

Student Name

Mathematics Stage 4

PROBABILITY

Book 2 Determine probabilities for
complementary events

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OVERVIEW

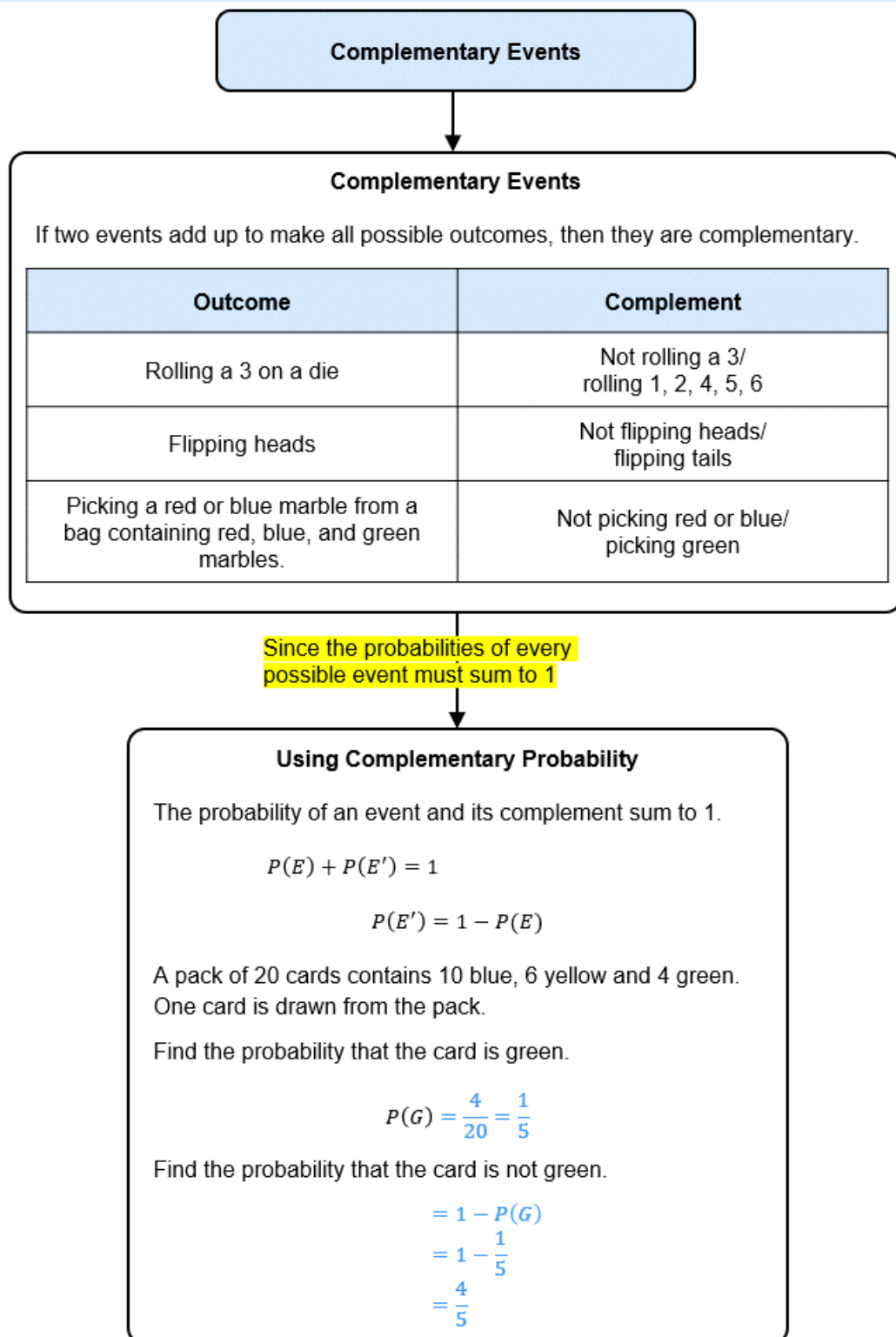
SYLLABUS CONTENT

MA4-PRO-C-01 solves problems involving the probabilities of simple chance experiments

Determine probabilities for complementary events

- Identify and describe the complement of an event
- Verify that the sum of the probability of an event and its complement is a total of 1
- Solve problems involving the probability of complementary events
- Represent the possible outcomes for complementary events in various forms

SUMMARY



REVIEW OF PRIOR KNOWLEDGE

1 Calculate the following

a $1 - 0.6$

b $1 - 0.45$

c $1 - (0.3 + 0.4)$

d $1 - \frac{5}{7}$

e $1 - \frac{2}{6} - \frac{1}{6}$

f $1 - \left(\frac{3}{10} + \frac{1}{5}\right)$

2 Solve these equations

a $x + 0.2 = 1$

b $x + \frac{3}{4} = 1$

c $2x + 0.6 = 1$

d $x + 0.2 + 0.7 = 1$

e $x + \frac{1}{4} + \frac{1}{2} = 1$

f $2x + \frac{2}{5} = 1$

COMPLEMENTARY EVENTS

- **Complementary events** cover all possible outcomes.
- The **complement** of an event is the event **not** taking place.

Example Identify the complement of an event.

The event is rolling a 3 on a fair six-sided die. The complementary event is: not rolling a 3: {1,2,4,5,6}	A bag contains red, green, and blue marbles. The event is picking red or blue. The complementary event is: Not picking red or blue: {green}
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Identify the complement of these events.

a Sunny day tomorrow.	b Winning a game of table tennis.
c Rolling an even number on a fair six-sided die.	d Picking fruit from a box containing chocolates, lollies, and fruit.
Not a sunny day tomorrow	Not winning a game / losing a game
Not rolling an even number / rolling an odd number	Not picking fruit / picking chocolates and lollies

FOUNDATION

1 Determine the complement of each event.

- | | |
|---|---|
| a Tossing a tails on a coin. | Not tossing a tails / tossing a heads |
| b Cloudy day tomorrow. | Not a cloudy day tomorrow |
| c Rolling a 6 on a die. | Not rolling a 6 / |
| d Winning a game of OzTag. | Not winning a game of OzTag / drawing or losing a game of OzTag |
| e Selecting a heart from a deck of cards. | Not selecting hearts / selecting diamond, club, or spade |
| f Being born in Winter. | Not being born in winter / born in spring, summer, or autumn |
| g Being under 15 years old. | Not being under 15 years old / being 15 or older |

2 Identify whether these are complementary events.

Event 1	Event 2	Complementary	Not Complementary	
Getting out of bed.	Not getting out of bed.	☐	☐	C
Walking home from school.	Riding a bike home from school	☐	☐	NC
Picking a picture card from a deck of 52 cards.	Picking a number card from a deck of 52 cards.	☐	☐	C
Rolling at least a three on a die.	Rolling three or less on a die.	☐	☐	NC

USING THE COMPLEMENT IN PROBABILITY PROBLEMS

- **Complements** simplify the **sample space** to two events.
- The probability of an event not happening is written as $P(E')$, meaning $P(\text{not } E)$.
- The probability of an event and its complement sum to 1.

Complementary Probabilities
$P(E) + P(E') = 1$ $P(E') = 1 - P(E)$

Example Using complementary outcomes in problem-solving.

A pack of 20 cards contains 10 blue, 3 yellow, 3 red, and 4 green. One card is drawn from the pack. Find the probability that the card is green. $P(G) = \frac{4}{20} = \frac{1}{5}$ Find the probability that the card is not green. $P(G') = 1 - P(G)$ $= 1 - \frac{1}{5}$ $= \frac{4}{5}$	Calculate the probability that the card is not green by adding up $P(\text{blue}) + P(\text{yellow})$ What is the advantage of using complementary outcomes? <i>In using $1 - P(G)$ instead of $P(B) + P(Y) + P(R)$, we only had to calculate probability instead of three. The more there are, the more time saved.</i>
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- a** The probability of it raining is 0.2.
What is the probability that it doesn't rain?

0.8

- b** A pack of 20 cards contains 10 blue, 6 yellow and 4 green.
One card is drawn from the pack. Find the probability that the card is not yellow.

$\frac{7}{10}$

- c** A bag of 25 coloured cubes contains 3 colours.
You know there are 7 green cubes.
What is the probability of picking a cube that is not green?

$\frac{18}{25}$

Key ideas

1. The complement of an outcome is the outcome happening. 2. The probability of an outcome and the outcome not happening must sum to 3. We can write "the probability of an event happening" as $P(E')$ 4. We can calculate the $P(E')$ by $P(E)$ from 1.

1

The probability that it will rain this weekend is $\frac{8}{10}$.
What is the probability that it won't rain?

$\frac{2}{10}$

2

The probability of a bus arriving on time is 75%.
What is the probability the bus does not arrive on time?

25%

3

Levon buys 5 tickets in a raffle of 1000 tickets.
What is the probability that he doesn't win?

$\frac{995}{1000}$

4

There are 11 cards, 6 of which are red. What is the probability of not selecting a red card?

$\frac{5}{11}$

5

Here are some counters

O

O

O

O

O

P

P

P

P

P

P

B

B

B

B

B

B

B

B

A counter is selected at random. Calculate these probabilities,

a P(orange)	$\frac{5}{19}$	b P(pink)	$\frac{6}{19}$	c P(blue)	$\frac{8}{19}$
d P(orange or pink)	$\frac{11}{19}$	e P(not blue)	$\frac{11}{19}$		

6

A car park has 450 cars, 100 motor bikes, and 10 buses. One is selected at random.

a What is the probability that it is a car?

$\frac{45}{56}$

b What is the probability that it is not a car?

$\frac{11}{56}$

7

The letters of the word PROBABILITY are written on separate cards and one is selected a randomly. Calculate:

a P(Selecting P)

b P(Not selecting P)

c P(Selecting I)

d P(Not selecting I)

$\frac{1}{11}$

$\frac{10}{11}$

$\frac{2}{11}$

$\frac{9}{11}$

8 Complete this table.

Event	$P(\text{event})$	$P(\text{not event})$	Sum of probabilities
Rolling a fair 6-sided die and getting 3	$\frac{1}{6}$	$\frac{5}{6}$	1
Tossing a coin and getting heads	$\frac{1}{2}$	$\frac{1}{2}$	1
Rolling a fair 6-sided die and getting 2 or 5.	$\frac{1}{3}$	$\frac{2}{3}$	1
Selecting a letter from 'HEART' and getting a vowel	$\frac{2}{5}$	$\frac{3}{5}$	1

9 If the probability of selecting a vowel in a particular word is $\frac{3}{13}$, what is the probability of selecting a consonant?

$\frac{10}{13}$

10 A spinner has different 3 different coloured sections.

Colour	Red	Green	Blue
Probability	0.2	0.3	

What is the probability of spinning blue?

0.5

11 William plays soccer. The table shows the probability that he will win, lose or draw. Work out the missing probability.

a

Win	Lose	Draw
$\frac{1}{4}$	$\frac{1}{4}$	

b

Win	Lose	Draw
0.2	0.2	

0.6

c

Win	Lose	Draw
0.3	0.03	

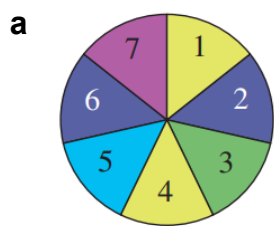
0.67

d

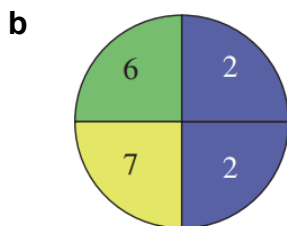
Win	Lose	Draw
0.7		0.07

0.23

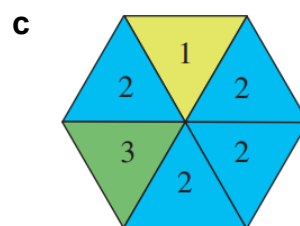
12 Calculate $P(\text{not } 2)$ for each of these spinners as a simplified fraction.



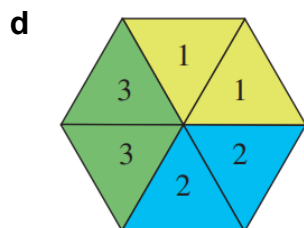
$$\frac{6}{7}$$



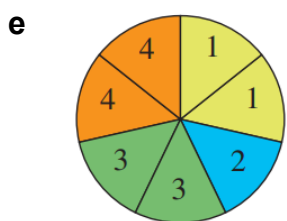
$$\frac{1}{2}$$



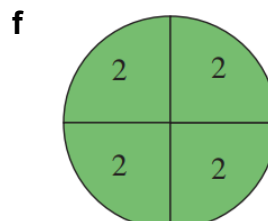
$$\frac{1}{3}$$



$$\frac{2}{3}$$



$$\frac{6}{7}$$



$$0$$

13 The probability of each outcome, A to D, is shown in the table.

Outcome	A	B	C	D
Probability	0.3	0.24		

The probability of outcome C is the same as outcome D.

What is the probability of getting outcome B or C?

Let probability of C and probability of D be x

$$0.3 + 0.24 + x + x = 1$$

$$x = 0.23$$

$$P(B \text{ or } C) = 0.24 + 0.23 = 0.47$$

14 The table shows the probabilities of three different outcomes. Explain why this table of outcomes must be incorrect.

Outcome	A	B	C
Probability	1.3	-0.2	-0.1

Probability can't be negative

15 A bag contains marbles. When one marble is chosen at random from the bag, the probability that it is red is $\frac{7}{19}$ and the probability that it is blue is $\frac{4}{19}$. What is the probability that it is neither red nor blue?

$$\frac{8}{19}$$

16 A bag contains coins. When a coin is chosen at random from the bag, the probability that it is \$1 is $\frac{3}{10}$ and the probability that it is \$2 is $\frac{2}{5}$. What is the probability that it is not a gold coin?

$$\frac{3}{10}$$

- 17 A spinner with three colours (black yellow white) is constructed with these probabilities:

$$P(\text{Black}) = 0.2 \quad P(\text{Yellow}) = 0.4$$

- a Calculate $P(\text{not black})$

0.8

- b Calculate $P(\text{white})$

0.4

- c Explain why $P(\text{not black nor yellow})$ is the same as $P(\text{white})$ with reference to complementary events.

Since there are only 3 colours, *not black nor yellow* is the same as *white* as they are complementary events.

- 18 Two teams play a game of football.
The probability that team A wins is 0.3.
The probability that team B wins is 0.23.
What is the probability of a draw?

0.47

- 19 A spinner has 20 sections. Each section is labelled A, B, C or D.
The table shows the probability of spinning each of the letters.

Letter	A	B	C	D
Probability	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{2}{5}$	

- a What is the probability of spinning the letter A or B?

$\frac{3}{10}$

- b What is the probability of spinning the letter D?

$\frac{3}{10}$

- c How many sections are labelled C?

8

- 20 The table shows the probabilities of students being picked at random from each year group.

Year 7	Year 8	Year 9	Year 10	Year 11
0.3	0.15	0.1	0.05	x

A student is selected at random. Find these probabilities.

- a $P(\text{Year 11})$

0.4

- b $P(\text{Year 7 or Year 8})$

0.45

- c $P(\text{not Year 9})$

0.9

21 A fair six-sided dice is rolled. What is the probability of not rolling a factor of 6?

$\frac{1}{3}$

22 There are 50 marbles in a bag. What is the probability of getting a white marble?
Give your answer as a percentage.

Marble	Number of marbles
Blue	20
Red	12
Orange	4
White	

28%

23 A box contains different coloured counters, with
 $P(\text{purple}) = 10\%$, $P(\text{yellow}) = 0.6$ and $P(\text{orange}) = \frac{1}{7}$.

Is it possible to obtain a colour other than purple, yellow or orange? If so, state the probability.

Yes, as the probabilities don't sum to 1.

24 A bag contains blue, green, and red counters.

a

Colour	Green	Red	Blue
P(colour)	0.3	0.15	???

P(green or red)

P(blue)

0.45

0.55

b

Colour	Green	Red	Blue
P(colour)	$\frac{3}{4}$	$\frac{1}{12}$	???

P(green or red)

P(blue)

$\frac{5}{6}$

$\frac{1}{6}$

c

Colour	Green	Red	Blue
P(colour)		0.7	

Green is twice as likely to be picked than blue

P(green)

P(blue)

0.2

0.1

d

Colour	Green	Red	Blue
P(colour)	11%	70%	19%

P(green or red)

P(blue)

P(not green nor red)

What do you notice?

88%

19%

19%

Not green nor red is the same as blue

- 25** A bucket containing a number of counters with four different colours (green, blue, orange, and red) are picked out of a container. The following probabilities are given.

- $P(\text{green}) = \frac{6}{15}$
- $P(\text{blue}) = \frac{1}{5}$
- $P(\text{red}) = \frac{1}{3}$

For this question, write all the probabilities as fractions:

- State the event complementary to picking blue.
- Calculate $P(B')$
- Calculate $P(O)$
- Calculate $P(O')$

Not blue

$\frac{4}{5}$

0

1

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The table shows the relative frequency of selecting each of the different coloured jelly beans from packets containing green, yellow, black, red and white jelly beans.

- What is the relative frequency of selecting a red jelly bean?

0.17

- Based on this table of relative frequencies, what is the probability of NOT selecting a black jelly bean?

<i>Colour</i>	<i>Relative frequency</i>
Green	0.32
Yellow	0.13
Black	0.14
Red	
White	0.24

0.86

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There are 100 tickets sold in a raffle. Justine sold all 100 tickets to five of her friends. The number of tickets she sold to each friend is shown in the table.

- Justine claims that each of her friends is equally likely to win first prize. Give a reason why Justine's statement is NOT correct.

Each person has a different number of tickets

- What is the probability that first prize is NOT won by Khalid or Herman?

<i>Friend</i>	<i>Number of tickets</i>
Danielle	45
Khalid	5
Nancy	10
Shani	14
Herman	26
Total	100

$\frac{69}{100}$

CHECK YOUR UNDERSTANDING

Complementary outcomes

- 1** A bag contains strawberry sweets and orange sweets. The probability of choosing a strawberry sweet is 0.8. What is the probability of choosing an orange sweet?

a 0.5 **b** 0.8 **c** 0.2 **d** Impossible to know

- 2** A bag contains strawberry sweets, orange sweets, and blackcurrant sweets. The probability of choosing a strawberry sweet is 0.8. What is the probability of not choosing a strawberry sweet?

a $\frac{1}{3}$ **b** 0.1 **c** 0.2 **d** Impossible to know

- 3** Some letter cards are placed in a box. Each card is labelled either P, Q, or R. The probabilities are shown on the table. What is the probability of getting an R?

Letter	P	Q	R
Probability	$\frac{3}{5}$	$\frac{1}{10}$	

a $\frac{4}{15}$ **b** $\frac{7}{10}$ **c** $\frac{6}{10}$ **d** $\frac{3}{10}$

- 4** A bag contains strawberry sweets, orange sweets, and blackcurrant sweets. The probability of choosing a strawberry sweet is 0.8. What is the probability of choosing an orange sweet?

a $\frac{1}{3}$ **b** 0.1 **c** 0.2 **d** Impossible to know