Student Name

Mathematics Stage 4

PROBABILITY

Book 1 Determine probabilities of chance experiments

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CONTENTS

OVERVIEW

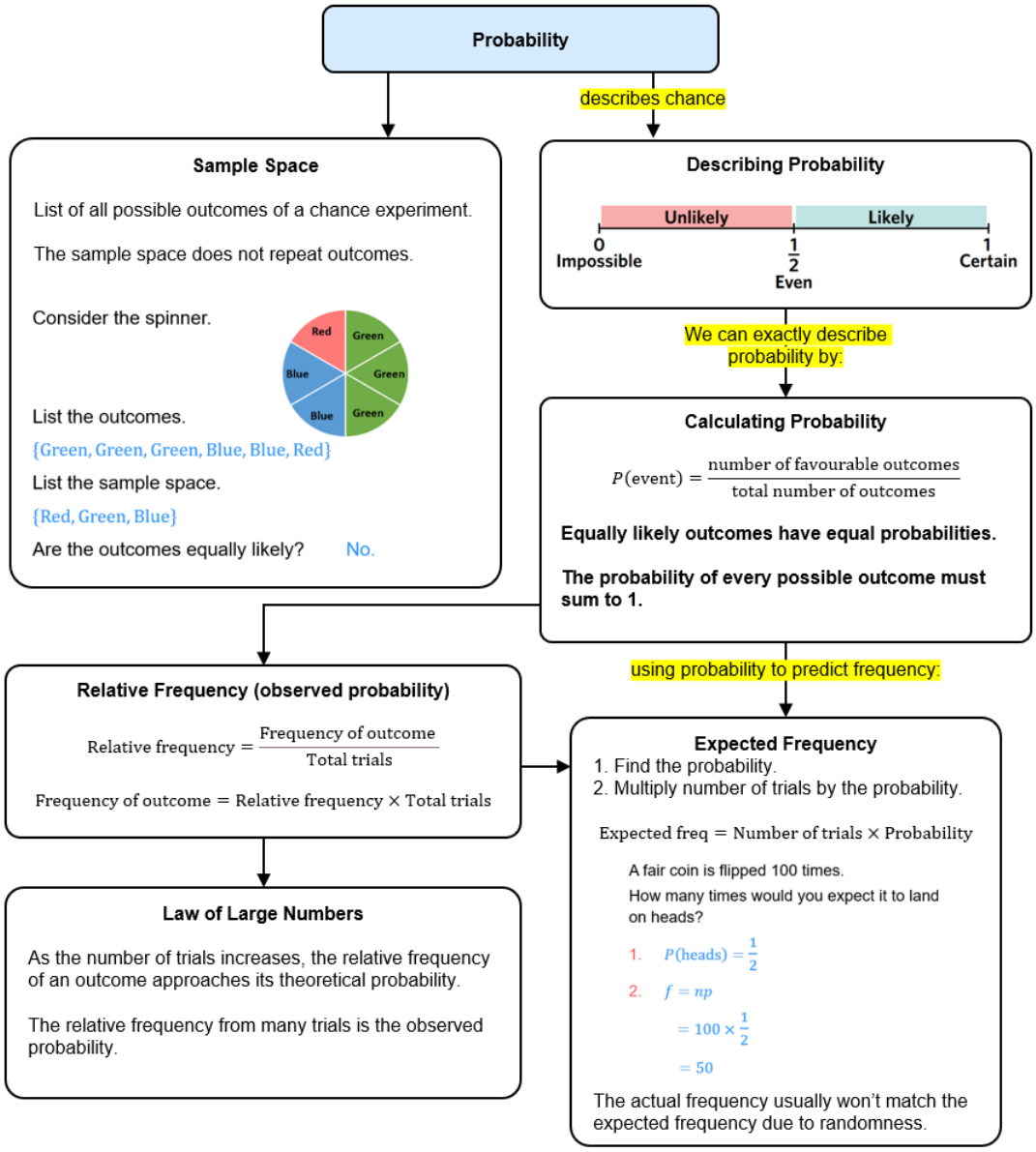
SYLLABUS CONTENT

MA4-PRO-C-01 solves problems involving the probabilities of simple chance experiments

Determine probabilities for chance experiments

- List the sample space for chance experiments
- Express the probability of an event, which has a finite number of equally likely outcomes, as $P(\text{event}) = \frac{\text{number of favourable events}}{\text{total number of outcomes}}$
- Recognise that probabilities range from 0 (impossible) to 1 (certain) and that equally likely outcomes have equal probabilities
- Verify that the total of the probabilities of all possible outcomes of an event is 1
- Identify and describe theoretical (expected) probabilities as being the likelihood of outcomes occurring under fair or unbiased conditions
- Explain that observed probability is the relative frequency resulting from repeated trials of a simulation and determine observed probabilities
- Explore relative frequencies by using a random number generator to repeat a chance experiment a number of times

SUMMARY

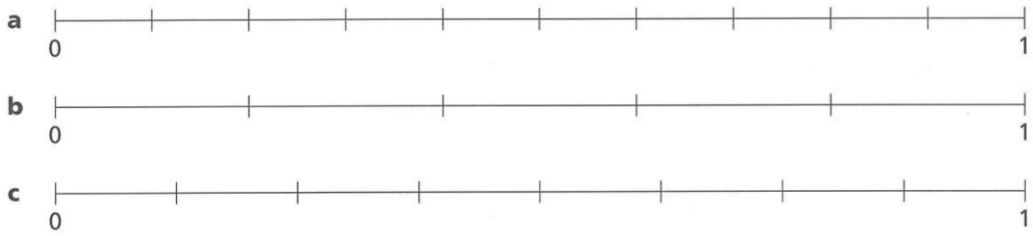


REVIEW OF PRIOR KNOWLEDGE

1 Simplify these fractions.

a $\frac{50}{100}$ **b** $\frac{4}{12}$ **c** $\frac{16}{64}$ **d** $\frac{25}{30}$

2 Complete the intervals on these number lines.



3 Work out the following:

a $\frac{4}{6}+\frac{2}{6}$ **b** $0.2+0.5$ **c** $0.36+0.4$

4 Calculate the following:

a $6\times\frac{2}{3}$ **b** $\frac{3}{5}\times 45$ **c** $\frac{1}{5}\times 35$ **d** $\frac{4}{5}\times 35$

e $\frac{3}{4}\times 28$ **f** $\frac{5}{6}\times 24$ **g** $\frac{1}{9}\times 72$ **h** $\frac{21}{30}\times 60$

5

a A box contains 20 counters. $\frac{1}{4}$ of the counters are blue. How many blue counters are there?

b A box contains 120 counters. $\frac{3}{5}$ of the counters are red. How many red counters are there?

OUTCOMES AND SAMPLE SPACE

- A **chance experiment** is a random activity with different results.
- **Random** means you can't predict the results with certainty.
- An **outcome** is a possible result of the chance experiment.
- The **sample space** lists all possible outcomes without repeats.
- **Equally likely** means two or more events have the same chance of happening.

Example List the sample space of a chance experiment.

A coin is flipped.

List all the outcomes.

Heads, Tails

List the sample space.

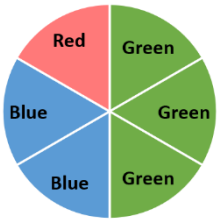
{Heads, Tails}

Is heads and tails equally likely? Yes.

Consider the spinner.

List all the outcomes.

G, G, G, B, B, R



List the sample space.

{G, B, R}

Is each colour equally likely? No.

Consider these chance experiments and answer the questions.

a A standard 6-sided die is rolled.

List the outcomes

1, 2, 3, 4, 5, 6

List the sample space.

{1, 2, 3, 4, 5, 6}

Is each number equally likely?

Yes

b A jar contains 5 red, 6 yellow, and 2 blue lollies. One is picked out at random.

List the outcomes

R, R, R, R ,R, Y, Y, Y, Y, Y, Y, B, B

List the sample space.

{R, Y, B}

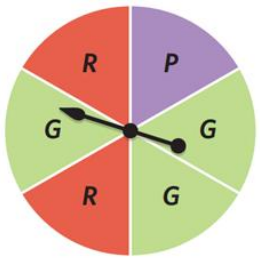
Is each colour equally likely?

No

Key ideas

1. An outcome is a possible …… of a chance experiment.
2. An …… is a collection of one or more outcomes.
3. The sample space is …… possible outcomes, without repeated outcomes.

1 This spinner is spun. The colour of the sector is recorded.



a List all the outcomes

P, G, G, R, G, R

b List the sample space.

$\{R, P, G\}$

c Is each colour equally likely?

no

2 This spinner is spun. The colour of the sector is recorded.

a List all the outcomes

G, G, R, B, B, R

b List the sample space.

$\{G, R, B\}$

c Is each colour equally likely?

yes

d Is purple a possible outcome?

no

3 The letters M A T H S are written on cards and put in a hat.
One card is selected at random.

a List each outcome of this chance experiment.

M, A, T, H, or S

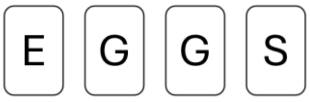
b List the sample space.

$\{M, A, T, H, S\}$

c Is each letter equally likely?

yes

4 The letters E G G S are written on cards and put in a hat.
One card is selected at random.



a List each outcome of this chance experiment.

E, G, G, S

b List the sample space.

$\{E, G, S\}$

c Is each letter equally likely?

no

5 A ball is selected at random from a bag containing 1 black, 1 white and 1 green ball.

a List all the outcomes of this chance experiment.

B, W, G

b List the sample space.

$\{B, W, G\}$

c Is each colour equally likely?

yes

6 A ball is selected at random from a bag containing 2 black, 3 white and 1 green ball.

a List all the outcomes of this chance experiment.

B, B, W, W, W, G

b List the sample space.

$\{B, W, G\}$

c Is each colour equally likely?

no

DESCRIBING PROBABILITY

- An **event** is a collection of one or more outcomes.
- We can describe the chance of an event using these terms:

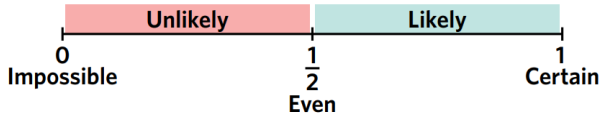
Impossible	Unlikely	Even Chance	Likely	Certain
Never happens	Happens less than half the time	Happens exactly half the time	Happens more than half the time	Always happens

- **Equally likely** means two or more events have the same chance of happening.

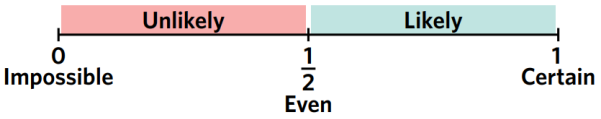
Example Describe the probability of these outcomes.

Circle impossible, unlikely, even, likely, or certain for these events.

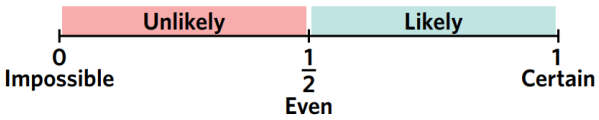
a Tossing 6 heads in a row on a fair coin.



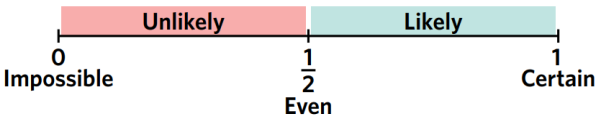
b 60 seconds passing in one minute.



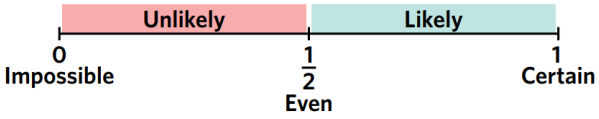
c Rolling an even number on a standard die.



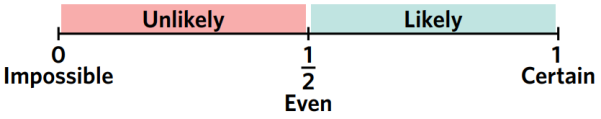
d A person jumping over the moon.



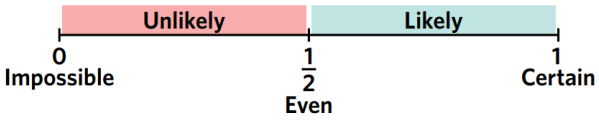
e Flipping a coin and getting tails.



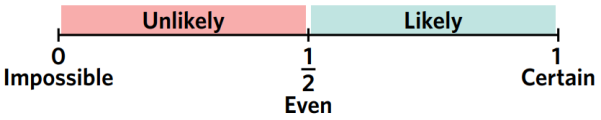
f Picking whether a person has an object in their left or right hand.



g The sun rises tomorrow morning.



h Winning the lottery.



Key ideas

1. We can describe chance using the words ……

1 State whether these events are impossible, unlikely, even chance, likely, or certain.

a Rolling a fair 6-sided die and landing on a 4.

unlikely

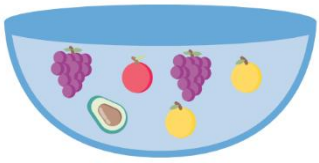
b The time being 11:00 am one hour after 10:00 am.

certain

c Getting 13 questions correct on a test containing 10 questions.

impossible

d Picking a banana at random out of the following fruit bowl.



unlikely

2 Consider a fair 6-sided die with the numbers 1 to 6 on it.

Answer true or false to each of the following.

a Rolling a 3 is unlikely.

true

b Rolling a 5 is likely.

false

c Rolling a 4 and rolling a 5 are equally likely events.

true

d Rolling an even number is likely.

false

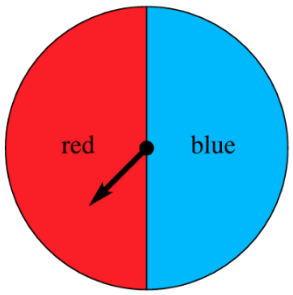
e There is an even chance of rolling an odd number.

true

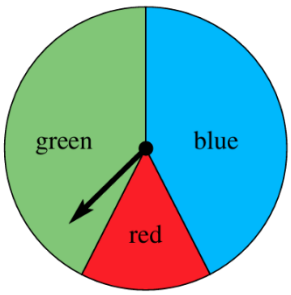
f There is an even chance of rolling a multiple of 3

false

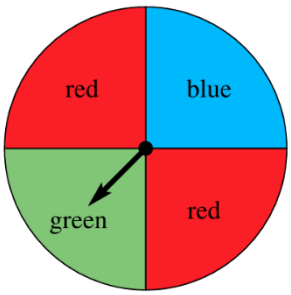
3 Match each spinner with the description.



Spinner 1



Spinner 2



Spinner 3

a Has an even chance of red, but blue is unlikely.

Spinner 3

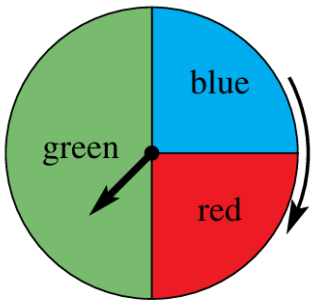
b Blue and green are equally likely, but red is unlikely.

Spinner 2

c Has an even chance of blue, and green is impossible.

Spinner 1

4 Consider the spinner shown, which is spun and could land with the arrow pointing to any of the three colours.



a State whether each of the following is true or false.

i There is an even chance that the spinner will point to green.

true

ii It is likely that the spinner will point to red.

false

iii It is certain that the spinner will point to purple.

false

iv It is equally likely that the spinner will point to red or blue.

true

v Green is twice as likely as blue.

true

b Use the spinner to give an example of:

i an impossible event

any colour other than green, blue or red

ii a certain event

landing on either green, blue, or red

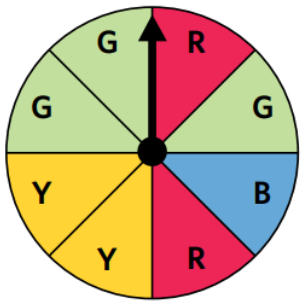
iii a likely event

landing on green or blue / landing on green or red

iv two events that are equally likely

landing on blue, landing on red

5 The spinner below is spun.



a List all the possible outcomes

R, R, G, G, G, B, Y, Y

b List the sample space

$\{R, G, B, Y\}$

Answer true or false to the following statements.

c Green is three times more likely than red.

false

d Yellow and green have equal chance.

false

e Yellow and red have equal chance.

true

f Green is likely.

false

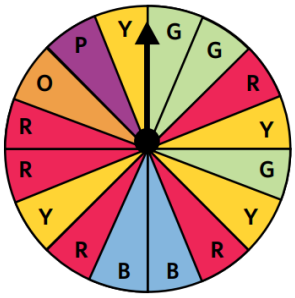
g Green is unlikely.

true

h Red is twice as likely as blue.

true

6 Fill in the blanks for the statements related to the spinner.



a Landing on green is …… times as likely than landing on purple.

3

b Landing on red is …… times as likely than landing on orange.

5

c Landing on …… is two times as likely than landing on blue.

yellow

d Landing on orange has the same chance as landing on …….

purple

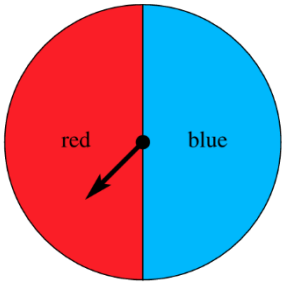
e Landing on either red or …… has an even chance of occurring.

green

f Landing on red has the same chance as landing on either green or ……

blue

7 Explain why tossing a coin and this spinner are equivalent chance experiments.



Both have only two outcomes that are equally likely

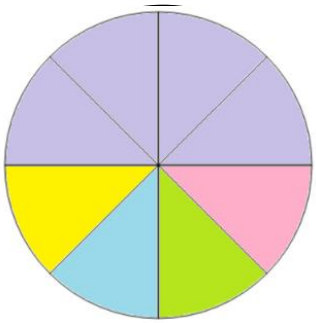
8 Explain the difference between an outcome having *even chance* and two or more outcomes being equally *likely*.

Even chance means a probability of exactly 50%.

Equally likely means that multiple outcomes have the same probability as each other, but that probability can be any value, not necessarily 50%.

9 Create a spinner with all these conditions.

- 8 equally sized sectors.
- 5 colours, purple, pink, green, blue, yellow.
- An even chance of landing on purple.
- The chance of blue and yellow are the same.



CALCULATING PROBABILITY

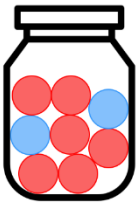
- An **event** is a collection of one or more outcomes.
- **Theoretical probability** is a number showing how likely an event is.
- $P(\text{event})$ means "probability of the event happening".

Theoretical Probability
$P(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
Each outcome must be equally likely.

- Probability of an event is always between **0 and 1**.
- All probabilities **add up to 1**.

Example Calculate theoretical probability.

A jar of marbles contains
6 red marbles and
2 blue marbles.



Find the probability of picking a red marble.

$$P(\text{red}) = \frac{6}{8} = \frac{3}{4}$$

Find the probability of picking a blue marble.

$$P(\text{black}) = \frac{2}{8} = \frac{1}{4}$$

The total number of outcomes is 8. Why?

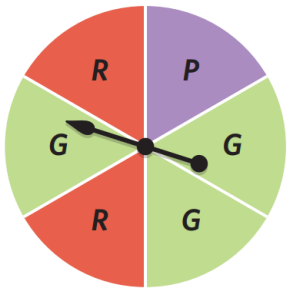
There are …… marbles in total.

Why is $P(\text{Red}) = \frac{6}{8}$?

What do you notice about $P(\text{red}) + P(\text{blue})$?

The sum of $P(\text{red}) + P(\text{blue})$ is ……, this is because it is certain you will pick either …… or ……

A spinner with equal sections is spun. Find:



a $P(\text{G})$ $\frac{1}{2}$

b $P(\text{P})$ $\frac{1}{6}$

c $P(\text{R})$ $\frac{1}{3}$

d Sum of probabilities 1

A jar contains 8 blue, 4 orange, and 6 red marbles. Calculate:

a $P(\text{blue})$ $\frac{4}{9}$

b $P(\text{orange})$ $\frac{2}{9}$

c $P(\text{red})$ $\frac{1}{3}$

d $P(\text{blue}) + P(\text{orange}) + P(\text{red})$ 1

Key ideas

1. We can express probability as a number between …… and ……
2. The probability of every possible outcome must sum to ……
3. …… …… outcomes have the same probability.

1 Calculate the probability of these events.

a Event: selecting a black ball

Black balls: ☐ Total balls: 10

Fraction: $\frac{\square}{\square}$

$\frac{3}{10}$

b Event: selecting a black card

Black cards: 5 Total cards: ☐

Fraction: $\frac{\square}{\square}$

$\frac{5}{8}$

2 The spinner below is fair. Find the probability of each event.

a ..... out of ..... sections are red. $P(\text{red}) =$

$\frac{3}{8}$

b ..... out of ..... sections are green. $P(\text{green}) =$

$\frac{2}{8} = \frac{1}{4}$

c ..... out of ..... sections are yellow. $P(\text{yellow}) =$

$\frac{3}{8}$

d Which events are equally likely?

Landing on Red, Landing on Yellow

3 Consider a fair six-sided die.

a List all the outcomes.

$\{1, 2, 3, 4, 5, 6\}$

b List the odd numbers on the die.

$\{1, 3, 5\}$

c Calculate the probability of rolling an odd number.

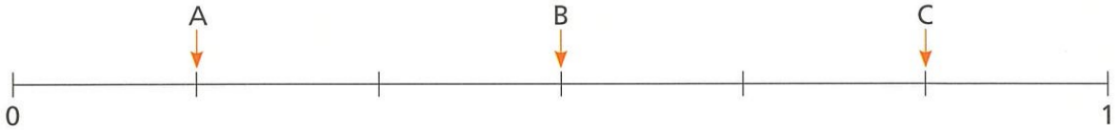
$P(\text{odd}) =$

$\frac{3}{6} = \frac{1}{2}$

4 Here is a probability scale. Three probabilities are labelled A, B and C.

A fair six-sided dice numbered 1 to 6 is rolled.

Write down the letter that matches each of these probabilities.



a The probability of rolling an even number

B

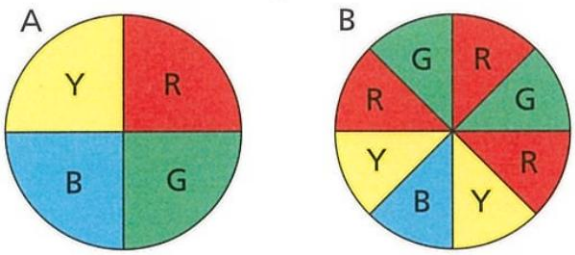
b The probability of rolling a 3.

A

c The probability of rolling a number greater than 1.

C

7 Here are two fair spinners.

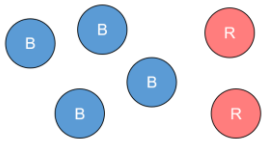
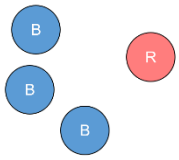
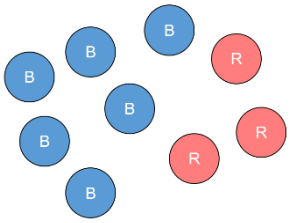
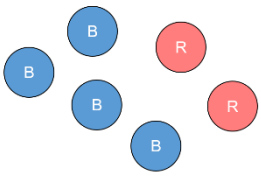


Jackson says, “The chance of spinning blue on each spinner is the same because there is one blue section on each spinner.” Explain why Jackson is incorrect.

The size of the section are different.

For spinner A, $P(B) = \frac{1}{4}$, for spinner B, $P(B) = \frac{1}{8}$

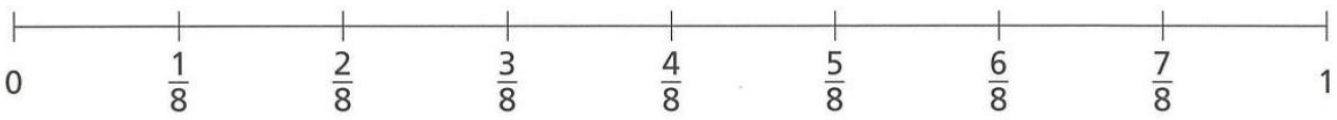
5 Which box you do you have more chance of picking a red marble out of?
Determine the answer by calculating the probability.

a	Box A  $\frac{2}{6} = \frac{1}{3}$	Box B  $\frac{1}{4}$	More chance of red: A
b	Box A  $\frac{3}{9} = \frac{1}{3}$	Box B  $\frac{2}{6} = \frac{1}{3}$	More chance of red: Both have same chance

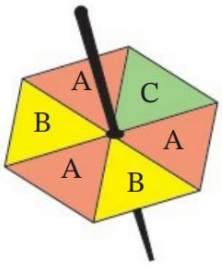
6 A bag contains 8 counters. 5 of the counters are red and 3 of the counters are blue.
On the probability scale, mark:

a $P(\text{red})$ **b** $P(\text{green})$ **c** $P(\text{blue})$

P(Green) P(blue) P(red)



7 The spinner is spun. Calculate:



a $P(\text{A}) =$	**b** $P(\text{C}) =$	**c** $P(\text{B}) =$	**d** $P(\text{D}) =$
$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	0

e Do all probabilities sum to 1?

yes

f Which event is the most likely?

Getting an A

8 A jar of lollies contains 6 red, 4 green, 7 yellow, and 3 blue lollies.

A lolly is selected from the jar at random.

Calculate the probabilities of these events as fractions, decimals, and percentages.

	Fraction	Decimal	Percentage
a $P(\text{green})$	$\frac{4}{20} = \frac{1}{5}$	0.2	20%
b $P(\text{yellow})$	$\frac{7}{20}$	0.35	35%
c $P(\text{red})$	$\frac{6}{20} = \frac{3}{10}$	0.3	30%
d $P(\text{blue})$	$\frac{3}{20}$	0.15	15%

e Do all probabilities sum to 1?

yes

9 A bag of marbles contains 3 red marbles, 2 green marbles and 5 blue marbles.
A marble is chosen at random.

a What is the probability that a red marble is chosen?

$\frac{3}{10}$

b Explain why your answer to part **a** is not $\frac{1}{3}$.

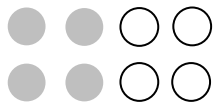
Each colour is not equally likely

c What is the probability that a blue marble is chosen? Answer as a percentage.

50%

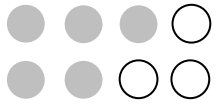
10 For each of these scenarios, calculate the probability of choosing a grey counter as a decimal (you may use a calculator)

a



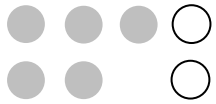
0.5

b



0.625

c



$0.\overline{714285}$

d

$0.\dot{6}$

e

1

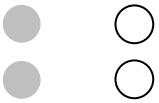
f

1

g

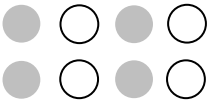
$0.\dot{6}$

h



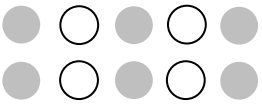
0.5

i



0.5

j



0.6

11

a What two numbers must the probability of a number be between?

0 and 1

b Circle which of these numbers could represent a probability.

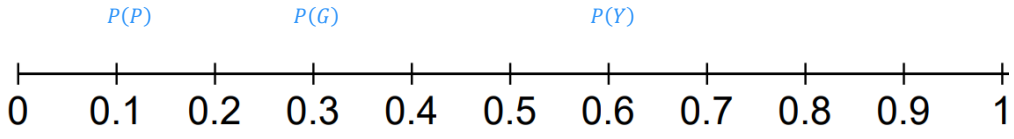
0.3	1	0.43045783	$1\frac{2}{3}$
-0.3	2	1.43045783	$-\frac{2}{3}$
1.3	-1	-0.43045783	$\frac{3}{2}$
0.000003	$\frac{2}{3}$	$0.\dot{4}$	$\frac{43}{51}$

0.3, 1, 0.43045783

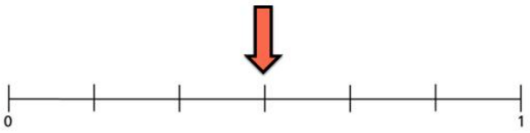
$0.000003, \frac{2}{3}, 0.\dot{4}, \frac{43}{51}$

12 Consider this spinner. Mark on the number line the probability of spinning:

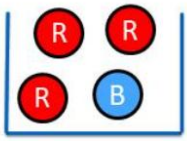
a Yellow | **b** Green | **c** Purple



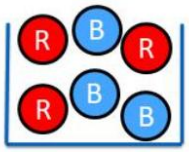
13 Tom and Katie want to place counters in a bag so the probability of picking a red is here on the number line.



Tom's bag



Katie's bag



Who do you agree with, and why?

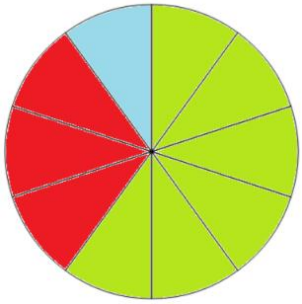
The arrow points to $\frac{1}{2}$

Katie is correct as half the counters in her bag are red.

14 A fair spinner is split into 10 sections that are either red, blue or green.

The probability of the spinner landing on each colour is shown on the scale.

Complete the diagram to show what the spinner might look like.



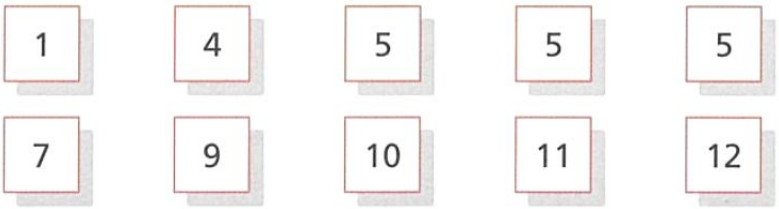
15 **NAPLAN A**

A standard six-sided die is rolled. What is the probability that it lands on a factor of 4?

Factors of 4 are 1, 2, 4

$\frac{3}{6} = \frac{1}{2}$

16 Mandy puts these numbers cards into a bag.



She removes a card without looking. First calculate the probability then use an appropriate word to describe the probability of each outcome:

a a number greater than 6

$\frac{1}{2}$, even chance

b the number 1

$\frac{1}{10}$, unlikely

c a number less than 20

1, certain

d the number 30

0, impossible

e a number above 2

$\frac{9}{10}$, likely

f a number below 8

$\frac{3}{5}$, likely

17 Some cards have some letters and symbols on them.

A card is chosen at random. What is the probability that the card:

a has the letter A on it

$\frac{1}{2}$

b has a square on it

$\frac{3}{8}$

c has both an A and a square on it

$\frac{1}{4}$

18 Adam has spread out a pack of playing cards.

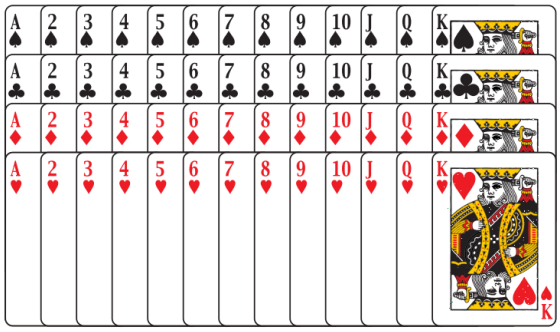
He picks a card at random.

Hearts and diamonds are red.

Clubs and spades are black.

Picture cards are king, queen and jack.

Find:



a P(spade)

$\frac{1}{4}$

b P(4 of hearts)

$\frac{1}{52}$

c P(club)

$\frac{1}{4}$

d P(black 10)

$\frac{1}{26}$

e P(ace)

$\frac{1}{13}$

f P(picture card)

$\frac{3}{13}$

g P(5 or 6)

$\frac{2}{13}$

h P(green card)

0

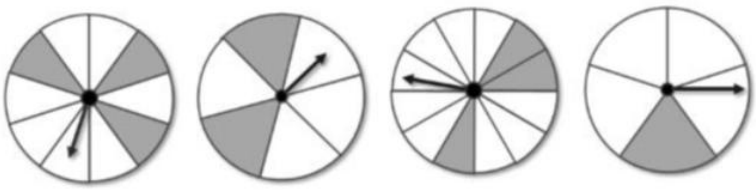
i P(not a picture card)

$\frac{10}{13}$

19 20000 people enter a competition. There are going to be 10 winners.
Jakub says that there is less than a 1% chance of winning the competition. Is Jakub correct?

Yes, $\frac{10}{20\,000}$ is $\frac{1}{2000}$, which is less than 1%

20 Which spinner has the highest probability of landing on grey?



A B C D

$A: \frac{3}{10} = 0.3$

$B = \frac{2}{5} = 0.4$

$C = \frac{3}{12} = 0.25$

$D = \frac{1}{5} = 0.2$

Spinner B

21 A bag contains various coloured marbles. Some are red, some are blue, some are yellow and some are green. You are told that P(red) = $\frac{1}{2}$, P(blue) = $\frac{1}{4}$ and P(yellow) = $\frac{1}{6}$.
You are not told the probability of selecting a green marble.

a If there are 24 marbles:

i Find how many marbles there are of each colour.

Red = 12, Blue = 6, Yellow = 4, Green = 2

ii What is the probability of getting a green marble?

$\frac{1}{12}$

b If there are 36 marbles:

i Find how many marbles there are of each colour.

Red = 18, Blue = 9, Yello = 6

ii What is the probability of getting a green marble?

$\frac{1}{12}$

c What is the minimum number of marbles in the bag?

LCM of 2, 4, 6, is 12

22 **NAPLAN A**

Sharon made 24 milkshakes at a party. The milkshakes were either vanilla or chocolate.
All the milkshakes were served in an aluminium cup and looked the same.
Murray took one milkshake and had a 1 in 8 chance of taking a vanilla milkshake.
How many chocolate milkshakes did Sharon make?

Vanilla: $\frac{1}{8} \times 24 = 3$

So, there are 24 – 3 = 21 choccy milkshakes

23 Ally has 20 numbered cards.
The probability of choosing a card less than 3 is shown on the scale.

How many more cards less than 3 do you need to add so that the probability of getting a number less than 3 is 0.5? Show all your working.

Currently the probability is 0.3, $0.3 \times 20 = 6$

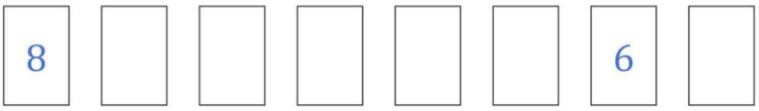
There are 6 cards less than 3.

Currently the fraction is $\frac{6}{20}$

$\frac{6+x}{20+x} = \frac{1}{2}$

Need to add 6 more cards to make $\frac{12}{24} = \frac{1}{2}$

24 Elliott has eight numbered cards.



One of the cards is chosen at random. Elliott says:

- The probability of an 8 is $\frac{1}{4}$
- The range of numbers is 5.
- The probability of a number greater than 10 is 0.
- The probability of 7 is $\frac{1}{2}$

Find the six missing numbers.

Missing numbers are 8, 3, 7, 7, 7, 7

25 A bag contains some green marbles, blue marbles, and white marbles.

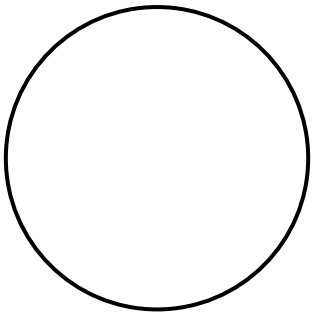
- The probability of picking blue is $\frac{1}{2}$
- There are 5 green marbles.
- There are 4 white marbles.

How many blue marbles are there?

The 5 green and 4 white marbles make up the other $\frac{1}{2}$ of the bag. So, 9 blue.

26 Construct a spinner with 4 colours (red, green, blue, and yellow) that meets these conditions:

- The probability of landing on blue is $\frac{1}{3}$
- There is an equal chance of landing on red and green.
- Landing on yellow is twice and likely to occur as landing on red.



Let $P(R) = P(G) = x$

$P(B) + P(R) + P(G) + P(Y) = 1$

$\frac{1}{3} + x + x + 2x = 1$

$x = \frac{1}{6}$

Therefore the spinner should be

$\frac{1}{3}$ Blue, $\frac{1}{6}$ Red, $\frac{1}{6}$ Green, $\frac{1}{3}$ Yellow

27 **2011 HSC Standard 2 Band 4**

Which of the following could be the probability of an event occurring?

(A) 1

(B) $\frac{6}{5}$

(C) 1.27

(D) 145%

A

28 2008 HSC Standard 2 Band 4

A bag contains some marbles.
The probability of selecting a blue marble at random from this bag is three eighths.
Which of the following could describe the number of marbles in the bag?

(A) 3 blue, 8 red

(B) 6 blue, 11 red

(C) 3 blue, 4 red, 4 green

(D) 6 blue, 5 red, 5 green

D

For option D,

$P(B) = \frac{6}{16} = \frac{3}{8}$

29 2005 HSC Standard 2 Band 4

Four radio stations reported the probability of rain as shown in the table.

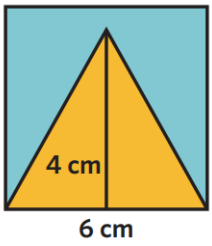
Which radio station reported the highest probability of rain?

Radio station	*Probability of rain*
2AT	0.53
2BW	17%
2CZ	$\frac{13}{25}$
2DL	0.6

Convert all probabilities to decimals.

2DL has the highest probability.

30 A dartboard is shown. Juna throws a dart, and it hits the board.
What is the probability of it landing in the triangular area?



$P(\text{triangle}) = \frac{\text{Triangle Area}}{\text{Square Area}} = \frac{\left(\frac{(4)(6)}{2}\right)}{6^2} = \frac{12}{36} = \frac{1}{3}$

EXPECTED FREQUENCY

- A **trial** is a repetition of a chance experiment; rolling a die 100 times means 100 trials.
- **Frequency** is how often something happens.
- **Expected frequency** is how many times we think an event will happen in certain trials.
- Actual frequency may differ from expected frequency due to randomness.

Expected Frequency
$\text{Expected freq} = \text{Number of trials} \times \text{Probability}$ $f = np$

- To calculate the expected frequency:
  1. Find the probability.
  2. Multiply number of trials by the probability.

Example Calculate expected frequency of an outcome.

A fair coin is flipped 100 times.

How many times would you expect it to land on heads?

1. $P(\text{heads}) = \frac{1}{2}$
2. $f = np$
   $= 100 \times \frac{1}{2}$
   $= 50$

How did we know the probability of heads is one half?

For a fair coin, there are ..... outcomes that are ..... likely.

How did we know to do $100 \times \frac{1}{2}$?

The formula for expected frequency is:

.....

a A fair die is rolled.

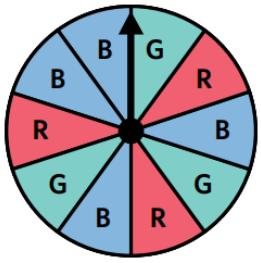
i What is the probability of rolling a 4?

$\frac{1}{6}$

ii What is the expected frequency of it landing on a 4 in 120 trials?

20

b Consider the spinner.



i Calculate the probability of landing on blue.

$\frac{2}{5}$

ii Calculate the expected frequency of the spinner landing on blue from 20 spins.

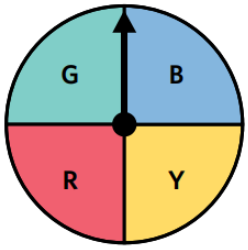
8

Key ideas

1. Expected frequency is how many times we expect an outcome to happen in a given number of .....
2. We calculate the expected frequency of an outcome by multiplying its ..... by the number of trials.

1 Calculate the expected frequency of:

a Landing on red in 20 spins.



Red segments: 1

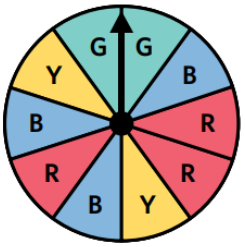
Total number of segments: 4

$Pr(red) = \frac{1}{4}$

Expected occurrence: $20 \times \frac{\square}{4} = \square$

5

b Landing on blue in 20 spins.



Blue segments: 3

Total number of segments: 10

$Pr(blue) = \frac{3}{10}$

Expected occurrence: $20 \times \frac{\square}{10} = \square$

6

2 A fair 10-sided die has each face numbered from 1–10.
Calculate the expected frequency of each event from the given number of trials.

a Rolling a 1 (30 trials)

$P(\text{Rolling } 1) = \ldots\ldots\ldots\ldots\ldots\ldots$

Expected freq = …… × ……

= ……

$\frac{1}{10} \times 30 = 3$

b Rolling an odd number (50 trials).

$P(\text{Rolling odd}) =$

Expected freq =

$\frac{1}{2} \times 50 = 25$

c Rolling a 12 (100 trials).

0

d Rolling a number 8 or greater (150 trials).

$\frac{3}{10} \times 150 = 45$

e Rolling a 2, 4, 6 or 8 (200 trials).

$\frac{4}{10} \times 200 = 80$

f Rolling a multiple of 3 (600 trials).

$\frac{3}{10} \times 600 = 180$

3 A fair 10-sided die has faces with the numbers: 3 3 3 5 8 8 8 9 10 10.

a Calculate the theoretical probability, as a fraction, of rolling a 3.

$\frac{3}{10}$

b Calculate the expected number of occurrences of rolling a 3 from 20 rolls.

$\frac{3}{10} \times 20 = 6$

c Calculate the theoretical probability, as a decimal, of rolling an odd number.

$\frac{5}{10} = 0.5$

d Calculate the expected number of occurrences of rolling an odd number from 150 rolls.

$0.5 \times 150 = 75$

4

a If I throw a fair coin 10 times, how many times would you expect it to land on heads?

5

b If I throw a fair coin 20 times, how many times would you expect it to land on heads?

10

c If I throw a fair coin 60 times, how many times would you expect it to land on heads?

30

d If I throw a fair coin 600 times, how many times would you expect it to land on heads?

300

e If I roll a fair dice 600 times, how many times would you expect it to land on the number 1?

100

f If I roll a fair dice 300 times, how many times would you expect it to land on the number 1?

50

g If I roll a fair dice 150 times, how many times would you expect it to land on the number 1?

25

h If I roll a fair dice 150 times, how many times would you expect it to land on the number 2?

25

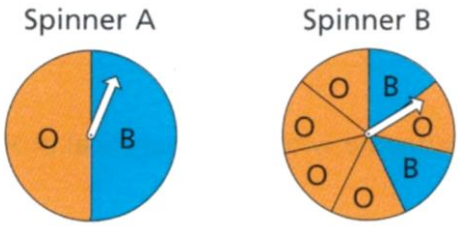
i If I roll a fair dice 1500 times, how many times would you expect it to land on the number 2?

250

j If I throw a fair coin 1500 times, how many times would you expect it to land on heads?

750

5 Each spinner is spun 1400 times.



Estimate how many times

a spinner A will land on blue

700

b spinner B will land on orange.

1000

6 A spinner is divided into sections, with colours orange, blue, green and yellow. Complete the table.

Outcome	Probability	Expected frequency after 800 rolls
orange	$\frac{1}{5}$	
blue	$\frac{9}{25}$	
green	0.37	
yellow	7%	

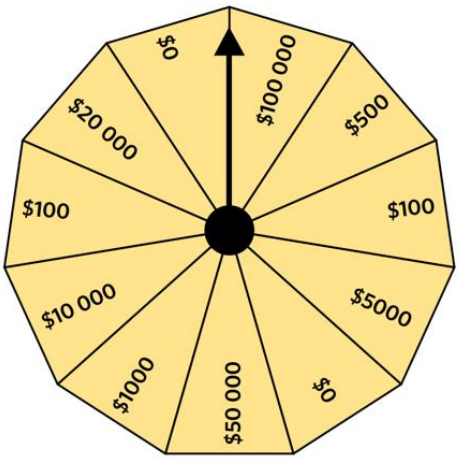
160

288

296

56

7 Wheel of Fortune is a quiz show where winners get to spin a wheel to determine their prize. The spinner is fair and has an equal chance of landing on each segment.



a What is the probability that a winner gets a prize more than $1000?

$\frac{5}{11}$

b In 110 spins of the Wheel of Fortune, how many winners are expected to receive a prize that is $5000 or less?

70

c Out of the first 110 winners, 104 have received a prize that is $5000 or less. Is there a reason to believe that the wheel may be rigged?

It seems likely that the wheel may be rigged as 104 is very different to the 70 that we expect.

d Assuming the wheel is fair, would you rather a $1000 prize or spin the wheel? Why?

Assuming the wheel is fair, it is better to spin the wheel as on average, you expect to gain $16972.73.

8 **2018 HSC Standard 2 Band 3**

An experiment has three distinct outcomes, *A*, *B* and *C*. Outcome *A* occurs 50% of the time. Outcome *B* occurs 23% of the time.

What is the expected number of times outcome *C* would occur if the experiment is conducted 500 times?

$0.27 \times 500 = 135$ times

RELATIVE FREQUENCY

- **Theoretical probability** is what we calculate.
- **Relative frequency,** or **observed probability**, is what actually happens in an experiment**.**
- It is the frequency **related** to total trials.

Relative Frequency
$\text{Relative frequency} = \dfrac{\text{Frequency of outcome}}{\text{Total trials}}$
$\text{Frequency of outcome} = \text{Relative frequency} \times \text{Total trials}$

Example Calculate relative frequency.

Worked Solution	**Your Turn**
A die is rolled 10 times. These numbers came up: $1, 4, 1, 3, 3, 1, 4, 3, 2, 3$	A coin is tossed 13 times. These results came up: $\text{H}, \text{H}, \text{H}, \text{T}, \text{T}, \text{H}, \text{T}, \text{H}, \text{H}, \text{H}, \text{T}, \text{H}, \text{H}$
Find the relative frequency of rolling a 3. $rf = \dfrac{\text{Frequency of 3}}{\text{Total trials}} = \dfrac{4}{10} = \dfrac{2}{5}$	Find the relative frequency of getting heads. $\frac{9}{13}$
Find the relative frequency of rolling an even number. $rf = \dfrac{\text{Frequency of even}}{\text{Total trials}} = \dfrac{3}{10}$	Find the relative frequency of getting tails. $\frac{4}{13}$
If the same experiment were repeated with 500 trials, what would you expect the frequency of 3 to be? $\text{f} = \text{Relative Frequency} \times \text{Trials}$ $= \dfrac{2}{5} \times 500 = 200$	If the same experiment were repeated with 325 trials, how many times would you expect heads to occur? 225

Key ideas

1. Relative frequency is the frequency of an outcome related to the total number of ……
2. We can use relative frequency to …… the expected frequency for a different number of trials.

1 A 6-sided die is rolled 10 times and the following numbers come up:

2 4 6 4 5 1 6 4 3 3

a What is the relative frequency of getting a 3?

$\frac{2}{10} = \frac{1}{5}$

b What is the relative frequency of getting an odd number?

$\frac{4}{10} = \frac{2}{5}$

2 When a coin is tossed 100 times, the results are 53 heads and 47 tails.

a What is the relative frequency of getting a tail?

$\frac{47}{100}$

b What is the theoretical probability of getting a tail?

$\frac{1}{2} = \frac{50}{100}$

c Explain why the theoretical probability and relative frequency do not match.

Random chance means you are unlikely to get the frequencies you expect.

3 A box contains black pens, blue pens and green pens. Abdullah selects a pen at random, notes its colour then puts it back in the box. He does this 20 times. The results are shown in the table.

Outcome	black	blue	green
Frequency	7	5	8

a Explain why the relative frequency of selecting a blue pen is 0.25.

$\frac{5}{20} = \frac{1}{4} = 0.25$

b What is the relative frequency of selecting a pen that is

i Black?

$\frac{7}{20} = 0.35$

ii Green?

$\frac{8}{20} = \frac{2}{5} = 0.4$

c What do you notice about the sum of your relative frequencies?

Adds up to 1. The proportion of every outcome adds up to 100%

4 A survey is conducted on people's television viewing habits.

Number of hours per week	0–5	5–10	10–20	20–30	30+
Number of people	20	10	15	5	0

a How many people participated in the survey?

50

b What is the relative frequency of a participant watching less than 5 hours of television?

$\frac{2}{5}$

c What is the relative frequency of a participant watching 5–20 hours of television?

$\frac{1}{2}$

5 The number of children in some families is recorded in the table shown.

Number of children	0	1	2	3	4
Number of families	5	20	32	10	3

a How many families have no children?

5

b How many families have an even number of children?

40

c How many families participated in the survey?

70

d Based on this experiment, what is the relative frequency of a family having an even number of children?

$\frac{4}{7}$

e If 140 families were surveyed, how many would you expect to have an even number of children?

$\frac{4}{7} \times 140 = 80$

6

a The relative frequency of a student throwing a pen in the bin is 0.5.
A student throws a pen 50 times. How many will be successful?

25

b The relative frequency of a student throwing a pen in the bin is 0.1.
A student throws a pen 50 times. How many will be successful?

5

c The relative frequency of a student throwing a pen in the bin is 1.
A student throws a pen 50 times. How many will be successful?

50

d The relative frequency of a student throwing a pen in the bin is 0.1.
A student throws a pen 100 times. How many will be successful?

10

e The relative frequency of a student throwing a pen in the bin is 0.900.
A student throws a pen 100 times. How many will be successful?

90

f The relative frequency of a student throwing a pen in the bin is 0.15.
A student throws a pen 100 times. How many will be successful?

15

g The relative frequency of a student throwing a pen in the bin is 0.015.
A student throws a pen 100 times. How many will be successful?

1.5

7 Complete the frequency tables and include the relative frequency.

a Coin toss results

Face	*f*	*r.f.*
H	49	$\frac{49}{105} = \frac{7}{15}$
T	56	$\frac{56}{105} = \frac{8}{15}$
$\Sigma f =$	105	

b Dice roll results

Number	Frequency	Relative frequency
1	30	1/4
2	16	2/15
3	14	7/60
4	16	2/15
5	16	2/15
6	28	7/30

8 The table shows how many products a store sold in a day. Complete the table.
Based on the data, if 350 products were sold,
how many would be:

Product	*f*	*r.f.*
Toaster	15	$\frac{15}{70} = \frac{3}{14}$
Kettle	18	$\frac{18}{70} = \frac{9}{35}$
Phone	12	$\frac{12}{70} = \frac{6}{35}$
PC/ Tablet	8	$\frac{8}{70} = \frac{4}{35}$
Other	17	$\frac{17}{70}$
$\Sigma f =$		

a Toasters?

$\frac{3}{14} \times 350 = 75$

b Phones?

$\frac{6}{35} \times 350 = 60$

c Kettles or toasters?

$\frac{9}{35} \times 350 = 90$
$90 + 75 = 165$

d Not toasters?

$350 - 75 = 275$

9 The table to the right shows how many products a store sold in a day.
Pete is running a promotion next week and expects to sell 1300 items.
How much of each product should he make?
Round to the nearest whole.

Product	*f*	*r.f.*
Pies	95	$\frac{95}{440}$
Sausage Rolls	66	$\frac{66}{440}$
Pastries	125	$\frac{125}{440}$
Bread	106	$\frac{106}{440}$
Other	48	$\frac{48}{440}$
$\Sigma f =$		

a Pies

281

b Sausage Rolls

195

c Pastries

369

d Bread

313

e Other

142

10 The numbers of children in the families of a class of students are summarised in the given frequency distribution table.

Number of children	Frequency
1	3
2	9
3	7
4	4
5	2

a What is the relative frequency of a family having 3 children?

$\frac{7}{25}$

b If 100 families were surveyed, how many families would you expect to have 3 children?

28

c What is the relative frequency of a family having 4 children?

$\frac{4}{25}$

d If 500 families were surveyed, how many families would you expect to have 4 children?

80

11 A fair dice is rolled 20 times, and an odd number is rolled every time.
Explain why the 21st roll is equally likely to be either odd or even.

The chance of odd and even for a fair die is always $\frac{1}{2}$

12 A bag contains red, green, and blue marbles.

A marble is selected and replaced from the bag 20 times. The results are:

Red: 8 Green: 10 Blue: 2

Answer these questions using your relative frequency knowledge.

a There are more red marbles than blue. True False Can't tell

Can't tell

b There are 20 marbles in the bag. True False Can't tell

Can't tell

c Are there definitely more green marbles than blue marbles? Yes No

no

d Is it possible that there are more marbles blue than green or red? Yes No

yes

13 A handful of 10 marbles of different colours is placed into a bag. A marble is selected at random, its colour recorded and then returned to the bag. The results are presented in the table.

Red marble chosen	Green marble chosen	Blue marble chosen
21	32	47

a Based on this experiment, how many marbles of each colour do you think there are? Justify your answer mathematically.

Using relative frequencies:

$\text{Red} = \frac{21}{100} \times 10 \approx 2, \text{Green} = \frac{32}{100} \times 10 \approx 3, \text{Blue} = \frac{47}{100} \approx 5$

b For each of the following, state whether it is a possible set of colours for the 10 marbles.

i 3 red, 3 green, 4 blue

yes

ii 2 red, 4 green, 4 blue

yes

iii 2 red, 3 green, 4 blue, 1 purple

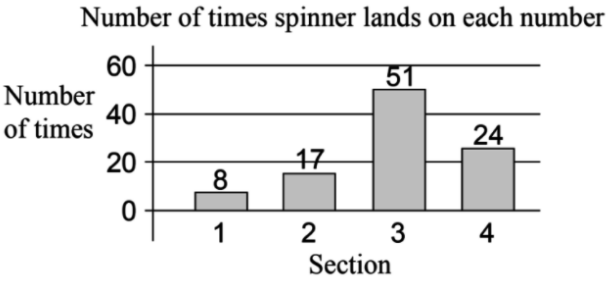
yes

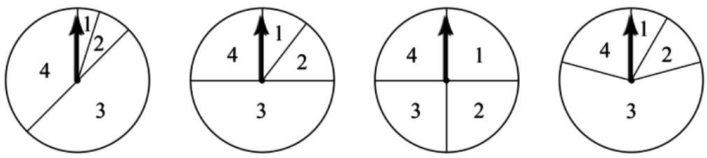
iv 2 red, 0 green, 8 blue

no

14 **NAPLAN Band 7**

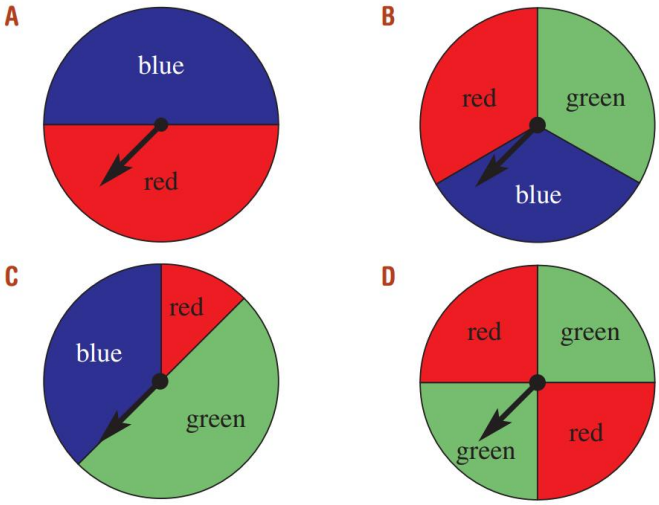
A spinner can land in any of 4 sections, labelled 1 to 4.
The spinner is spun 100 times and the results are recorded in the bar chart below.
Which of these spinners is most likely to give results shown in the graph?





B, 3 is around half and 4 is around a quarter.

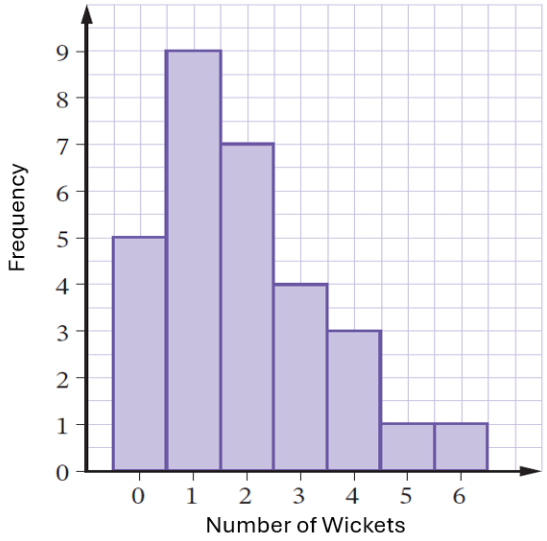
15 Match each of the experiment results **a** to **d** with the most likely spinner that was used (**A** to **D**)



	red	green	blue
a	18	52	30
b	27	23	0
c	20	23	27
d	47	0	53

a – C, b – D, c – B, d – A

16 The graph shows the frequency of wickets taken in a match of cricket.
What is the relative frequency of bowling
2 wickets in a match?



$\frac{7}{30}$

17 NAPLAN A

Which flavour did 4 out of 10 students choose as their favourite?

Favourite flavour	Number of students
Chocolate	75
Vanilla	100
Strawberry	35
Caramel	40

$\frac{4}{10} \times 250 = 100$

Vanilla

18 NAPLAN A

Based on the survey, predict how many people would choose rugby if 2000 people were surveyed.

Sport	Number of people
Netball	49
Aussie Rules	19
Rugby League	135
Hockey	13

$\frac{135}{216} \times 2000 = 1250$

19 2012 HSC Standard 2 Band 5

A spinner with different coloured sectors is spun 40 times. The results are recorded in the table.

What is the relative frequency of obtaining the colour orange, as a decimal?

Colour obtained	*Frequency*
Red	2
Yellow	4
Blue	6
Orange	
Green	10
Purple	12

Frequency of orange = 40 – (the rest) = 6

$\frac{6}{40} = \frac{3}{20} = 0.15$

20 2018 HSC Standard 2 Band 5

Jeremy rolled a biased 6-sided die a number of times. He recorded the results in the table. How many times would he expect to roll a 3 if he rolled the dice 1500 times?

Number	1	2	3	4	5	6
Frequency	23	19	48	20	21	19

$\frac{48}{150} \times 1500 = 480$

21 2011 HSC Standard 2 Band 5

A die was rolled 72 times. The results for this experiment are shown in the table.

Number obtained	*Frequency*
1	16
2	11
3	***A***
4	8
5	12
6	15

a Find the value of A

10

b What was the relative frequency of obtaining a 4?

$\frac{8}{72} = \frac{1}{9}$

c If the die was unbiased, which number was obtained the expected number of times?

You expect each outcome $72 \times \frac{1}{6} = 12$ times

5 was obtained expected number of times

22 2008 HSC Standard 2 Band 5

A die has faces numbered 1 to 6. The die is biased so that the number 6 will appear more often than each of the other numbers. The numbers 1 to 5 are equally likely to occur.

The die was rolled 1200 times and it was noted that the 6 appeared 450 times.

Which statement is correct?

A. The probability of rolling the number 5 is expected to be $\frac{1}{7}$.

B. The number 6 is expected to appear 2 times as often as any other number.

C. The number 6 is expected to appear 3 times as often as any other number.

D. The probability of rolling an even number is expected to be equal to the probability of rolling an odd number.

$rf \text{ of } 6 = \frac{450}{1200} = \frac{3}{8}$

Therefore, all the other numbers have probability $\frac{5}{8}$, so each of the other numbers have probability $\frac{1}{8}$

C

LAW OF LARGE NUMBERS

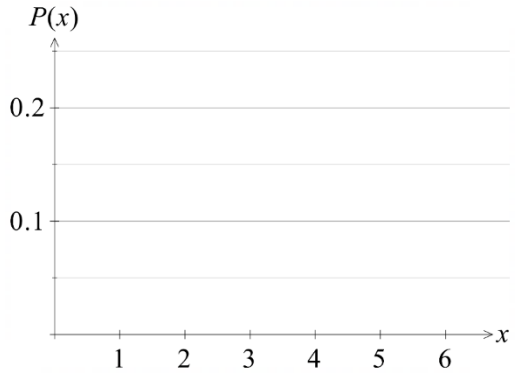
Investigation The Law of Large Numbers

A standard 6-sided die is rolled.

a List the set of possible outcomes: ……

b What is the theoretical probability of each outcome? ……

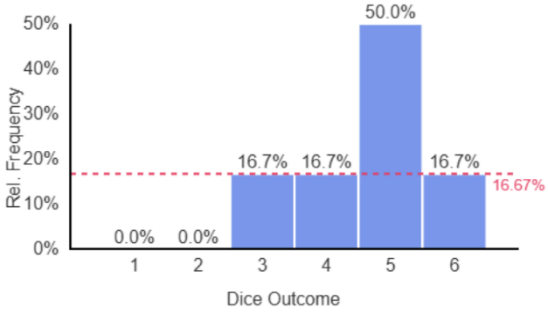
c Draw a histogram of the outcomes and their probabilities.



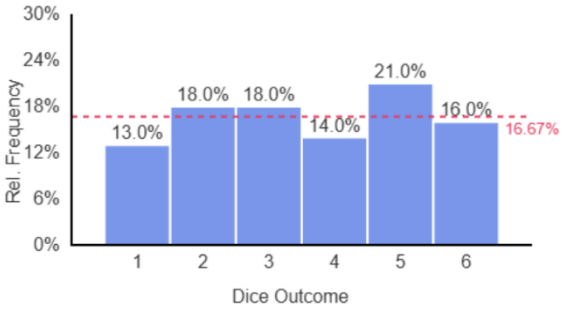
d What would you ***expect*** to happen if you roll a dice 6 times, according to the theoretical probabilities?

https://mrdingmaths.github.io/LawofLargeNumbers/

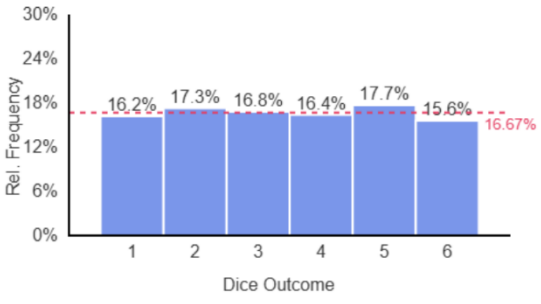
6 rolls



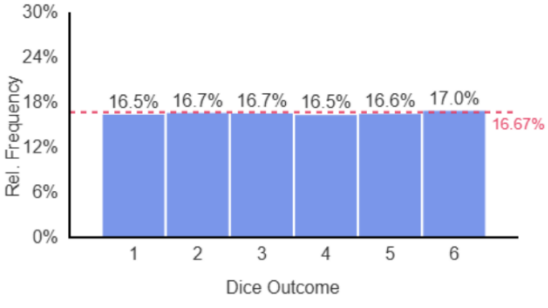
100 rolls



1000 rolls



50000 rolls



ⓘ Law of Large Numbers

As number of trials increase, relative frequency approaches theoretical probability.

Interpret The Law of Large Numbers

The result of a single random experiment is u……

With few trials, randomness means the relative frequency differs to the

t…… p……

The Law of Large Numbers says that as you …… the number of trials, the relative frequency gets closer to the theoretical probability.

Therefore, if we don't know the theoretical probability of an outcome, we can repeat an experiment many times and use the …… …… as an estimate for the theoretical probability. This is called observed probability.

The …… trials, the better the estimate.

1 Experiment A: 10 coins are flipped.
Experiment B: 200 coins are flipped.
The relative frequency of getting heads and tails of each experiment is calculated.
Which survey would have relative frequency closest to the theoretical probability?

a Experiment A **b** Experiment B **c** They would be the same. **d** Impossible to say.

2 One day Nicole sees 6 cars pass her house. 2 of which are blue.

Nicole says: I estimate probability of seeing a blue car pass my house is $\frac{2}{6}$

How can she improve her estimate?

a Simplify her fraction **b** Watch more cars and record her results **c** Find more blue cars **d** Go to a different house and repeat the observations

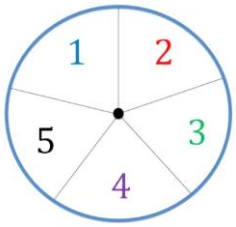
CHECK YOUR UNDERSTANDING

Language of probability

1 Tom says the sample space is $S = \{1,2,3,4,5\}$

Katie says the sample space is $S = \{5,4,3,2,1\}$

Who is correct?

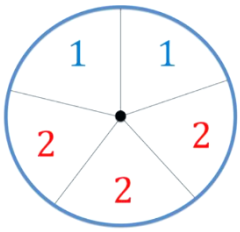


A Only Tom **b** Only Katie **c** Both **d** Neither

2 Tom says the sample space is $S = \{1,2\}$

Katie says the sample space is $S = \{1,1,2,2,2\}$

Who is correct?

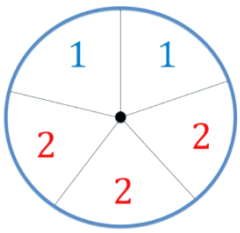


A Only Tom **b** Only Katie **c** Both **d** Neither

3 Tom says there are 2 outcomes.

Katie says there are 5 outcomes.

Who is correct?

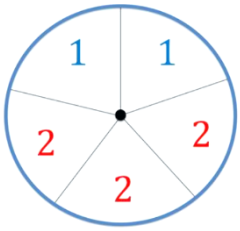


A Only Tom **b** Only Katie **c** Both **d** Neither

4 Tom says you are equally likely to spin a 1 or a 2.

Katie says you are more likely to spin a 1.

Who is correct?



A Only Tom **b** Only Katie **c** Both **d** Neither

5 The sample space for these cards is $S = \{2,4,6\}$

Tom says the last card could be a 6

Katie says the last card could be a 2.

Who is correct?

A Only Tom **b** Only Katie **c** Both **d** Neither

6 Some cards are labelled with numbers. The sample space is $S = \{1,2,3\}$.

How many cards have odd numbers?

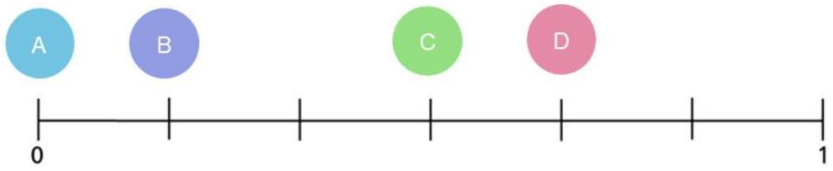
A 1 **b** 2 **c** 3 **d** Impossible to know

Calculating probability

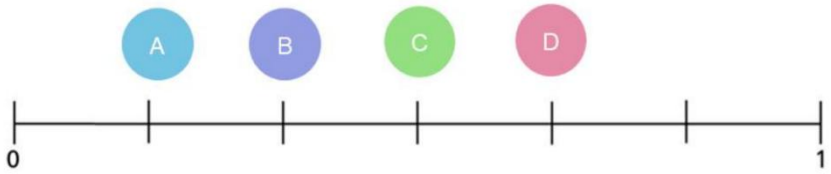
1 Isaac is trying to solve a problem about rolling a dice. What does $P(3)$ mean?

A There were 3 dice **b** The dice was rolled 3 times **c** The number of 3s rolled **d** The probability of rolling a 3

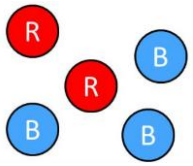
2 Here is a number line from 0 to 1.
Where would the probability of rolling a 4 on a fair die be?

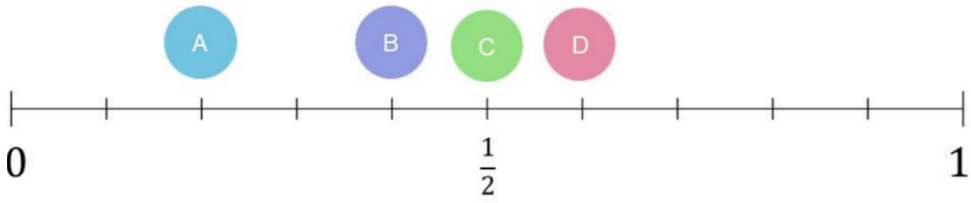


3 Where would the probability of rolling a number greater than 4 on a fair die be?



4 A cup contains some red and blue counters.
Where is the probability of picking a red counter?





Expected frequency

1 If you toss a coin 200 times, how many times would you expect to get heads?

a $\frac{1}{2}$ **b** 50 **c** 100 **d** Even chance

2 If you roll a standard 6-sided die 60 times, how many times would you expect to land on a three?

a 10 **b** 30 **c** 1 **d** 60

3 Sally rolls a fair die 90 times. How many 5s would she expect to get in total?

a $\frac{1}{6}$ **b** 18 **c** 20 **d** 15

4 The probability of tossing a head on a biased coin is $\frac{2}{3}$.
If this coin is tossed 60 times, how many heads would you expect to get?

a 20 **b** 30 **c** 40 **d** 90

5 The probability of winning a game is 0.6.
If you played this game 200 times, how many times would you expect to win?

a 80 **b** 160 **c** 120 **d** 40

6 The probability of winning a game is 0.7.
If you played this game 40 times, how many times would you expect to win?

a 7 **b** 28 **c** 280 **d** 12

Relative frequency

1 Tom misses his morning bus 9 times out of 40 school days. What is the relative frequency of him missing the bus on a school day?

a 7 **b** $\frac{9}{40}$ **c** $\frac{1}{2}$ **d** $\frac{9}{31}$

2 A netball team wins 23 games, draws 10 games, and loses 15 games.

What is the relative frequency of winning?

a 23 **b** $\frac{23}{25}$ **c** $\frac{1}{3}$ **d** $\frac{23}{48}$

3 A die is rolled 40 times. 19 sixes are recorded.

What is the relative frequency of rolling a 6?

a $\frac{1}{6}$ **b** $\frac{19}{21}$ **c** $\frac{19}{40}$ **d** 19

4 A die is rolled 40 times. 19 sixes are recorded.

What is the expected frequency of sixes when rolling the dice 1000 times?

a 167 **b** 475 **c** 19 **d** 25

5 50 people in town are asked if they eat meat.

39 eat meat. 11 do not eat meat.

What is the relative frequency of someone in town not eating meat?

a 11 **b** $\frac{11}{39}$ **c** $\frac{11}{50}$ **d** $\frac{11}{28}$

6 50 people in town are asked if they eat meat.

39 eat meat. 11 do not eat meat.

How many people would you expect to answer no if you asked 300 people?

a 66 **b** 261 **c** 33 **d** 11