Student Name

Mathematics Stage 5 Paths

PROPERTIES OF GEOMETRICAL FIGURES

Book 3 Develop and apply the minimum conditions for triangles to be similar

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

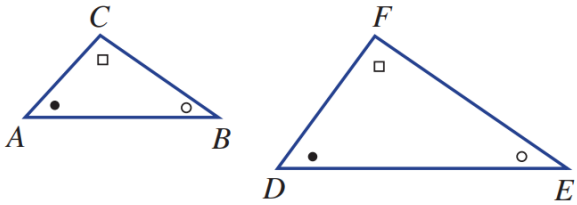
MA5-GEO-P-01 establishes conditions for congruent triangles and similar triangles and solves problems relating to properties of similar figures and plane shapes

Develop and apply the minimum conditions for triangles to be similar

- Examine the minimum conditions needed and establish the 4 tests for 2 triangles to be similar
- Apply the minimum conditions needed and determine whether 2 triangles are similar using an appropriate test

REVIEW OF PRIOR KNOWLEDGE

1 These two triangles are similar.



a Which vertex on ΔDEF corresponds to (matches) vertex B on ΔABC? .....

b Which vertex on ΔABC corresponds to (matches) vertex F on ΔDEF? .....

c Which side on ΔDEF corresponds to (matches) side AC on ΔABC? .....

d Which side on ΔABC corresponds to (matches) side EF on ΔDEF? .....

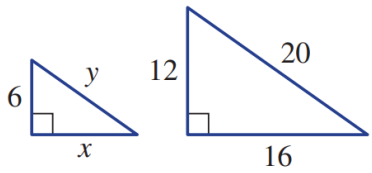
e Which angle on ΔABC corresponds to (matches) ∠D on ΔDEF? .....

f Which angle on ΔDEF corresponds to (matches) ∠B on ΔABC? .....

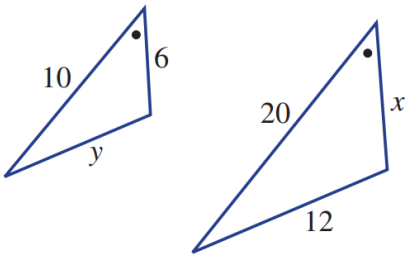
g Complete the similarity statement for these triangles ΔACB ~ Δ .....

2 Given that these pairs of triangles are similar, find the value of the pronumerals.

a



b



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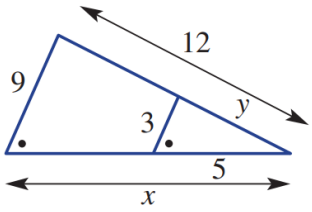
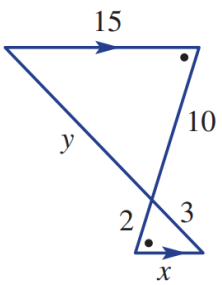
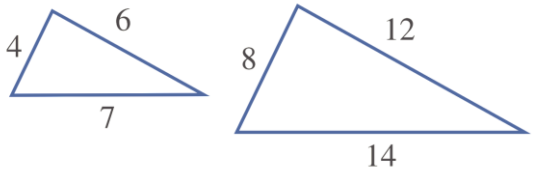
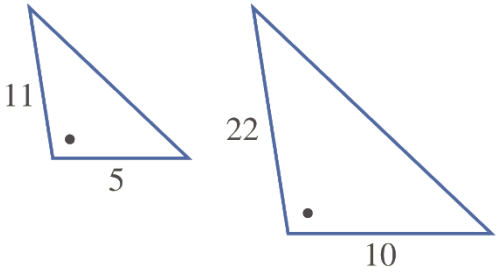
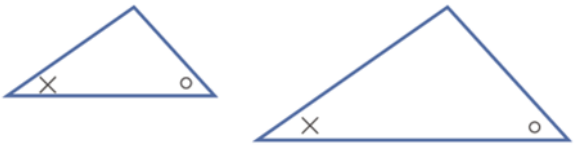
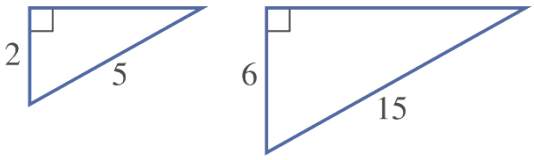


d



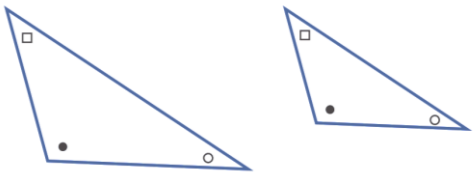
CONDITIONS FOR SIMILAR TRIANGLES

- There are four 'tests' that can be used to decide if two triangles are similar.
- Unlike the congruence tests, you must write the full reasoning every time.

SSS	SAS
	
Three pairs of corresponding sides in the same ratio.	Two pairs of corresponding sides in the same ratio and the included angle equal.
AAA	**RHS**
	
At least two pairs of equal corresponding angles. (If there are two equal pairs, the third must be equal)	For a right-angled triangle, the hypotenuses and another pair of corresponding sides in the same ratio.

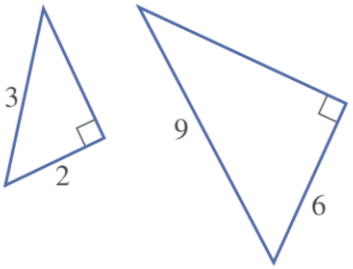
Example Identify congruent triangles using the four congruence tests.

Which of the tests (SSS, SAS, AAA or RHS) prove the similarity of these pairs of triangles?



AAA; Three pairs of corresponding angles.

$\frac{9}{3} = 3$; $\frac{6}{2} = 3$



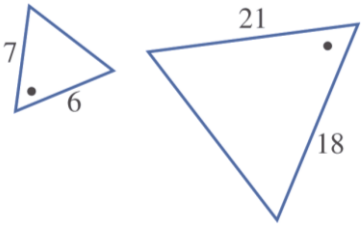
RHS; Hypotenuses and another pair of corresponding sides in the same ratio.

In the second example, what is the scale factor?

The scale factor is the ratio of $\frac{\text{image}}{\text{original}} = \ldots\ldots\ldots\ldots$

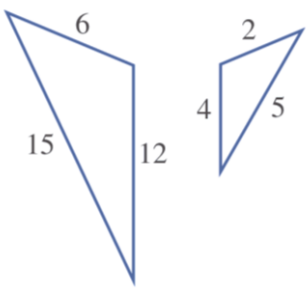
Which similarity test would be used to show that these pairs of triangles are similar?
You must write the full reasoning.

a



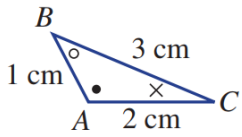
Two pairs of corresponding sides in the same ratio and the included angle equal.

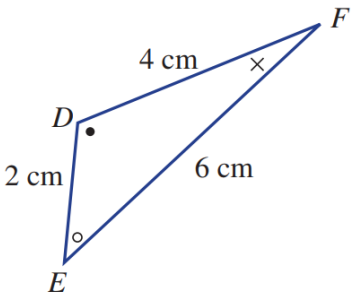
b



Three pairs of corresponding sides in the same ratio.

1 Consider these two triangles.





a Name the angle on the larger triangle that corresponds to:

i ∠A ..... ∠D **ii** ∠B ..... ∠E **iii** ∠C ..... ∠F

b Name the side on the smaller triangle that corresponds to:

i DE ..... AB **ii** EF ..... BC **iii** FD ..... CA

c Find these side ratios.

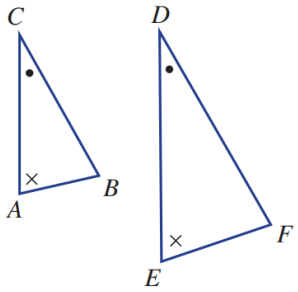
i $\frac{DE}{AB}$ ..... 2 **ii** $\frac{EF}{BC}$ ..... 2 **iii** $\frac{FD}{CA}$ 2

d Explain why the two triangles are similar.

Corresponding angles are equal. Corresponding sides are in the same ratio.

2 Write a similarity statement for these pairs of similar triangles; for example, △ABC ||| △DEF. Be careful with the order of letters and make sure each letter matches the corresponding vertex. Also state the reason.

a



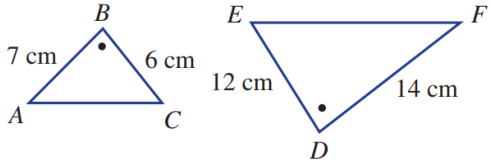
Δ ..... ||| Δ ..... (AAA)

ABC ||| EDF

At least ..... pairs of ..... angles.

2, equal

b



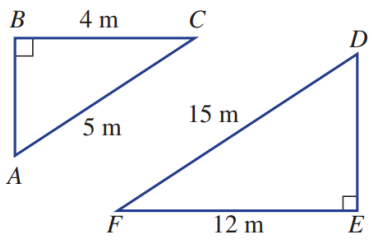
Δ ..... ||| Δ ..... (SAS)

ABC ||| FDE

..... pairs of corresponding sides in the same ..... and the ..... angle equal.

2, ratio, included

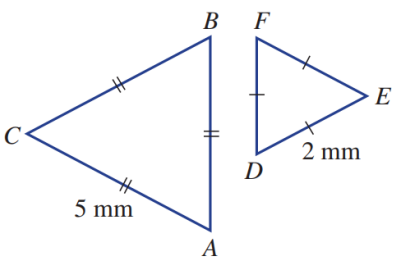
c



$\Delta ABC|||DEF$

the hypotenuses and another pair of corresponding sides in the same ratio.

d

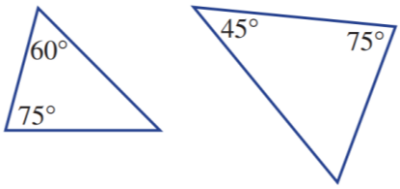


$\Delta ABC|||DEF$

Three pairs of corresponding sides in the same ratio.

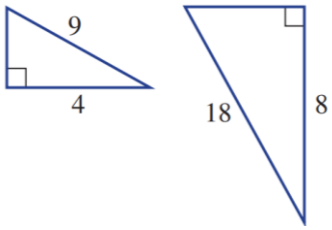
3 Choose the similarity test that proves that these pairs of triangles are similar.
You must write the full reasoning.

a



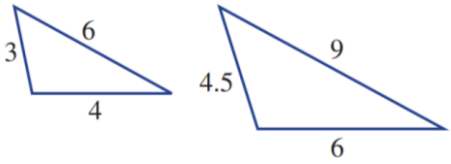
At least two pairs of equal corresponding angles.

b



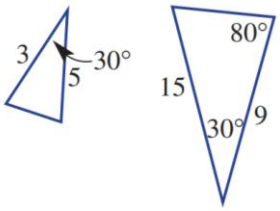
the hypotenuses and another pair of corresponding sides in the same ratio.

c



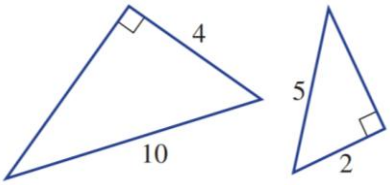
Three pairs of corresponding sides in the same ratio.

d



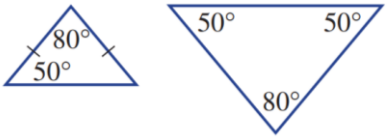
Two pairs of corresponding sides in the same ratio and the included angle equal.

e



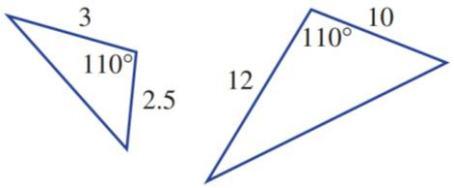
the hypotenuses and another pair of corresponding sides in the same ratio.

f



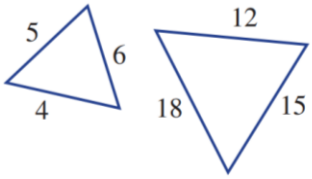
At least two equal pairs of corresponding angles.

g



Two pairs of corresponding sides in the same ratio and the included angle equal.

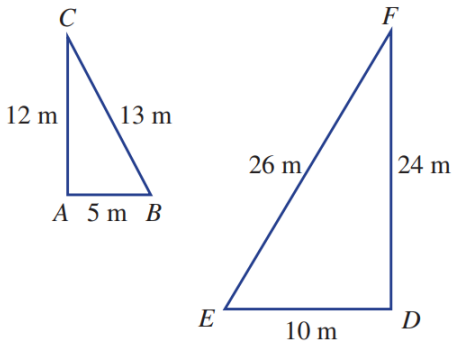
h



Three pairs of corresponding sides in the same ratio.

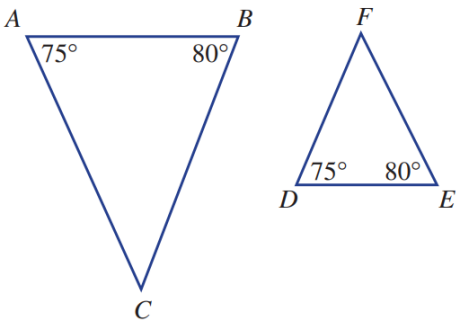
4 State, with reasons, why these triangles are similar.

a

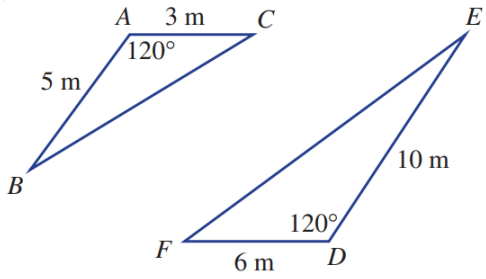


Three pairs of corresponding sides in the same ratio.

b

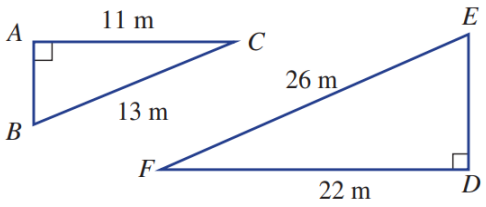


At least two pairs of equal corresponding angles.



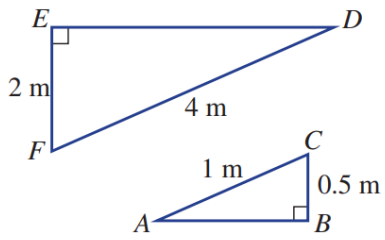
Two pairs of corresponding sides in the same ratio and the included angle equal.

d



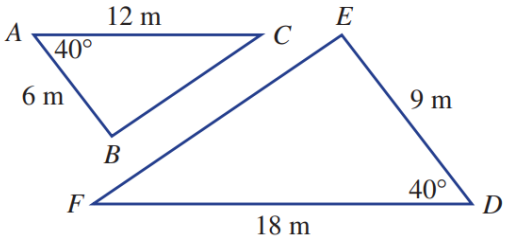
For a right-angled triangle, the hypotenuses and another pair of corresponding sides in the same ratio.

e



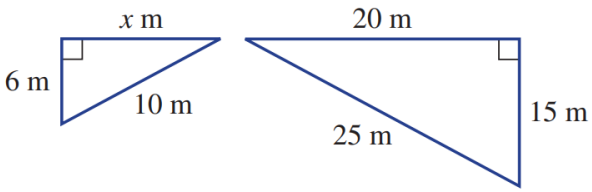
For a right-angled triangle, the hypotenuses and another pair of corresponding sides in the same ratio.

f



Two pairs of corresponding sides in the same ratio and the included angle equal.

5 For this pair of triangles:



a Identify why the two triangles are similar

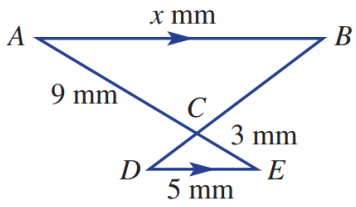
For a right-angled triangle, the hypotenuses and another pair of corresponding sides in the same ratio.

b find the value of x

$\frac{x}{20} = \frac{6}{15}$

$x = 8$ m

6 For this pair of triangles:



a Identify why the two triangles are similar

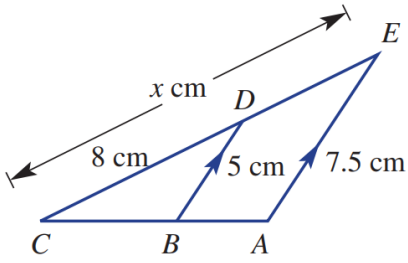
At least two pairs of equal corresponding angles.

b find the value of x

$\frac{x}{5} = \frac{9}{3}$

$x = 15$ mm

7 For this pair of triangles:



a Identify why the two triangles are similar

At least two pairs of equal corresponding angles.

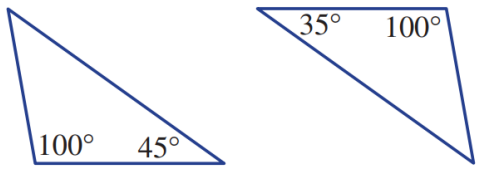
b find the value of x

$\frac{x}{8} = \frac{7.5}{5}$

$x = 12$

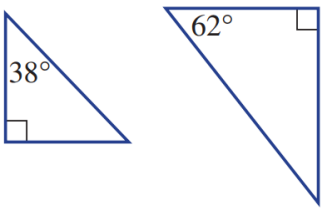
8 By using the angle sum of a triangle, decide if the given pairs of triangles are similar.

a



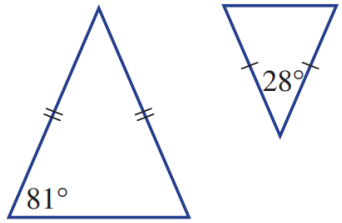
yes

b



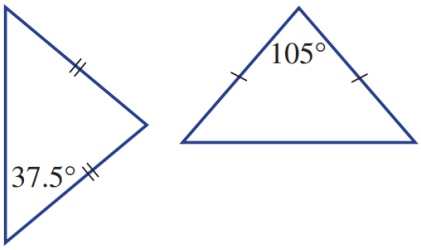
no

c



no

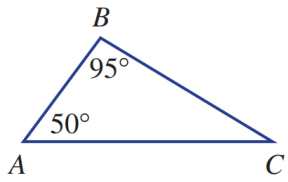
d

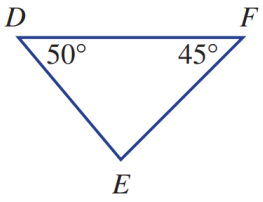


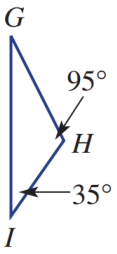
yes

9 For each set of three triangles, choose the two that are similar.
Choose the appropriate test and write a similarity statement (e.g. ΔABC ~ ΔFGH).

a

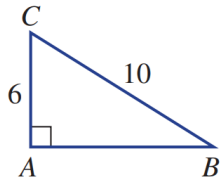


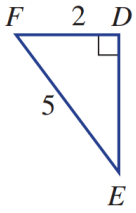


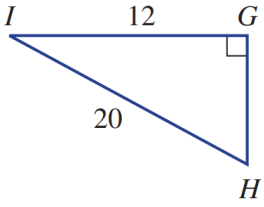


ABC ||| GHI
At least two pairs of corresponding angles.

b

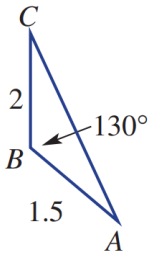


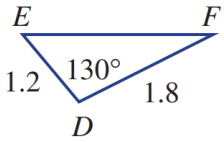


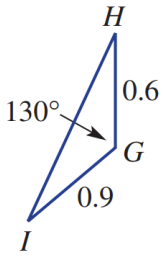


ABC ||| GHI
For a right-angled triangle, the hypotenuses and another pair of corresponding sides in the same ratio.

c

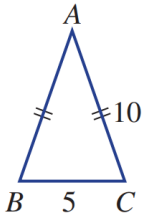


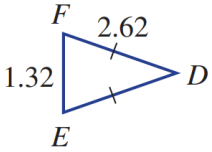


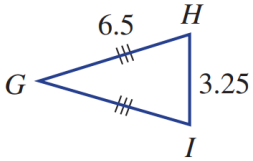


DEF ||| GHI
Two pairs of corresponding sides in the same ratio and the included angle equal.

d

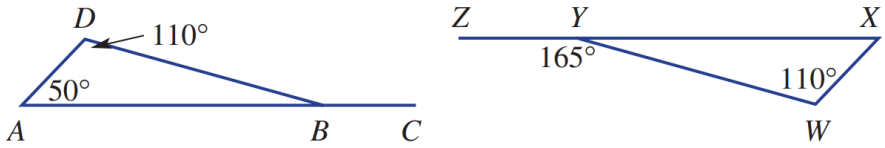






ABC ||| GHI
Three pairs of corresponding sides in the same ratio.

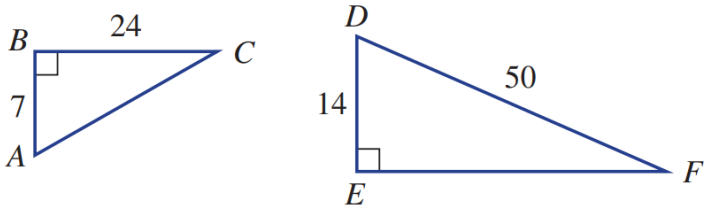
10 Give reasons why the two triangles in these diagrams are not similar.



$\angle DBA = 20°$, but $\angle XYW = 15°$

Corresponding angles are not equal

11 Using Pythagoras' theorem, show that these two triangles are similar.



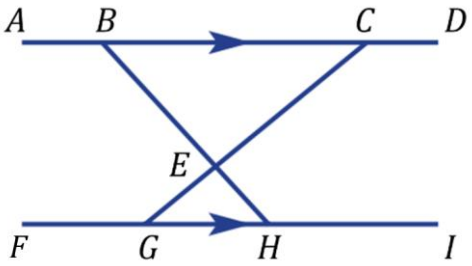
$AC = \sqrt{7^2 + 24^2} = 25$

$EF = \sqrt{50^2 - 14^2} = 48$

$\frac{AC}{DF} = \frac{AB}{DE} = \frac{BC}{EF} = 2$

Three pairs of corresponding sides in the same ratio.

12 When two intersecting transversals join parallel lines, two triangles are formed.
Explain why these two triangles are similar. Show full reasoning.



$\angle CBE = \angle EHG$ (alternate angles on || lines)

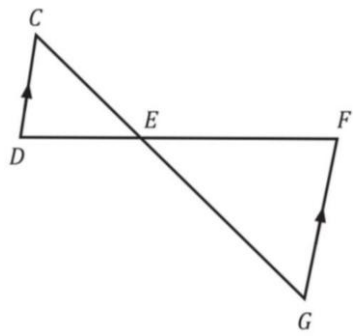
$\angle BCE = \angle EGH$ (alternate angles on || lines)

$\therefore \Delta BEC \,|||\, \Delta HEG$ at least two pairs of corresponding angles.

$\angle BEC = \angle GEH$ (vertically opposite) (not necessary, but another option)

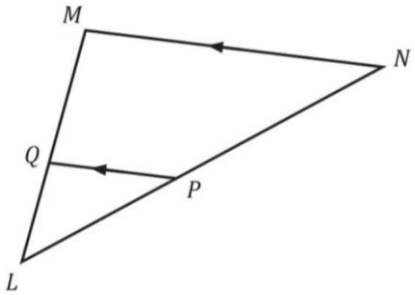
13 Explain why these triangles are similar. Show full reasoning.

a



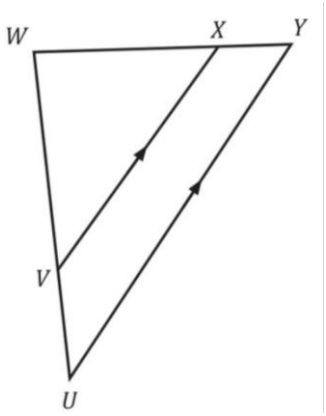
$\angle DCE = \angle FGE$ (alternate angles on || lines)
$\angle CDE = \angle GFE$ (alternate angles on || lines)
ΔCDE ||| ΔGFE
At least two pairs of equal corresponding angles.

b



$\angle L$ is common
$\angle LQP = \angle LMN$ (corresp angles on || lines)
ΔLQP ||| ΔLMN
At least two pairs of equal corresponding angles.

c



$\angle W$ is common
$\angle WVX = \angle WUY$ (corresp angles on || lines)
ΔWVX ||| ΔWUY
At least two pairs of equal corresponding angles.

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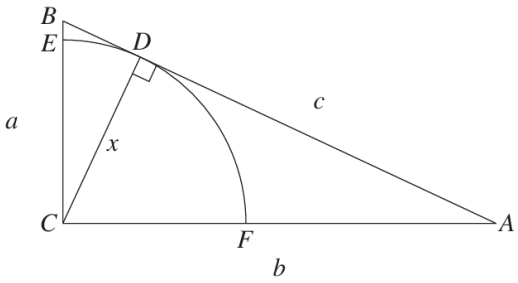
In triangle ABC, BC is perpendicular to AC.

Side BC has length a, side AC has length b and side AB has length c.

A quadrant of a circle of radius x, centred at C, is constructed. The arc meets side BC at E.

It touches the side AB at D, and meets side AC at F. The interval CD is perpendicular to AB.

a Show that ΔABC is similar to ΔACD.



In △ABC and △ACD

∠ACB = ∠ADC (CD perpendicular to AB, △ABC right-angled at C)

∠BAC = ∠CAD (common)

∴ △ABC ||| △ACD (equiangular)

b Show that $x = \frac{ab}{c}$

$\frac{x}{a} = \frac{b}{c}$ (corresponding sides of similar triangles)

$\therefore x = \frac{ab}{c}$