Student Name

Mathematics Stage 5 Core

AREA AND SURFACE AREA

Book 2 Develop and apply the formula for surface areas of cylinders

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-ARE-C-01 solves problems involving the surface area of right prisms and practical problems involving the area of composite shapes and solids

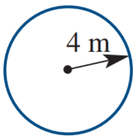
Develop and apply the formula for surface areas of cylinders

- Recognise the curved surface of a cylinder as a rectangle and apply this knowledge to calculate the area of the curved surface
- Develop and apply the formula to find the surface area of a closed cylinder: $A = 2\pi r^2 + 2\pi rh$, where r is the length of the radius and h is the perpendicular height

REVIEW OF PRIOR KNOWLEDGE

1 Calculate the circumference of these circles. $A = 2\pi r$. Round to 2 d.p.

a



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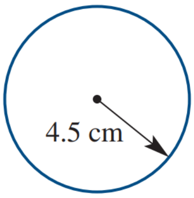

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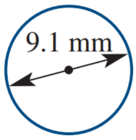

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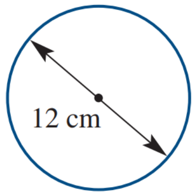
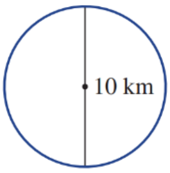


2 Calculate the area of these circles. $A = \pi r^2$. Round to 2 d.p.

a



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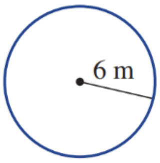

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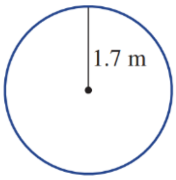

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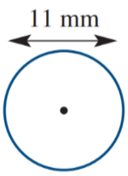
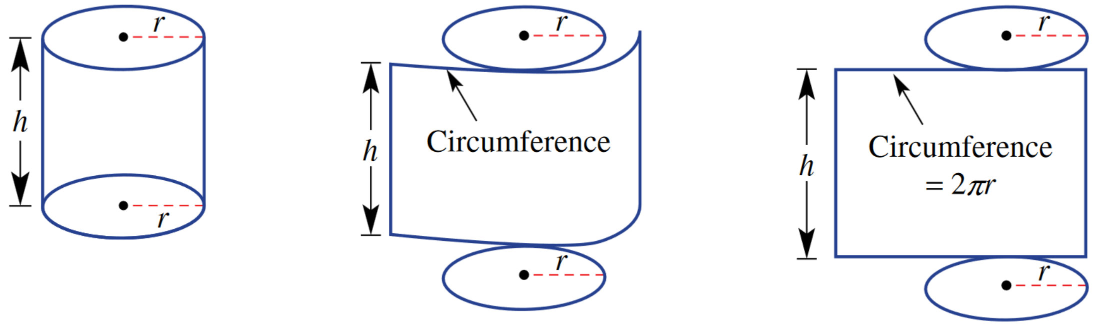


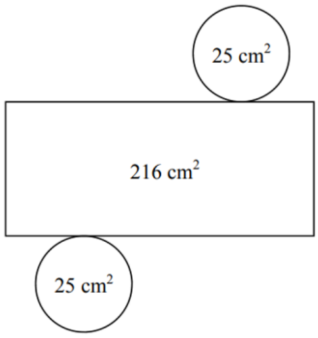
SURFACE AREA OF CLOSED CYLINDERS

- A **cylinder's net** has two circles and one rectangle.



Calculate the surface area of these cylinders by adding the areas of each face.

a



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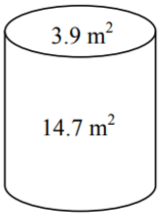

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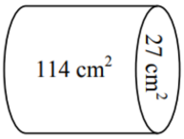


Investigation Surface area of a cylinder.

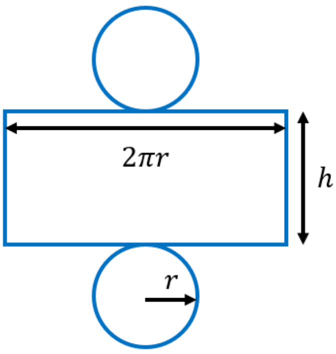
A cylinder is made up of ………… faces.

The top and bottom are circles, with area ………………

The curved part can be unrolled into a rectangle,

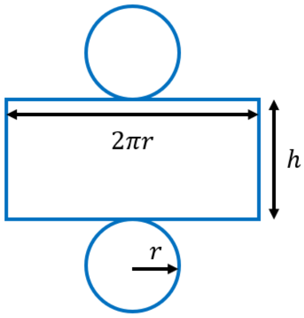
with length ………… and width ………………

So the total surface area is $A =$ ……………… + ………………



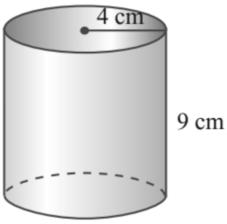
Surface Area of a Closed Cylinder

$$SA = \underbrace{2\pi r^2}_{\text{2 circles}} + \underbrace{2\pi rh}_{\text{curved surface}}$$



Example Calculate surface area of a cylinder.

Calculate the surface area of the cylinder, rounded to 2 decimal places.



$A = 2\pi r^2 + 2\pi rh$

$= 2\pi(4)^2 + 2\pi(4)(9)$

$= 326.73\ \text{cm}^2$

What does $2\pi r^2$ represent?

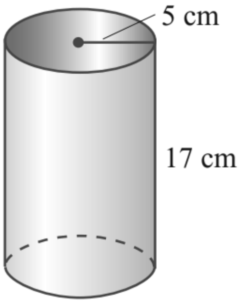
$2\pi r^2$ represents the ....

What does $2\pi rh$ represent?

$2\pi rh$ represents the ....

Calculate the surface area of these cylinders.

a



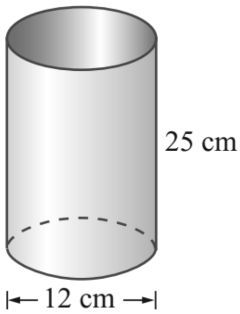
$A = 2\pi r^2 + 2\pi rh$

$=$ ....

$=$ ....

691.15 cm^2

b



$A =$ ....

$=$ ....

$=$ ....

1168.67 cm^2

Key ideas

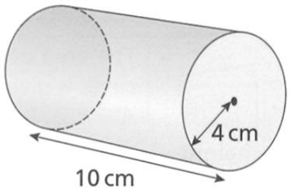
1. A cylinder is made up of two .... and a single ....
2. The rectangle's dimensions are the height of the cylinder and the .... of the circles that make up the bottom and top.

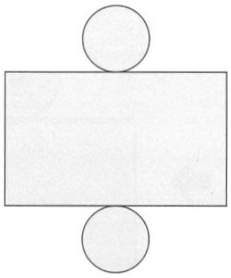
1 Consider the cylinder and net to the right.

a Label the net with correct lengths.

b Calculate the area of:

i The two circles.
Answer in exact form.





$2\pi(4)^2 = 32\pi$

ii The rectangle. Answer in exact form.

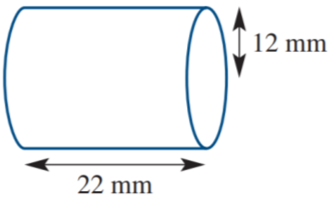
$2\pi(4)(10) = 80\pi$

iii Hence find the surface area of the cylinder to 1 d.p.

351.9 cm²

2 Find the surface area of these cylinders. Round your answer to 1 decimal place.

a



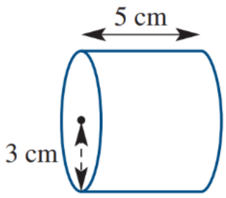
$A = 2\pi r^2 + 2\pi rh$

$= \dots\dots\dots\dots\dots\dots\dots$

$= \dots\dots\dots\dots\dots\dots\dots$

2563.5 mm²

b



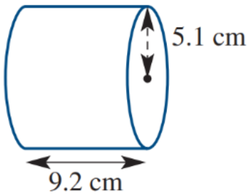
$A = \dots\dots\dots\dots\dots\dots\dots$

$= \dots\dots\dots\dots\dots\dots\dots$

$= \dots\dots\dots\dots\dots\dots\dots$

150.8 cm²

c



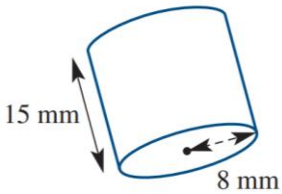
$A = \dots\dots\dots\dots\dots\dots\dots$

$= \dots\dots\dots\dots\dots\dots\dots$

$= \dots\dots\dots\dots\dots\dots\dots$

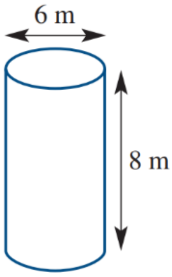
458.2 cm²

d



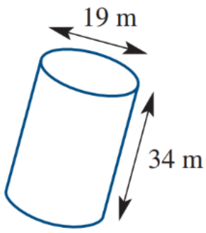
1156.1 mm²

e



207.3 cm²

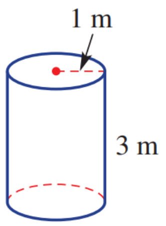
f



2596.5 cm²

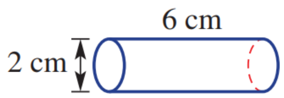
3 Work out the surface area of each cylinder. Answer in exact from in terms of π.

a



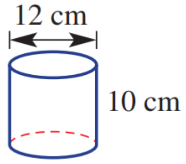
8π m²

b



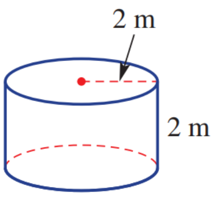
14π cm²

c



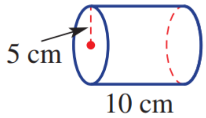
192π cm²

d



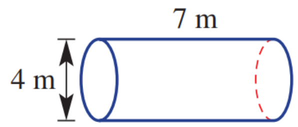
16π m²

e



150π cm²

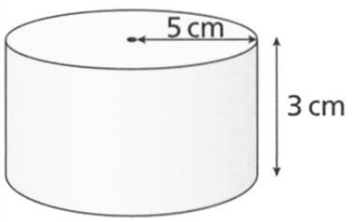
f



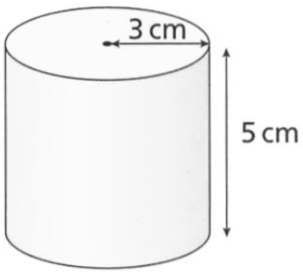
36π m²

4 Which shape has the larger surface area? Justify your answer.

A



B



A: 80π cm²

B: 48π cm²

A has larger surface area.

5 A cylinder has a radius of 12 cm and a height of 20 cm.

a Determine the surface area of the cylinder to 1 d.p.

2412.7 cm²

b The height of the cylinder is increased by 10%.

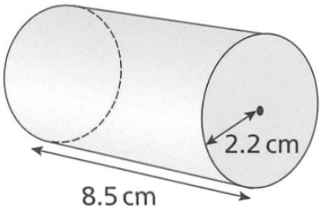
Work out the new total surface area of the cylinder.

2563.5 cm²

c Work out the percentage increase in the total surface area.

6.25% increase

6 Express the curved surface area of this cylinder as a percentage of the total surface area. Give your answer to 2 decimal places.



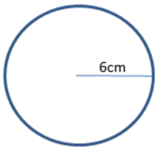
Curved: $2\pi rh$ = 117.5 cm²

Total = 147.9 cm²

Percentage = 79.44%

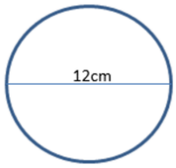
7

a The radius of a circle is 6 cm. Calculate the area of the circle.
Leave your answer in terms of π



36π cm²

b Shown is a face of a cylinder with a height of 10 cm. Calculate the exact surface area of the cylinder.



192π cm²

c The circumference of a circle shown is 7π cm. Calculate the area of the shaded region in terms of π.

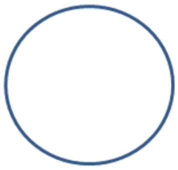
$C = 2\pi r$

$7\pi = 2\pi r$

$r = 3.5$ cm

$A = \dfrac{\pi(3.5)^2}{4} = \dfrac{49}{16}\pi \text{ cm}^2$

d The area of the circle is 16π cm². Find the circumference to 2 decimal places.



$A = \pi r^2$

$16\pi = \pi r^2$

$r = 4$ cm

$C = 2\pi r = 8\pi$ cm

8 A cylinder has radius 6 cm and height of h. The curved surface area of the cylinder is 754 cm^2

Work out the value of h.

$2\pi(6)h = 754$

$h = 20$ cm

9 A cylinder has radius 4 m and height h. The total surface area of the cylinder is 128π m^2.

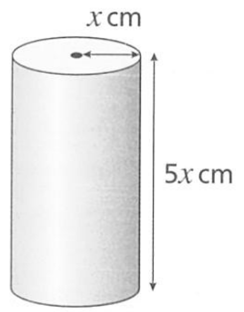
Work out the value of h.

$2\pi(4)^2 + 2\pi(4)(h) = 128\pi$

$8\pi h = 96\pi$

$h = 12$ m

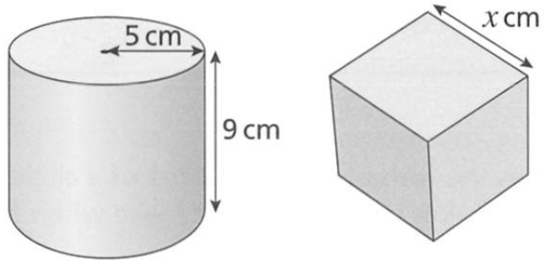
10 This cylinder has a surface area of 972π cm^2. Work out the value of x.



$972\pi = 2\pi x^2 + 2\pi x(5x)$

$x = 9$

11 The cylinder and cube have the same surface area. Work out the value of x to 2 d.p.



SA of cylinder = 140π cm²

$6x^2 = 140\pi$ cm²

$x = 8.56$ cm

12 If you double the height of a cylinder, do you double its surface area?

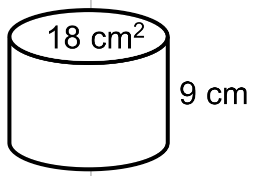
Explain why.

No. Only the curved surface is doubled, and the circular ends stay the same, so the total is not doubled.

13 Consider the cylinder below.

a Find the diameter of the cylinder to 2 d.p.

b Calculate the volume of the cylinder.



4.79 cm

162 cm³

c The top surface of a cylinder with the same volume has area of 50 cm^2. What is the height of the cylinder?

$V = \pi r^2 h$

$162 = 50h$

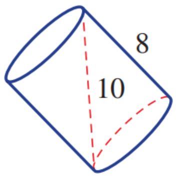
$h = 3.24$ cm

d Calculate the surface area of the cylinder.

171 cm²

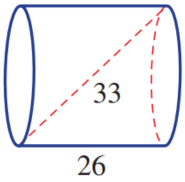
14 Use Pythagoras' Theorem $h^2 = a^2 + b^2$ to find unknown lengths and hence calculate the surface area of these solids. All units are in cm. Round your answer to 1 d.p.

a



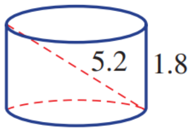
$r = 3$ cm
$SA = 207.4$ cm

b



$r = 10.1612\ldots$
$SA = 2308.7$ cm²

c



$r = 2.43926\ldots$
$SA = 65.0$cm²

15 Tennis balls are tightly packed in cylindrical cans which hold three balls.

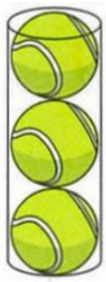
The radius of a ball is 37 mm.

The curved surface is manufactured from thin aluminium which costs \$4.80/m^2.

The top and bottom are made from stronger aluminium which costs \$5.60/m^2.

At the Australian Open Tennis Championships, about 48 000 balls are used.

Find the cost of producing the cans to hold this number of balls.



Dimensions of can: $r = 37$ mm, $h = 222$ mm

Circular ends = $2\pi(37)^2 = 2738\pi$ mm²

Curved surface = $2\pi(37)(222) = 16428\pi$ mm²

Cans needed = $48000 \div 3 = 16\,000$

Strong aluminium needed = $2738\pi \times 16000 = 43\,808\,000\pi$ mm² ÷ 1000² = 43.808π m²

Thin aluminium needed = $16428\pi \times 16000 = 262\,848\,000\pi$ mm² ÷ 1000² = 262.848π m²

Cost of thin aluminium = $43.808\pi \times \$5.60 = \$770.7105894\ldots$

Cost of strong aluminium = $262.848\pi \times \$4.80 = \$3963.65446\ldots$

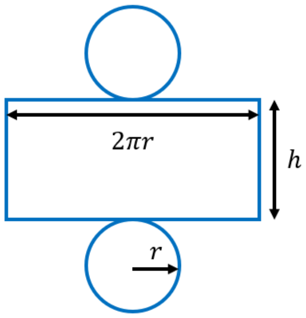
Total cost = \$4734.37

Check your answer using a faster method: Finding the cost of 1 can, then multiplying by 16 000.

SURFACE AREA OF CYLINDRICAL SOLIDS

Surface Area of a Closed Cylinder

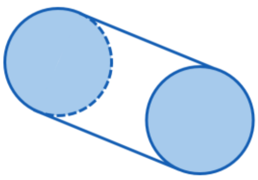
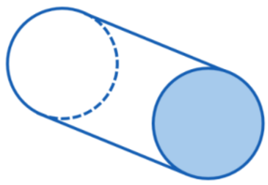
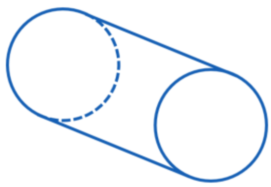
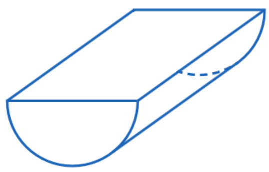
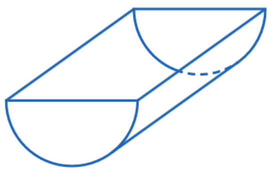
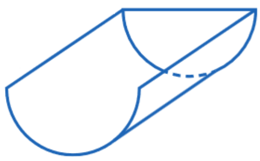
$$SA = \underbrace{2\pi r^2}_{\text{2 circles}} + \underbrace{2\pi rh}_{\text{curved surface}}$$



- Many questions will ask you to find a part of a cylinder.
- You need to decide how to adjust the formula:
  - Is there a full circular base, part of a base, or no base?
  - How much of the curved surface area remains?
  - Is there a new flat surface where the cylinder was "cut"?

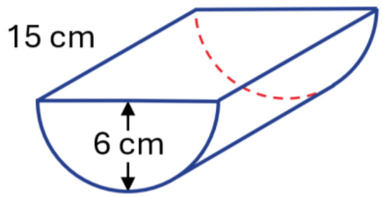
Example Identify formula to use to find surface area of cylindrical solid.

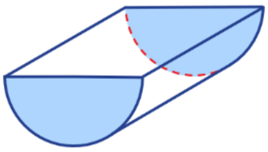
Write an expression for the surface area of these solids; the radius is r and height is h.

a Closed cylinder  Two end circles: Curved surface:	**b** Cylinder with one open end  One end circle: Curved surface:	**c** Cylinder open both ends  Curved surface:
d Closed half cylinder  Two end semicircles: Curved surface: Top rectangle:	**e** Half cylinder with open top  Two end semicircles: Curved surface:	**f** Half cylinder with open top, one end open  One end semicircle: Curved surface:

Example Calculate surface area of cylindrical solids.

Calculate the surface area of this **closed** half cylinder, rounded to 2 decimal places.

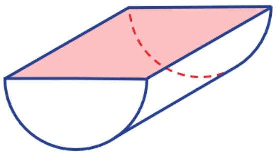




front and back semicircles

$\left(\frac{1}{2} \times \pi(6)^2\right) \times 2$

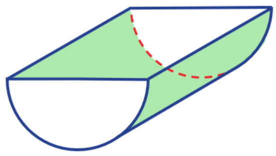
$= 113.0973 \ldots$



top rectangle

15×12

$= 180$



curved surface (rectangle)

$\frac{1}{2} \times 2\pi(6)(15)$

$= 282.7433 \ldots$

Total surface area $= 113.0972 + 180 + 282.7433$

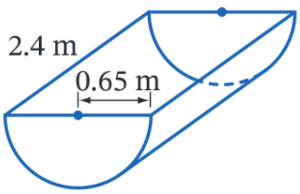
$= 575.84 \text{ cm}^2$

Explain why values were initially rounded to 4 d.p.

If you round too early, the final answer will be ……………………

Calculate the surface area of these solids. Answer to 2 d.p.

a Half-cylinder, **open top**



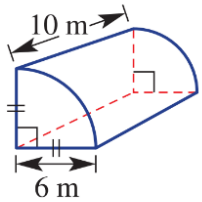
Front and back:

Curved side:

Total surface area:

6.23 m^2

b **Closed** quarter-cylinder



Front and back:

Curved side:

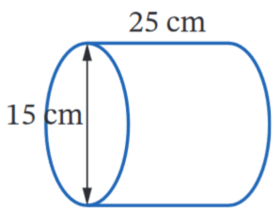
Side rectangles:

Total surface area:

270.80 m^2

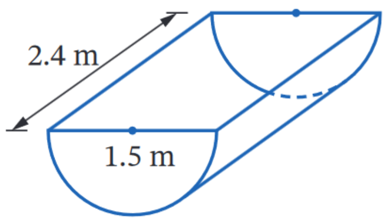
1 Calculate the surface area of the following solids to the nearest whole number.

a Cylinder with one open end



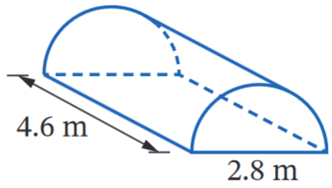
1355 cm²

b Half cylinder with open top



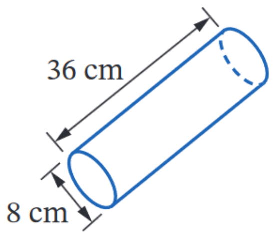
7 cm²

c Closed half-cylinder



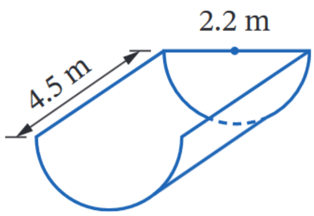
39 m²

d Cylinder open both ends



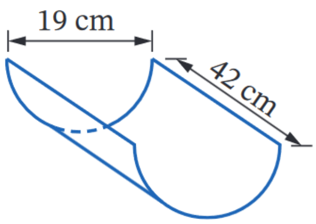
905 cm²

e Half cylinder with open top, one end open



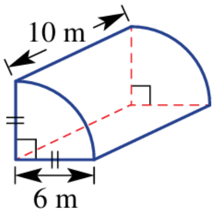
17 m²

f Half-cylinder with open top, open both ends



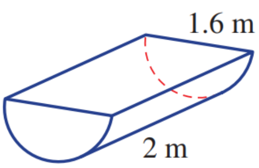
1253 cm²

g Closed quarter-cylinder



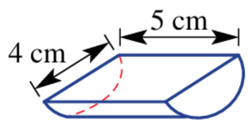
271 m²

h Half-cylinder with open top



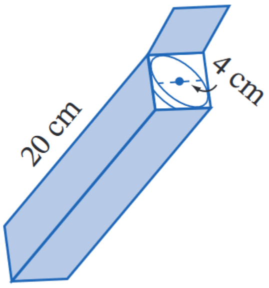
7 m²

i Closed half-cylinder



64 cm²

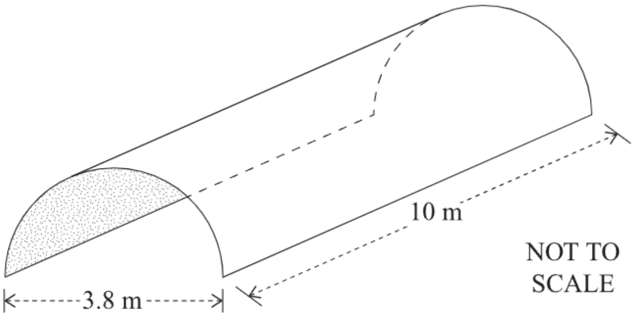
2 Circular cracker biscuits of diameter 4 cm are packed in a cardboard box of length 20 cm.
How much cardboard would be saved if the biscuits were packed into a cylindrical box instead?



76 cm²

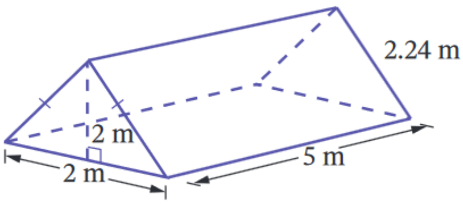
3 **2018 HSC Standard 2 Band 4**

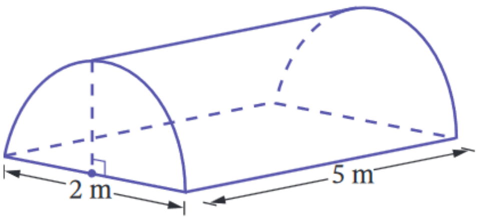
A shade shelter is to be constructed in the shape of half a cylinder with open ends.
The diameter is 3.8 m and the length is 10 m.
The curved roof is to be made of plastic sheeting.
What area of plastic sheeting is required, to the nearest m²?



$$A = \frac{1}{2}(2\pi r h)$$
$$= 2\pi(1.9)(10)$$
$$= 60 \text{ m}^2$$

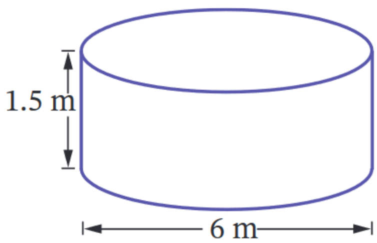
4 Which tent has the greater surface area? (include the floors).





The triangular prism tent has 7.6 m² more surface area.

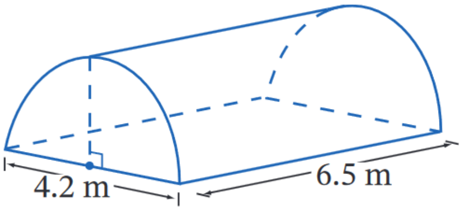
5 Jessica is repainting a swimming pool in the shape of a cylinder 1.5 m tall and 6 m in diameter.
The inside of the pool is to be repainted, including the floor.
If a one-litre can of paint covers 9 m^2, find the number of cans needed.



A = 56.5 m

6.3 L of paint required, so 7 cans to paint the pool.

6 The diagram shows a tent to be made in the shape of a half-cylinder.
The manufacturer knows that the flooring costs \$18.50/m^2 and the top canvas costs \$21/m^2.
How much would each tent cost to make? Answer to the nearest dollar.



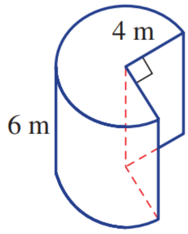
Floor: 27.3 m²

Surface Area: 56.7 m²

Cost = 27.3 × \$18.50 + 56.7 × \$21.75 = \$1738

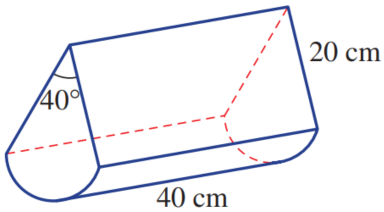
7 Find the surface area of these closed partial cylinders. Answer in exact form.
(hint: arc-length is $l = \frac{\theta}{360} \times 2\pi r$)

a



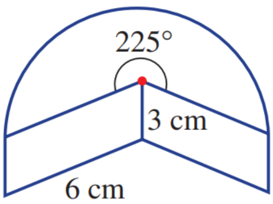
$(60\pi + 48)\ \mathrm{m}^2$

b



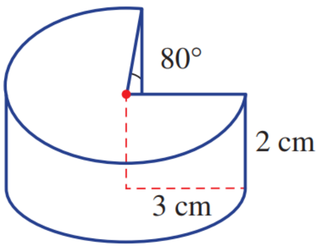
$\left(\frac{800\pi}{3} + 1600\right)\ \mathrm{cm}^2$

c



$\left(\frac{135\pi}{2} + 36\right)\ \mathrm{cm}^2$

d



$\left(\frac{70\pi}{3} + 12\right)\ \mathrm{cm}^2$