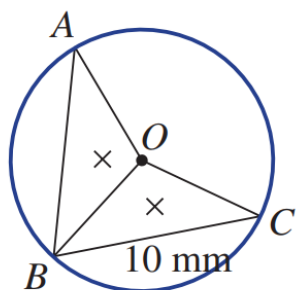
$\angle ADB = \angle CDB$ (corresponding sides on congruent triangles)

But $\angle ADB + \angle CDB = 180^\circ$ (angles on a straight line)

Since the two angles are equal and sum to 180, each angle must be 90° .

So AC is perpendicular to DB

12 In this diagram O is the centre of the circle and $\angle AOB = \angle COB$.



- a Give the reason why $\triangle AOB \equiv \triangle COB$

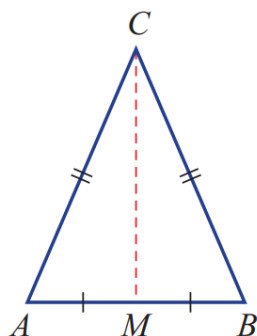
(OA, OB, OC) are all radii of a circle and equal.

SAS

- b Hence state the length of AB .

10 mm (corresponding sides of congruent triangles equal)

13 An isosceles triangle is cut as shown, using the midpoint of AB .



- a Name the two triangles formed.

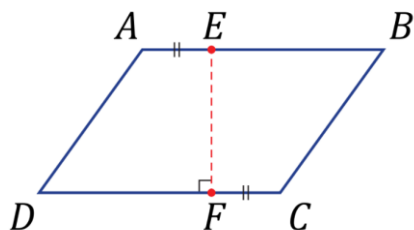
$\triangle ACM, \triangle BCM$

- b Will the two triangles be congruent? Give reasons.

Yes, by SSS

- 14** If this parallelogram is cut as shown, explain why the two pieces are congruent.

(hint: what are the properties of a parallelogram?)



$AE = FC$ (given)

$AD = BC$ (opposite sides of parallelogram equal)

$EB = FD$ (opposite sides of parallelogram equal)

EF is common

So, all corresponding sides are equal

$\angle ADF = \angle CBE$ (opposite angles of parallelogram equal)

$\angle DAE = \angle BCF$ (opposite angles of parallelogram equal)

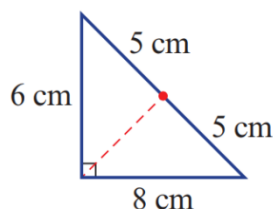
$\angle EFC = 90^\circ$ (angles on a straight line sum to 180)

$\angle FEA = \angle EFD = 90^\circ$ (co-interior angles between parallel lines sum to 180)

$\angle FEB = \angle EFC = 90^\circ$ (co-interior angles between parallel lines sum to 180)

All four sides equal and all angles equal, therefore congruent.

- 15** If this triangle is cut as shown, determine whether the two triangles will be congruent. Explain.



No, let the red line have length x

the sides of one triangle are 5, x , 6 and the other is 5, x , 8

So they cannot be congruent as the sides are not equal.