Student Name

Mathematics Stage 5 Paths

LINEAR RELATIONSHIPS C

Book 6 Describe translations, reflections in an axis, and rotations through multiples of 90 degrees on the Cartesian plane, using coordinates

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-LIN-P-01 describes and applies transformations, the midpoint, gradient/slope and distance formulas, and equations of lines to solve problems

Describe translations, reflections in an axis, and rotations through multiples of 90 degrees on the Cartesian plane, using coordinates

- Apply the notation P' to name the image resulting from applying a transformation to a point P on the Cartesian plane
- Determine and plot the coordinates for P' resulting from translating P one or more times
- Determine and plot the coordinates for P' resulting from reflecting P in either the x- or y-axis
- Determine and plot the coordinates for P' resulting from rotating P by a multiple of 90° about the origin

TRANSFORMATIONS

- A **transformation** changes a shape's position or orientation.
- The three main transformations are **translation** (slide), **reflection** (flip)**,** and **rotation** (spin).
- We call the **starting shape** the **original**, and the **changed shape** the **image**.
- These transformations are **isometric,** meaning the image is **congruent** to the original.

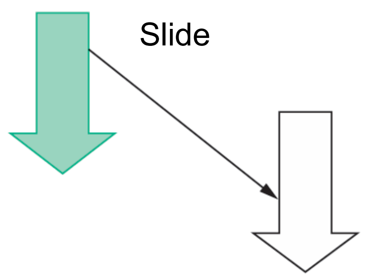
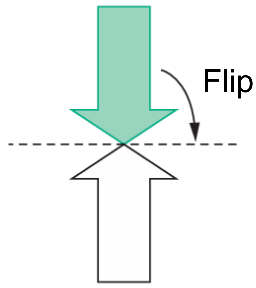
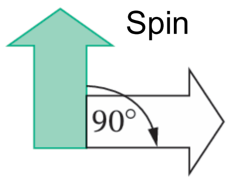
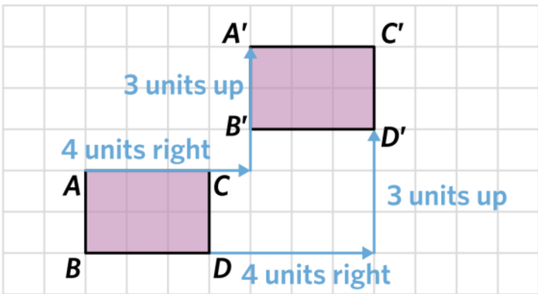
Translation	Reflection	Rotation
		

IMAGE NOTATION

- The image of a point A is written as A'. We pronounce this "A prime."



Key ideas

1. Three transformations are …………………, …………………, ……………………
2. The shape after transformation is called the ……………………
3. Isometric transformations mean the image is ………………… to the original.
4. We write the image of a point P using the notation ………, which is read "P prime"

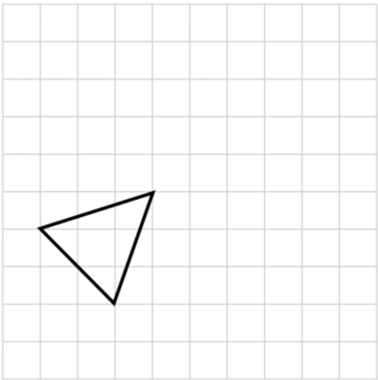
TRANSLATION

Translation: https://www.geogebra.org/m/qnbqfcmp

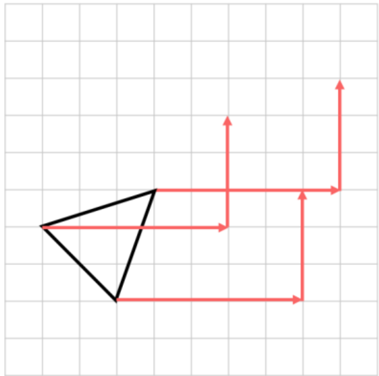
- To perform a translation, slide every point of a shape the same distance left/right and up/down.
- For diagonal translations, combine these movements.

Example Draw images after translation.

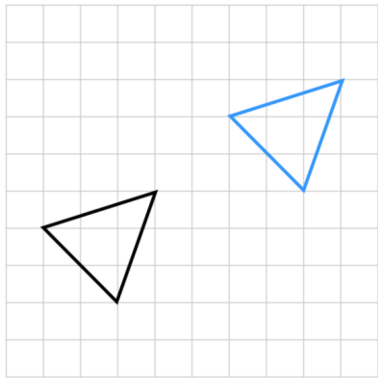
Draw the image after translation 5 units right and 3 units up.



Slide each vertex 5 right 3 up

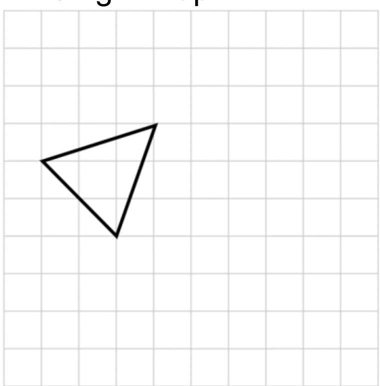


Draw the image

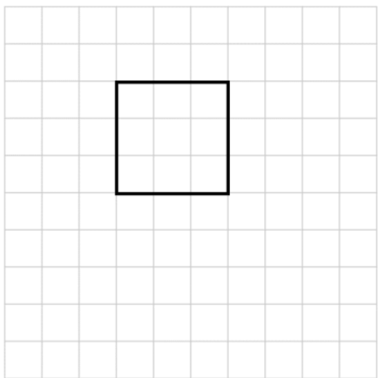


Draw the image after translation:

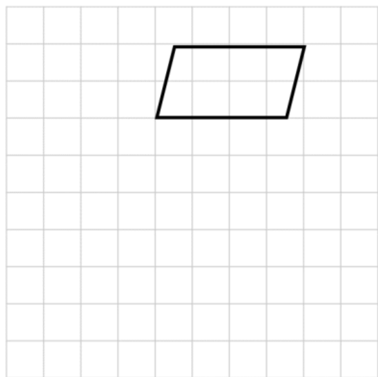
a 3 right 1 up



b 1 left, 3 down

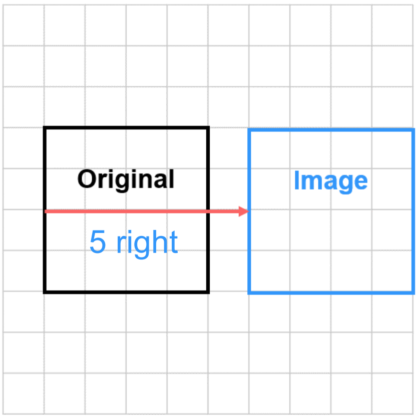


c 5 left, 6 down



1 Describe these translations. The first question has been done for you.

a



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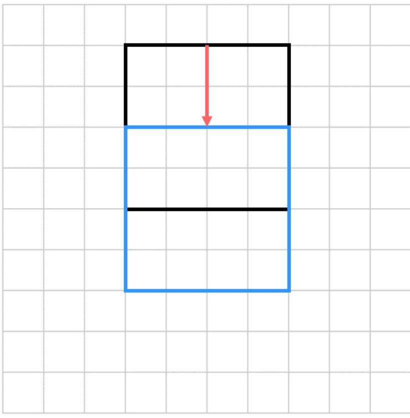

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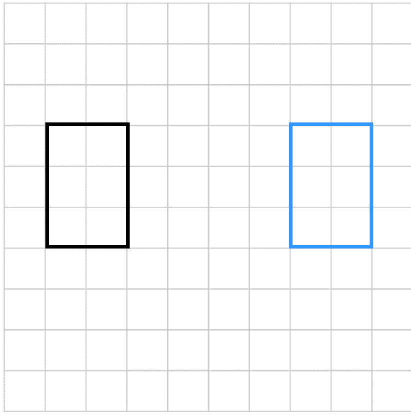
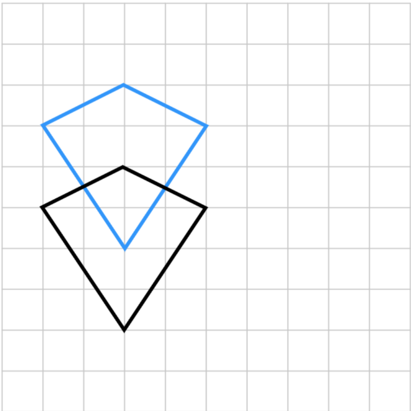


d



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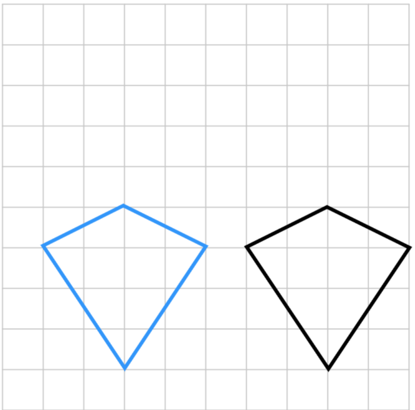

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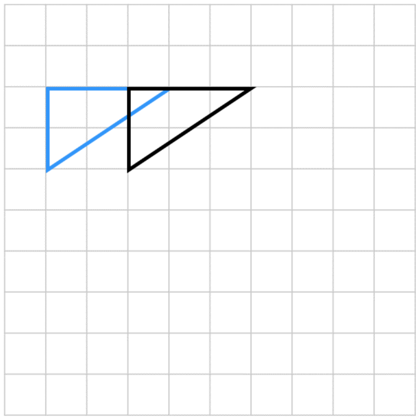
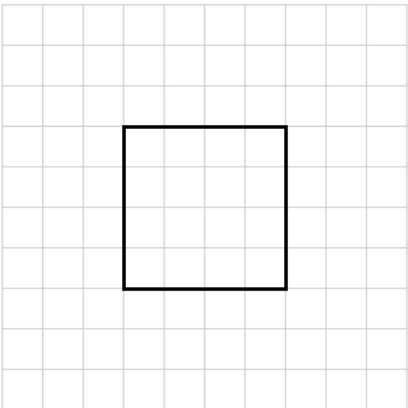
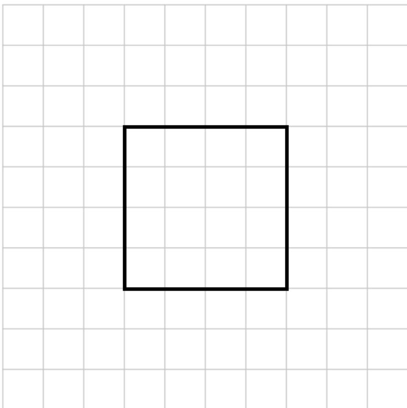


2 Draw these translations.

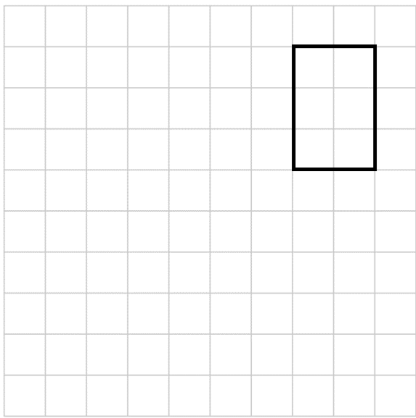
a 2 right



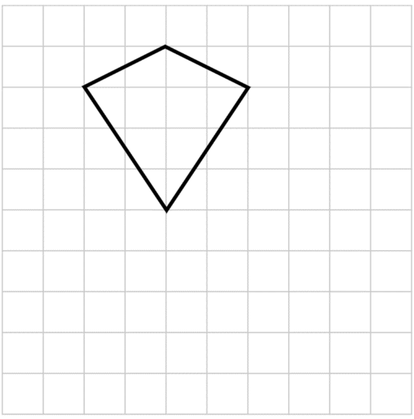
b 3 left



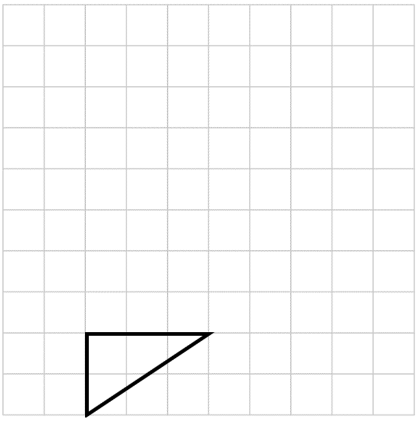
c 1 down



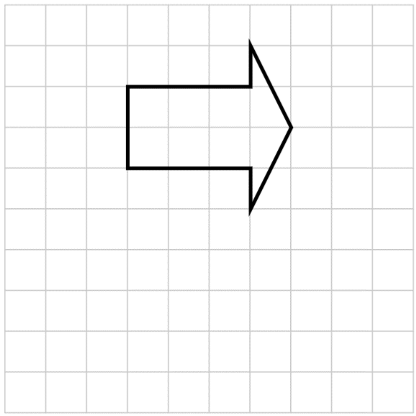
d 5 down



e 7 up

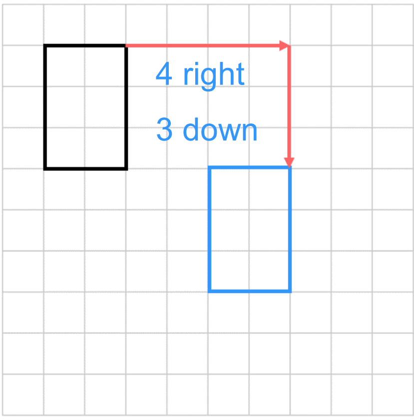


f 2 left

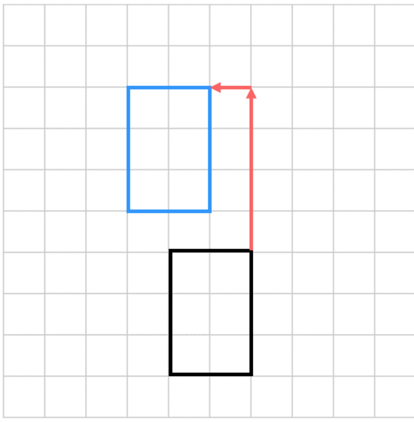


3 Describe these translations. The first question has been done for you.

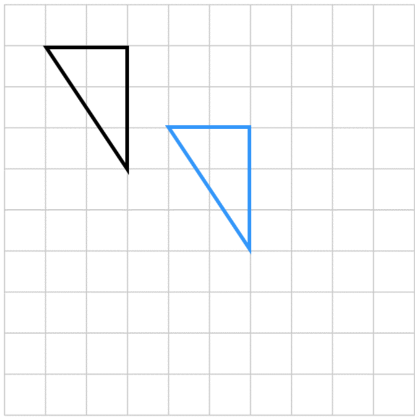
a



b



c



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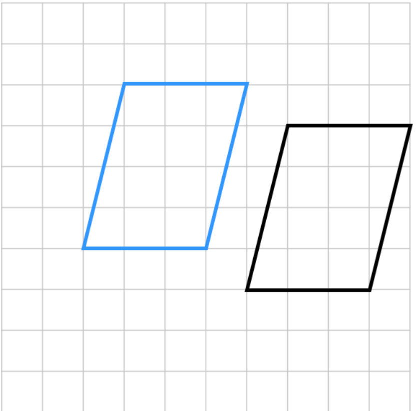

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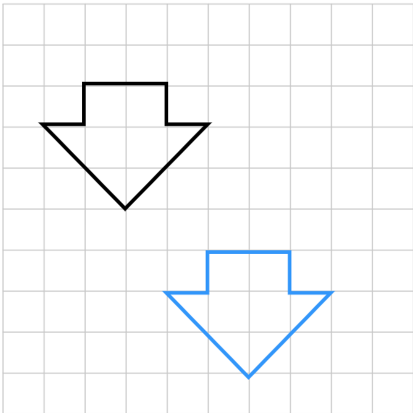

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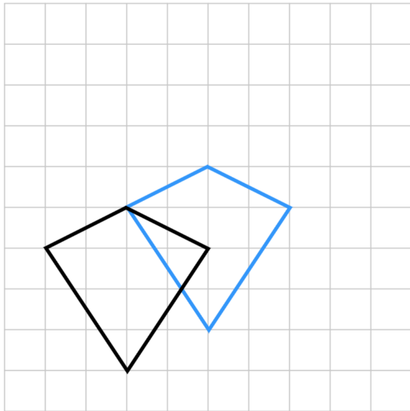
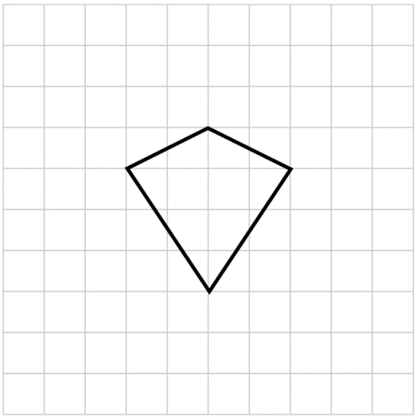
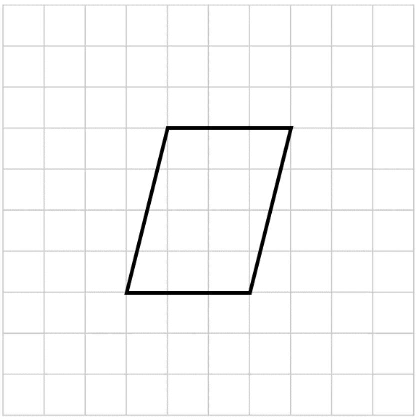


4 Draw these translations.

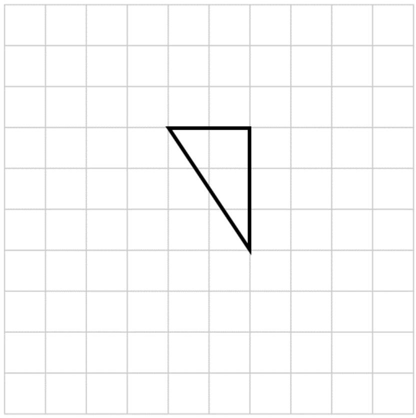
a 1 up, 2 down



b 2 left, 3 up

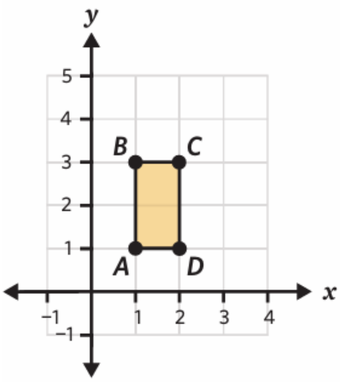


c 4 left, 4 down

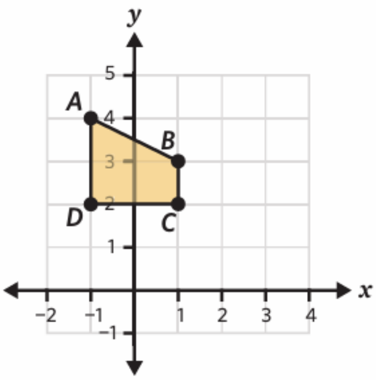


5 Translate the following and write the coordinates of the image.

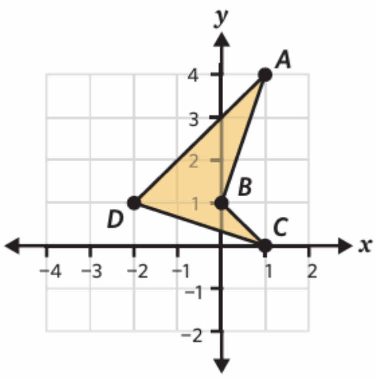
a 2 right 2 up



b 3 right 1 down



c 1 left 2 down



REFLECTION

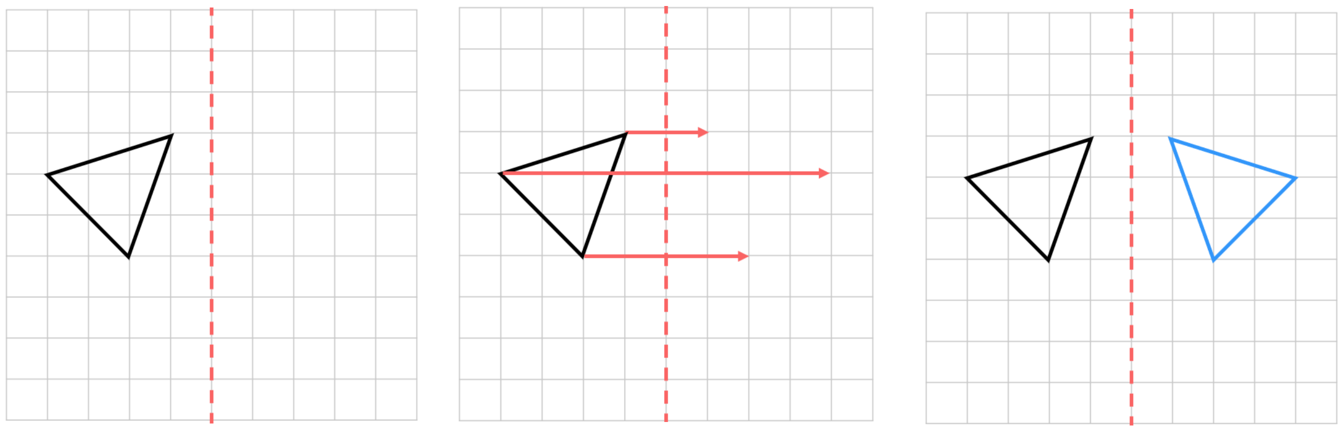
Reflection: https://www.geogebra.org/m/fxne4xqh

- To perform a reflection, flip each point across a mirror line so they are the same distance on the opposite side.

Example Draw images after translation.

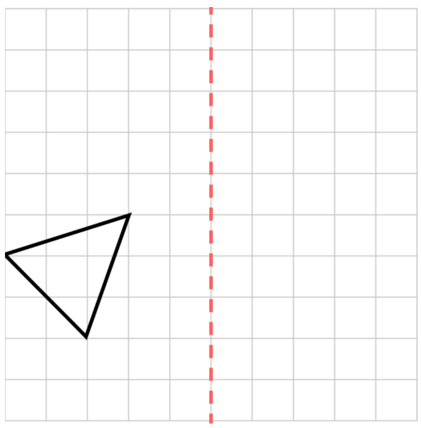
Draw the image after reflection

Flip each vertex by the same units   Draw the image

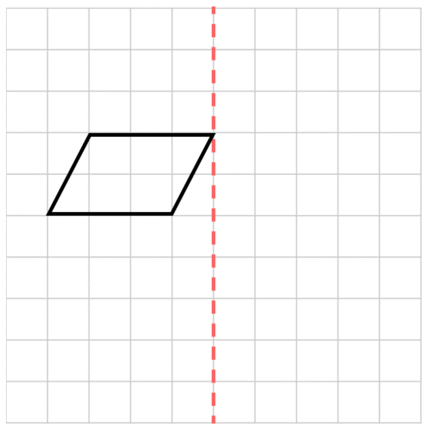


Draw the image after reflection:

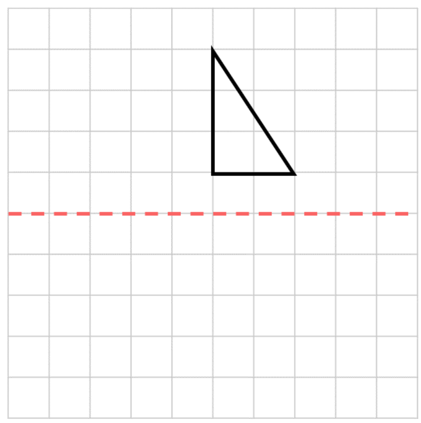
a



b

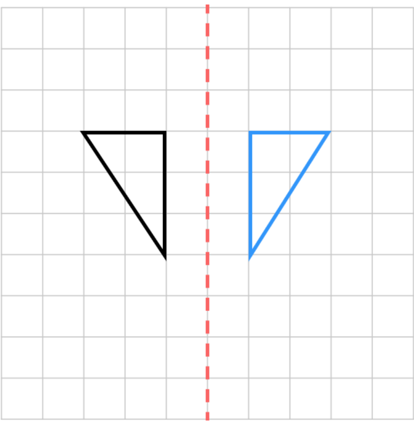


c

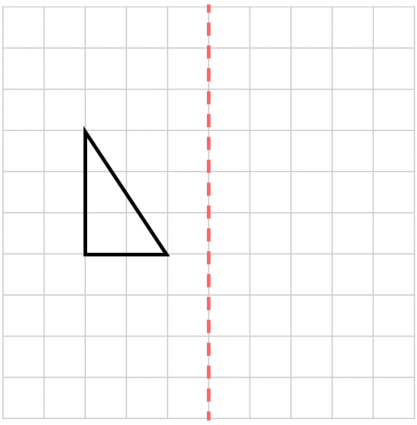


1 Reflect these shapes across the line shown. The first question has been done for you.

a



b



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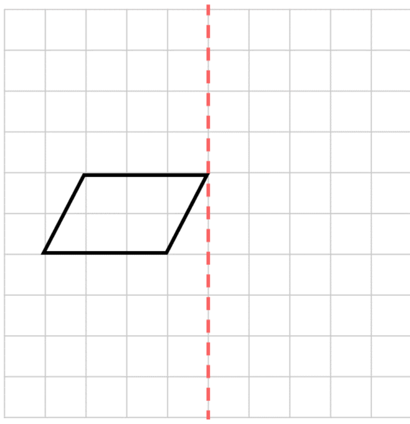

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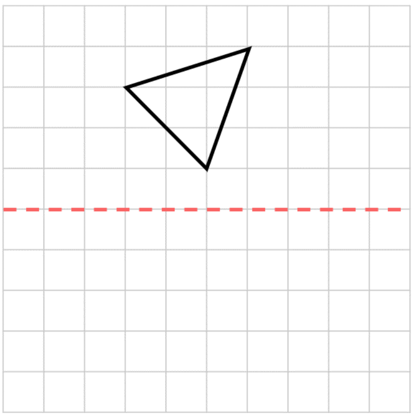

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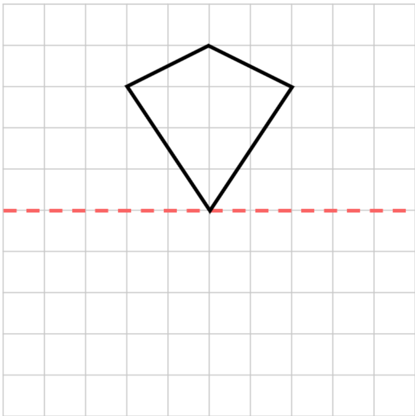

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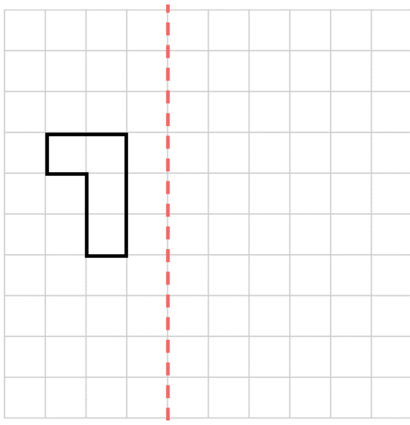
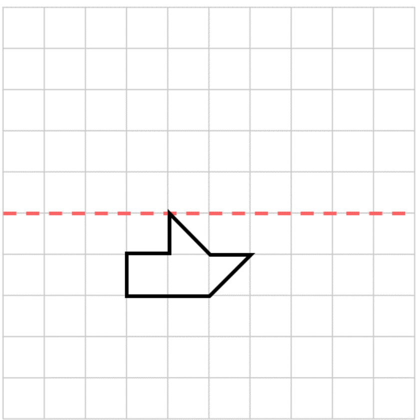


g



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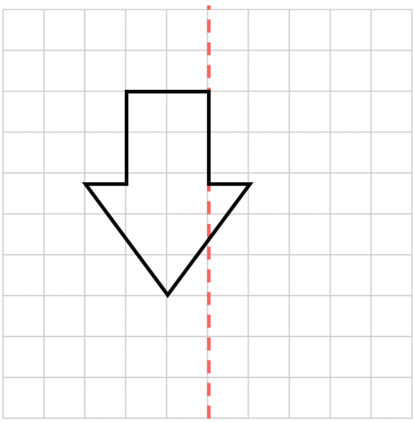

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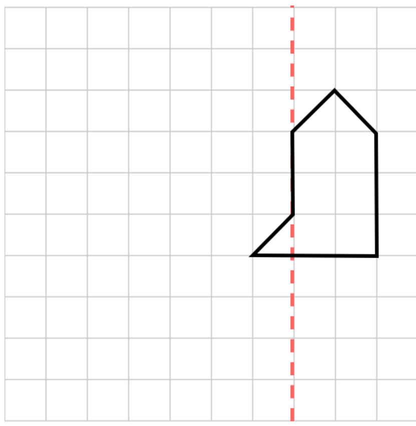

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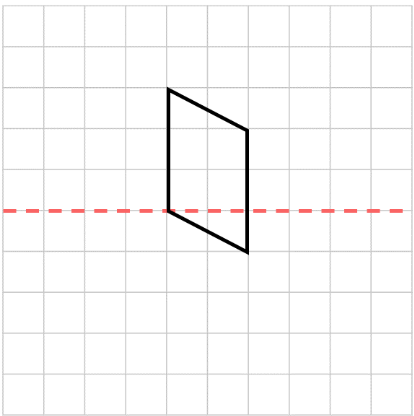

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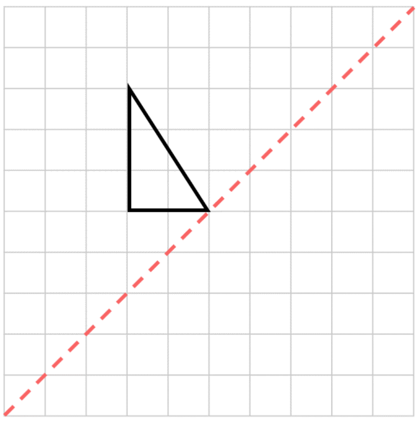

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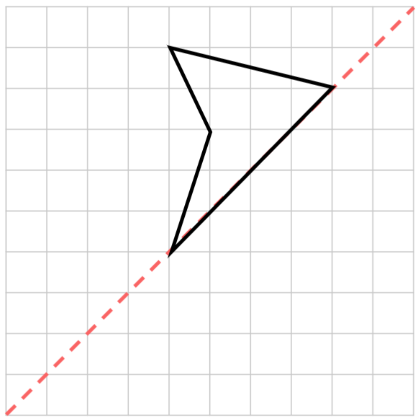
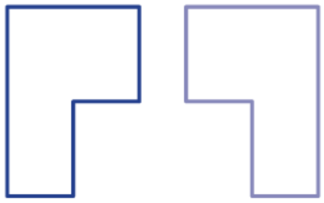


2 Draw the mirror line for these reflections.

a



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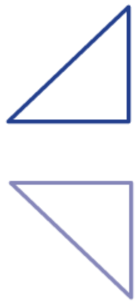

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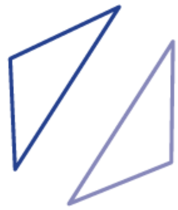

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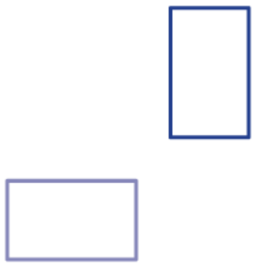

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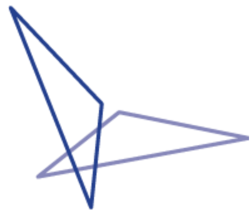
f



3 State the coordinates of the vertices after the shape is reflected:

a

Across x axis

$A'($, $)$

$B'($, $)$

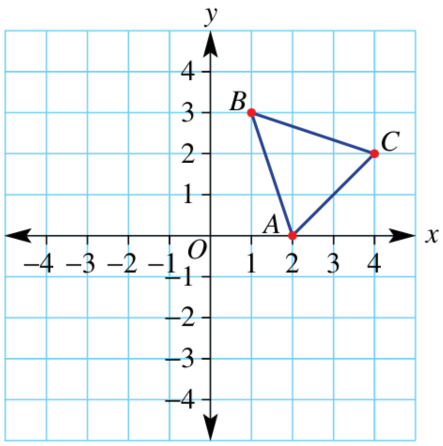
$C'($, $)$

Across y axis

$A'($, $)$

$B'($, $)$

$C'($, $)$



b

Across x axis

$A'($, $)$

$B'($, $)$

$C'($, $)$

$D'($, $)$

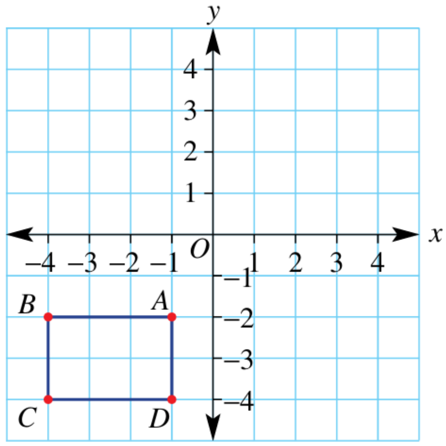
Across y axis

$A'($, $)$

$B'($, $)$

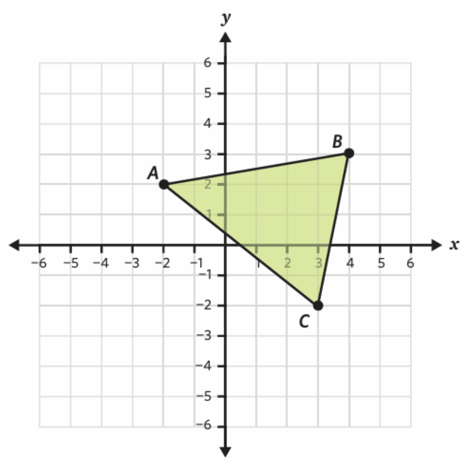
$C'($, $)$

$D'($, $)$

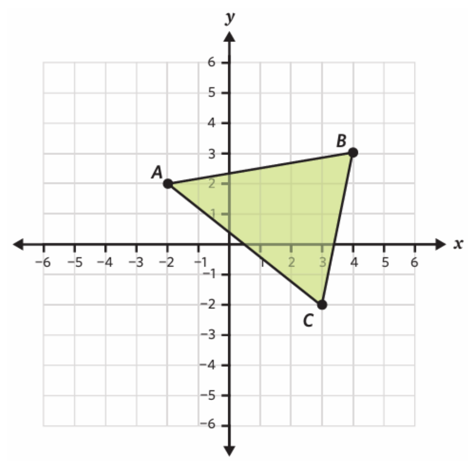


4 Reflect the shape:

a Across x axis



b Across y axis.



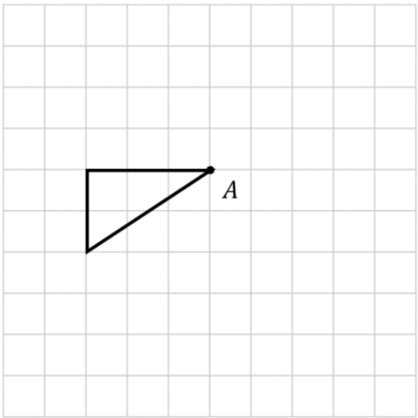
ROTATION

- To rotate a shape, for each point in the shape:

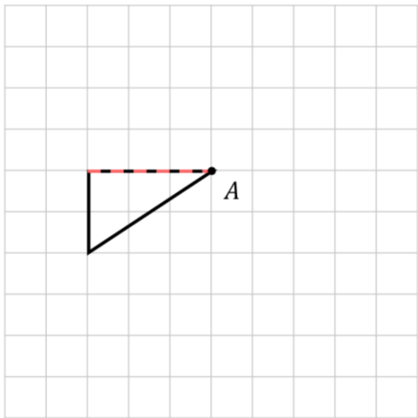
1. Draw a line from to the centre of rotation. Measure the length of this line.
2. Swing this line through the required angle and direction.

Example Draw images after translation.

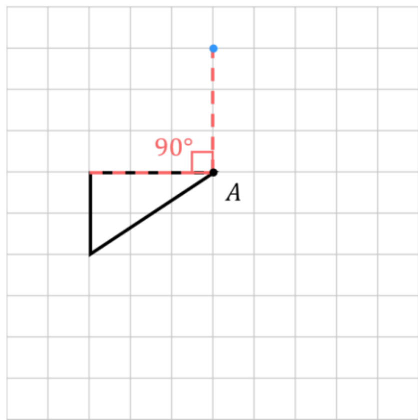
Draw the image after rotation by $90°$ clockwise around point A.



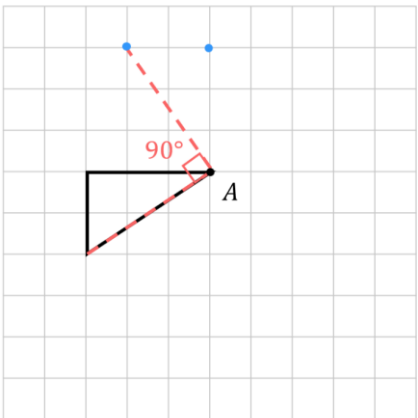
Draw line from a vertex to point



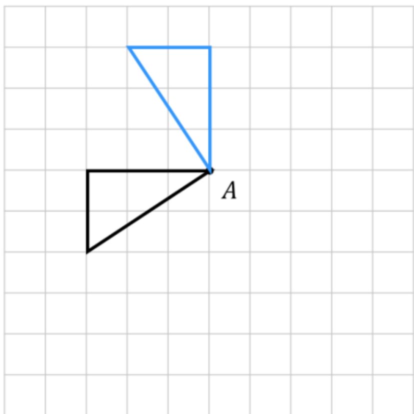
Rotate arm by $90°$



Repeat for other vertices

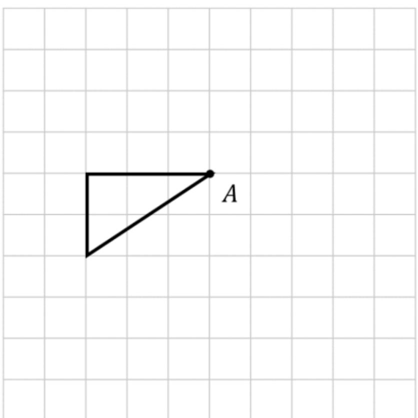


Draw image

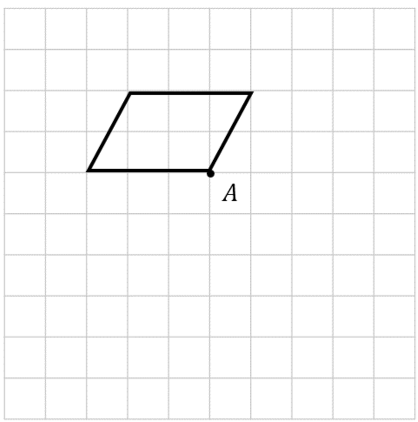


Draw the image after rotation:

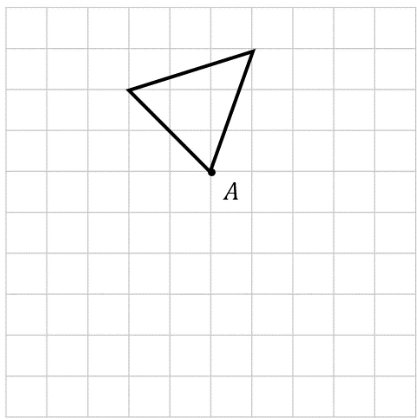
a $90°$ anticlockwise



b $90°$ clockwise

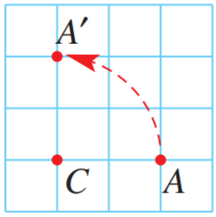


c $180°$

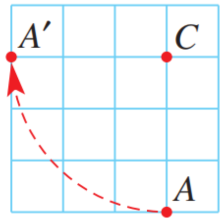


1 Point A has been rotated to its image point A′. For each part state whether the point has been rotated clockwise or anticlockwise and by how many degrees it has been rotated.

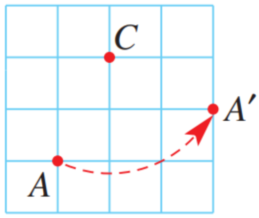
a



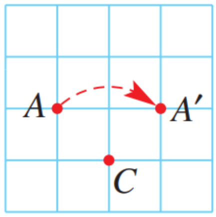
b



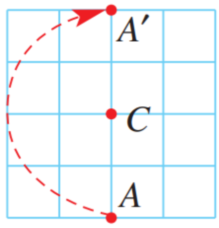
c



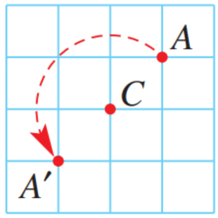
d



e



f



2 Complete these sentences.

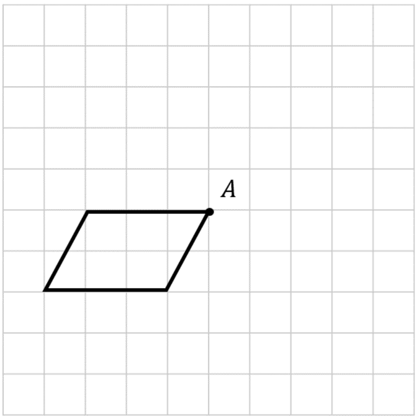
a A rotation clockwise by 270° is the same as a rotation anticlockwise by ……………

b A rotation anticlockwise by 180° is the same as a rotation clockwise by ……………

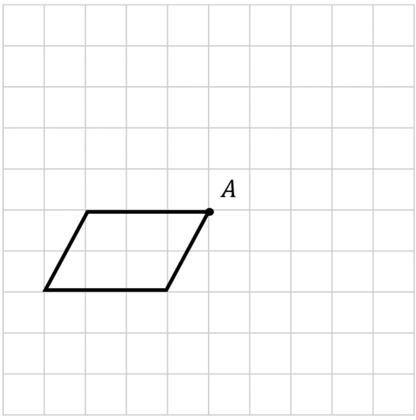
c A rotation clockwise by 90° is the same as a rotation anticlockwise by ……………

3 Draw the image after rotation:

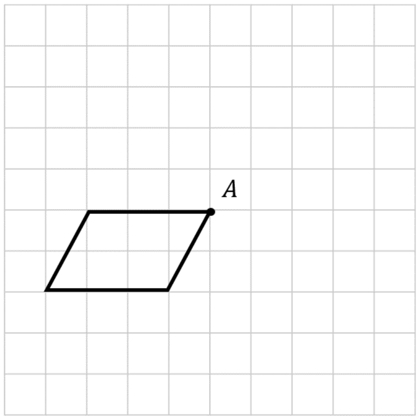
a 90° clockwise



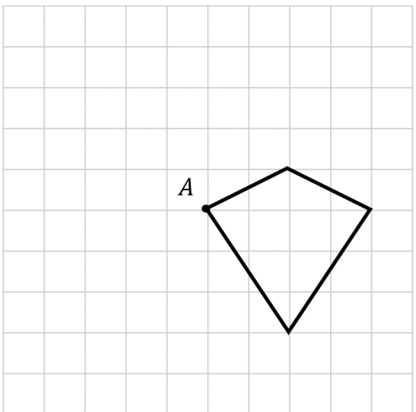
b 90° anticlockwise



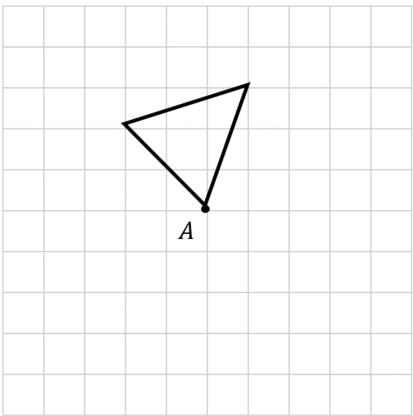
c 180°



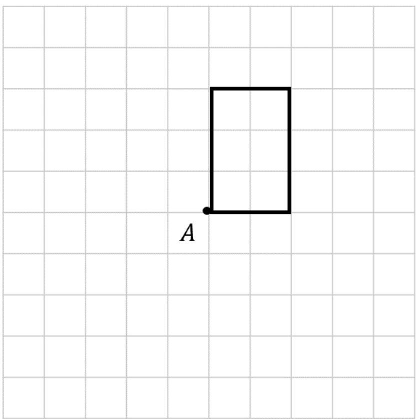
d 270° anticlockwise



e 270° clockwise



f 180°

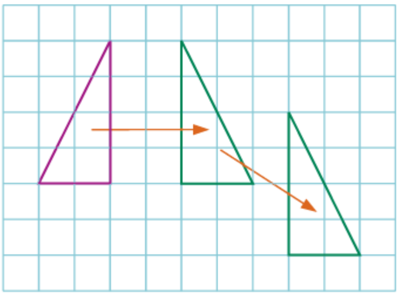


MIXED TRANSFORMATIONS

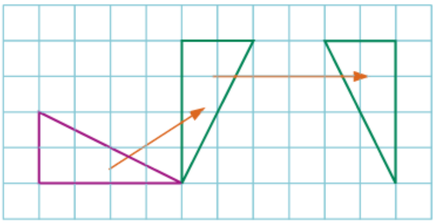
FOUNDATION

1 What combination of transformations is shown in each diagram?

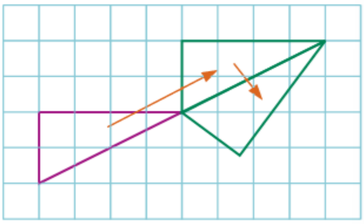
a



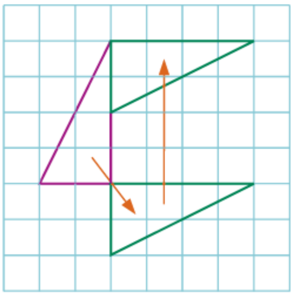
b



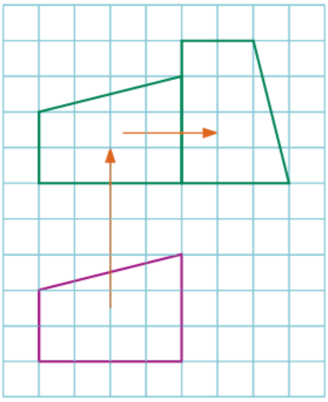
c



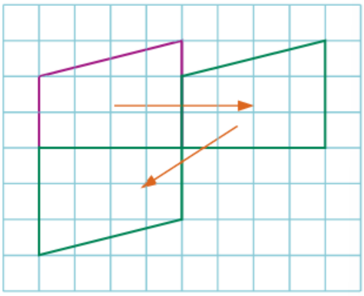
d



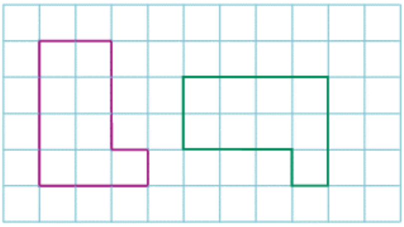
e



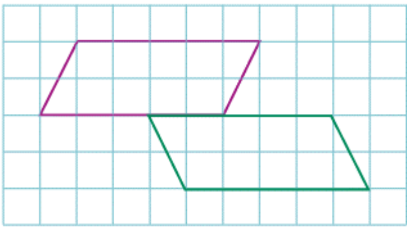
f



g

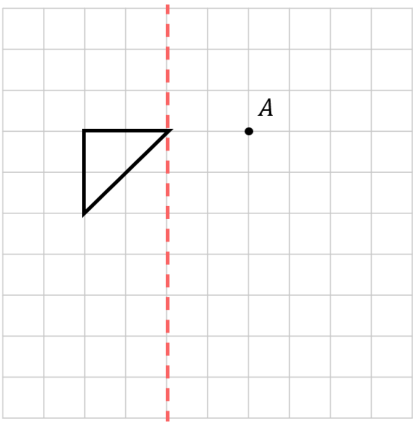


h

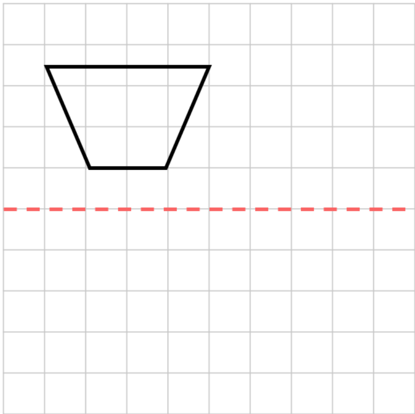


2

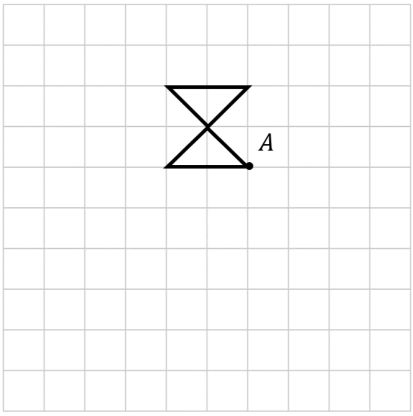
a Reflect across the line
Then rotate $90°$ clockwise about A



b Translate 3 units right
Then reflect across the line

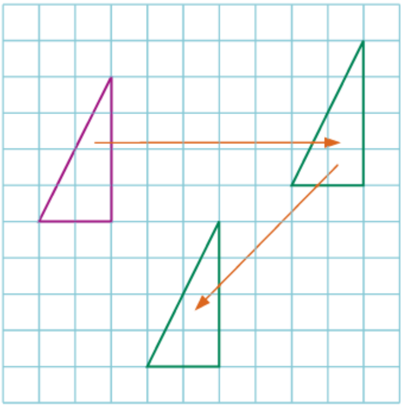


c Rotate $90°$ clockwise about A.
Then translate 3 units left.

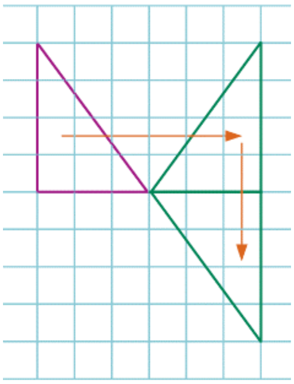


3 What combination of transformations is shown in each diagram?
What *single* transformation could produce the same result?

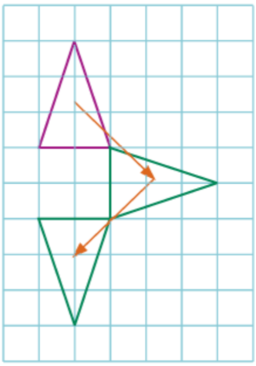
a



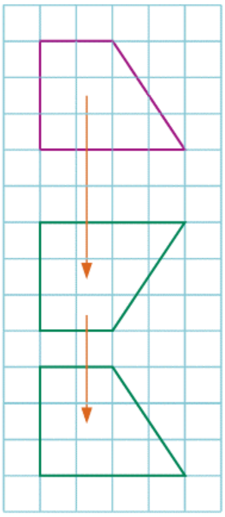
b



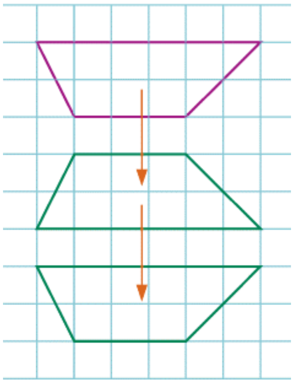
c



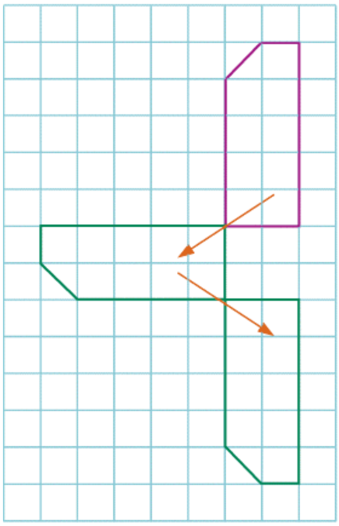
d



e



f

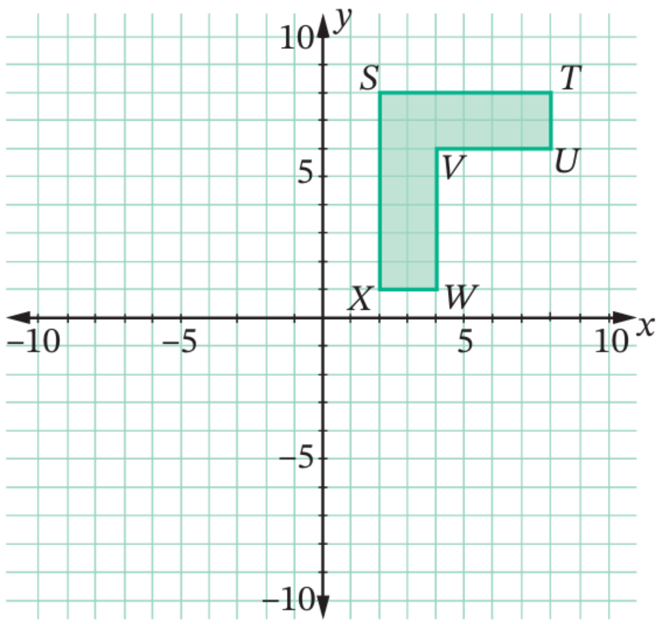


4

a Reflect the L-shape across the x-axis to create the image S' T' U' V' W' X'

b Write the coordinates of the original point and its image.

Original	Image
$S\ (2, 8)$	$S'\ (2, -8)$
T	
U	
V	
W	
X	



c What do you notice about the coordinates of the image compared to the original?

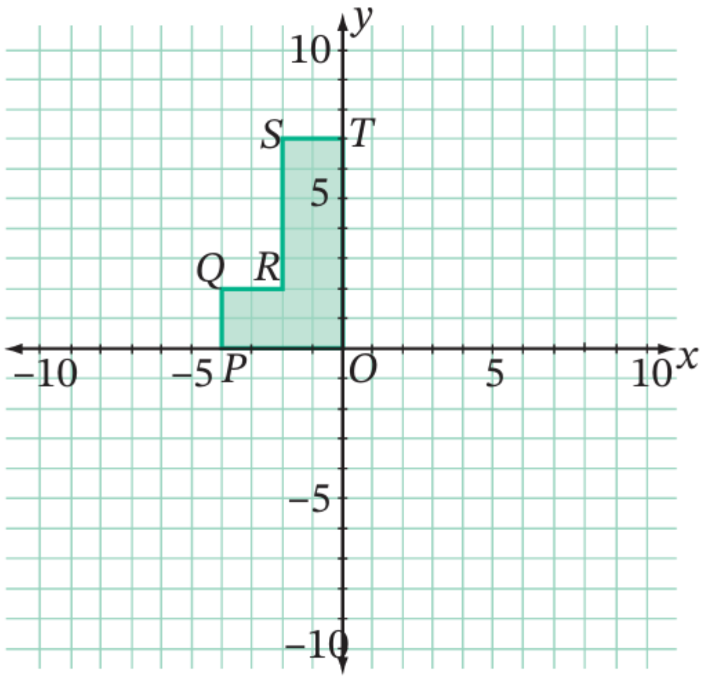
....

5

a Rotate OPQRST about O through an angle of 180°

b Write the coordinates of the original point and its image.

Original	Image
O	O'
P	P'
Q	Q'
R	R'
S	S'
T	T'



c What do you notice about the coordinates of the image compared to the original?

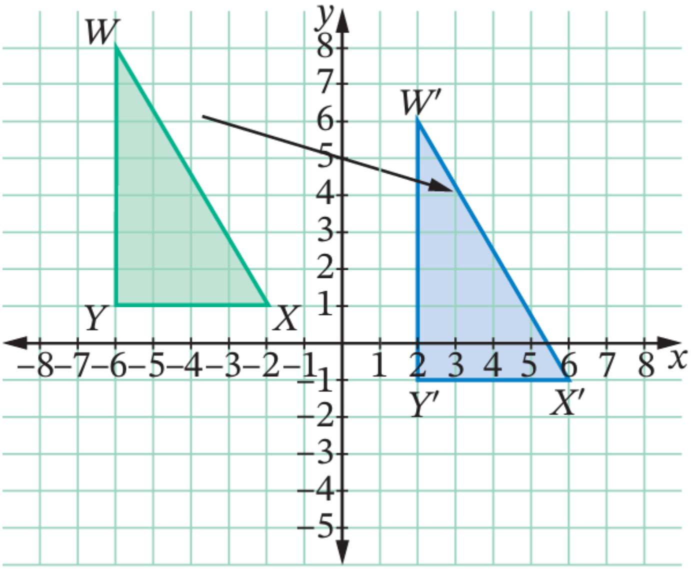
....

....

6

a What transformation has been performed to WXY? ....

b Write the coordinates of the original point and its image.



Original	Image
W	W'
X	X'
Y	Y'

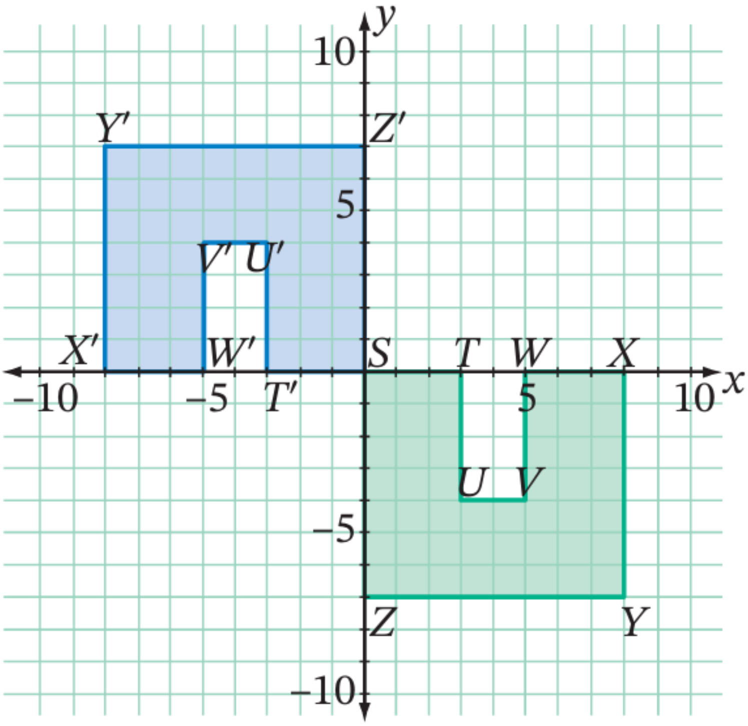
c What do you notice about the coordinates of the image compared to the original?

....

....

7 What transformation has been performed on STUVWXYZ? ....

a Write the coordinates of the original point and its image.



Original	Image
S	S'
T	T'
U	U'
V	V'
W	W'
X	X'
Y	Y'
Z	Z'

b What do you notice about the coordinates of the image compared to the original?

....

....

TRANSLATION VECTORS

- A **translation** shifts every point the same distance and direction.
- We describe this movement using a **translation vector**.
- A vector $\begin{pmatrix}x\\y\end{pmatrix}$ shows how far to move left/right (x) and up/down (y).

Translation Vector	
x $\leftarrow$	horizontal translation
y $\leftarrow$	vertical translation

Example Describe shifting of a translation vector.

Vector	Meaning	Vector	Meaning
$\begin{pmatrix}4\\3\end{pmatrix}$	4 right 3 up	$\begin{pmatrix}4\\-1\end{pmatrix}$	
$\begin{pmatrix}3\\2\end{pmatrix}$		$\begin{pmatrix}5\\-1\end{pmatrix}$	
$\begin{pmatrix}-3\\2\end{pmatrix}$		$\begin{pmatrix}6\\-1\end{pmatrix}$	
$\begin{pmatrix}-4\\2\end{pmatrix}$		$\begin{pmatrix}7\\-1\end{pmatrix}$	
$\begin{pmatrix}-1\\-1\end{pmatrix}$		$\begin{pmatrix}-6\\-2\end{pmatrix}$	
$\begin{pmatrix}-2\\-2\end{pmatrix}$		$\begin{pmatrix}4\\-2\end{pmatrix}$	
$\begin{pmatrix}-3\\-3\end{pmatrix}$		$\begin{pmatrix}-1\\0\end{pmatrix}$	
$\begin{pmatrix}3\\-1\end{pmatrix}$		$\begin{pmatrix}-5\\-2\end{pmatrix}$	
$\begin{pmatrix}-3\\-1\end{pmatrix}$		$\begin{pmatrix}7\\2\end{pmatrix}$	

1 Use the words left, right, up, or down to complete these sentences:

a	The vector $\begin{pmatrix}2\\4\end{pmatrix}$ means to move	2 units ……….,	4 units ……….
b	The vector $\begin{pmatrix}-5\\7\end{pmatrix}$ means to move	…… units ……….,	…… units up
c	The vector $\begin{pmatrix}3\\-1\end{pmatrix}$ means to move	………………………	………………………
d	The vector $\begin{pmatrix}-10\\-12\end{pmatrix}$ means to move	………………………	………………………

2 Write the vector that describes these transformations.

a 5 units to the right and 2 units down	**b** 2 units to the left and 6 units down
c 7 units to the left and 4 units up	**d** 9 units to the right and 17 units up

3 Match each vector to its description.

$\begin{pmatrix}-6\\1\end{pmatrix}$	1 right and 6 up
$\begin{pmatrix}6\\1\end{pmatrix}$	6 left and 1 down
$\begin{pmatrix}-6\\-1\end{pmatrix}$	1 left and 6 up
$\begin{pmatrix}6\\-1\end{pmatrix}$	6 right and 1 up
$\begin{pmatrix}-1\\6\end{pmatrix}$	6 right and 1 down
$\begin{pmatrix}1\\6\end{pmatrix}$	6 left and 1 up

4 Decide if these vectors describe vertical or horizontal translation.

a $\begin{pmatrix}2\\0\end{pmatrix}$	**b** $\begin{pmatrix}0\\7\end{pmatrix}$	**c** $\begin{pmatrix}0\\-4\end{pmatrix}$	**d** $\begin{pmatrix}-6\\0\end{pmatrix}$

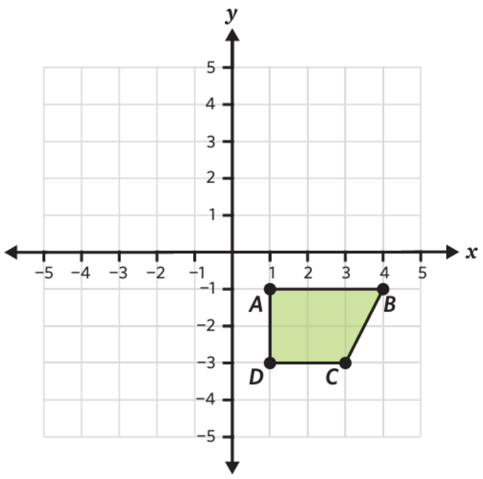
TRANSLATION USING VECTORS

Translation using vectors: https://www.geogebra.org/m/xugfvfqr

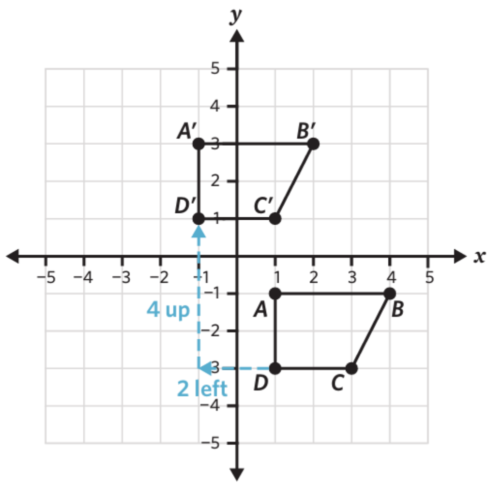
Translation using Vectors
$A\,(\,x, y\,)$
$\downarrow \binom{a}{b}$
$A'(x+a, y+b)$

Example Draw images using translation.

Translate the trapezium ABCD by vector $\binom{-2}{4}$



Slide every point 2 left and 4 up

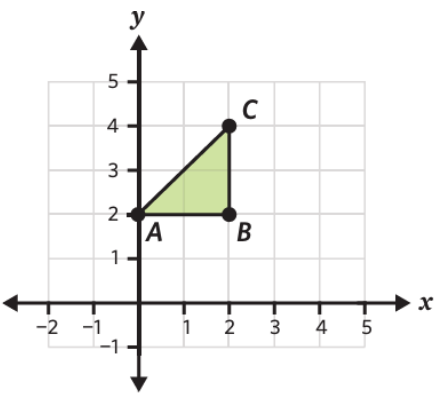


What does A' mean? *The ………… of A after …………………………*

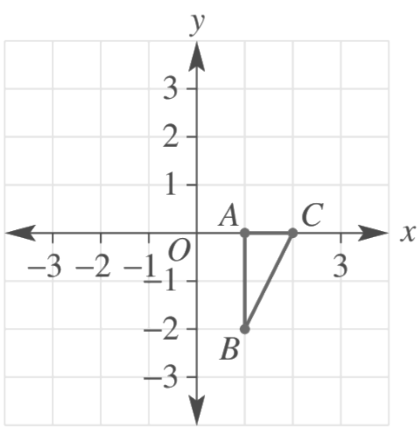
What instructions does a translation vector $\binom{-2}{4}$ communicate?

3 units ………, −3 units ……………

a Translate the triangle ABC by $\binom{2}{-1}$

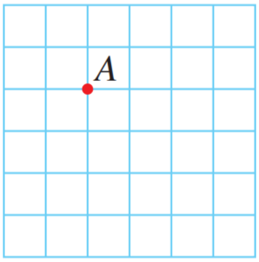


b Draw the image after translation by $\binom{-4}{3}$

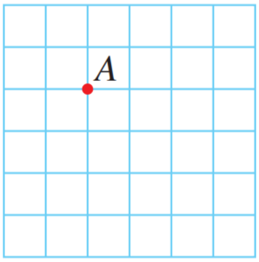


1 Show on the position of A′ after applying each of the following vector translations.

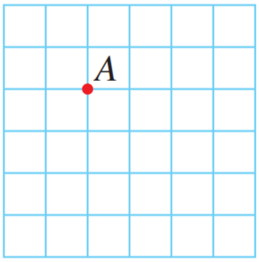
a $\begin{pmatrix}1\\-3\end{pmatrix}$



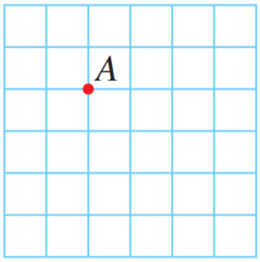
b $\begin{pmatrix}2\\1\end{pmatrix}$



c $\begin{pmatrix}-1\\2\end{pmatrix}$

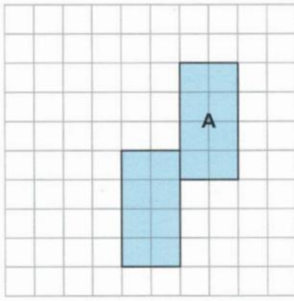


d $\begin{pmatrix}0\\-2\end{pmatrix}$

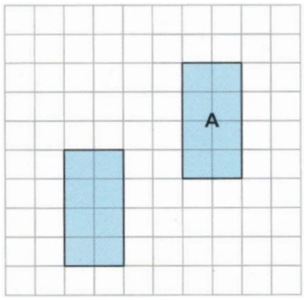


2 Ed has translated rectangle A 2 squares left and 3 squares down.

a Describe Ed's translation using vector notation.



b Beca has also tried to translate 2 squares left and 3 squares down. Here is her answer.

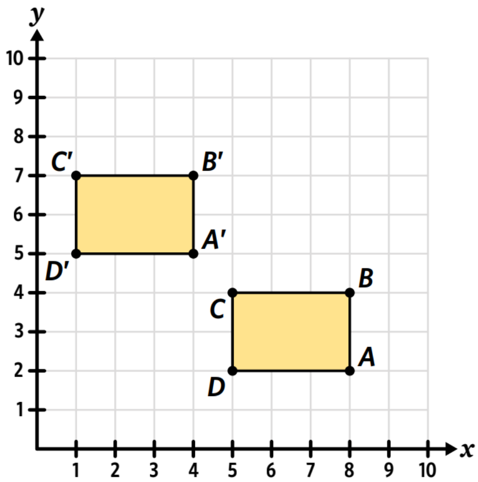


i Explain the mistake that Beca has made.

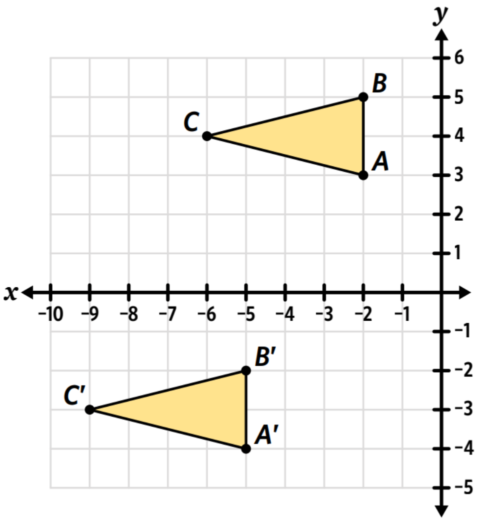
ii Use vector notation to describe Beca's translation.

3 Describe each translation using a vector.

a



b

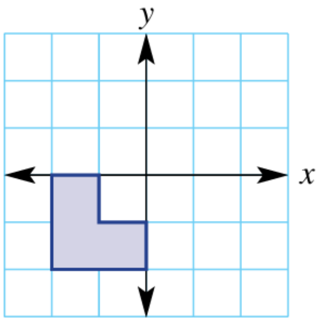


c Point $D(-2,-3)$ to $D'(2,5)$

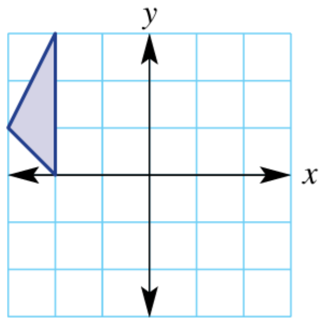
d Point $C(4,-2)$ to $C'(2,2)$

4 Draw the image of the shapes translated by the given vectors.

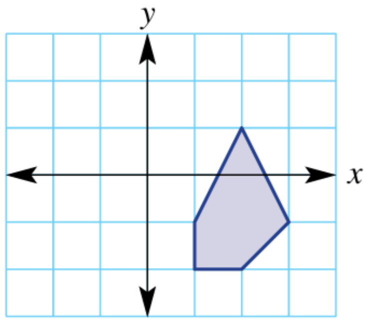
a $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$



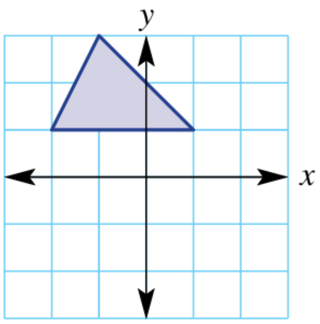
b $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$



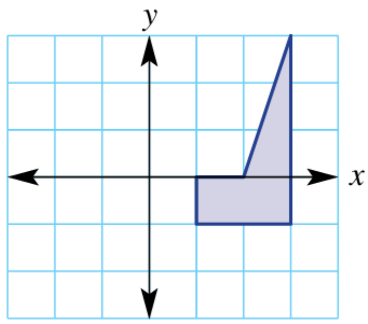
c $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$



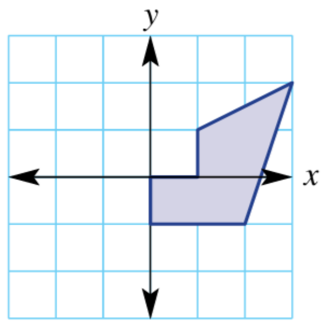
d $\begin{bmatrix} 0 \\ -3 \end{bmatrix}$



e $\begin{bmatrix} -4 \\ -1 \end{bmatrix}$

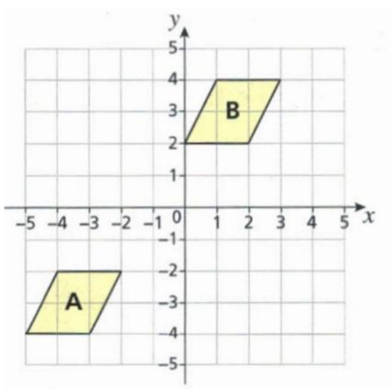


f $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$

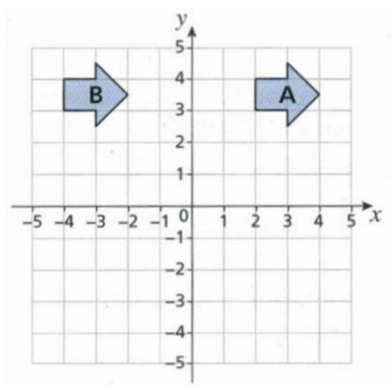


5 Use vector notation to describe each translation from shape A to shape B.

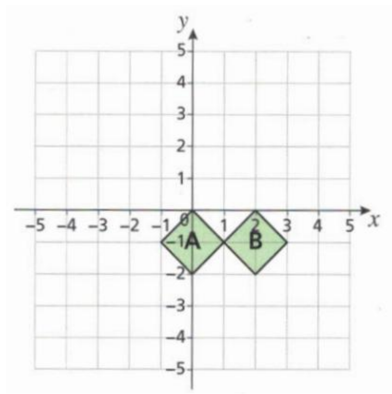
a



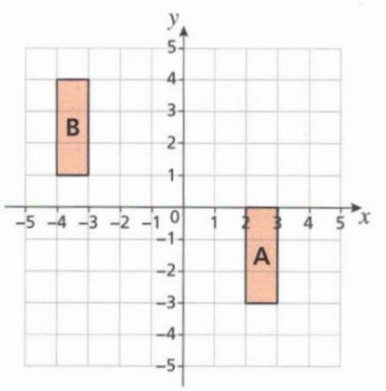
b



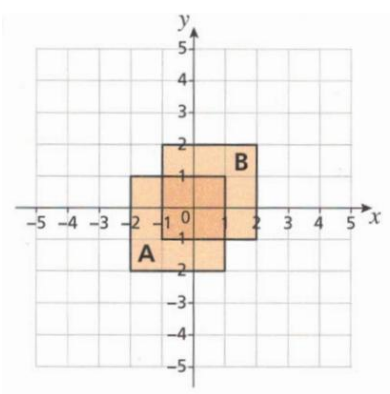
c



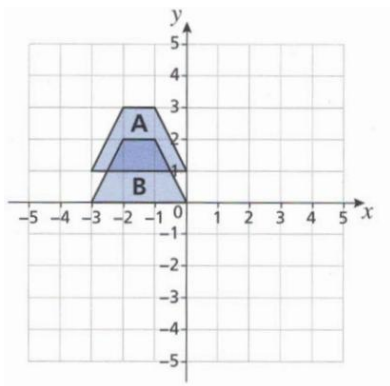
d



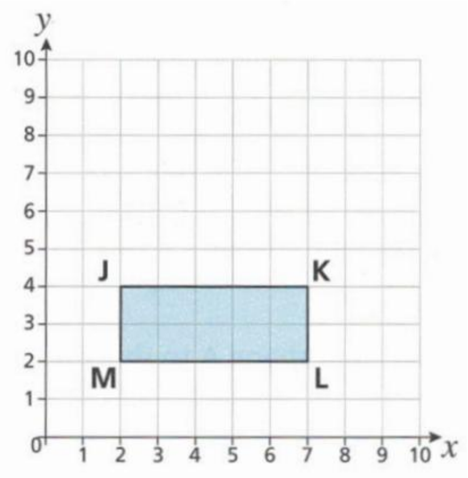
e



f



6



a translate rectangle $JKLM$ by vector $\binom{2}{4}$ and label the vertices $J'K'L'M'$

b Write the coordinates of the original and image.

Original	Image
J	J'
K	K'
L	L'
M	M'

c What do you notice about the coordinates of J, K, L, M and J', K', L', M'?

………………………………………………………………………………………………………

d A square has vertices with coordinates $A\ (3, 5), B\ (3, 8), C\ (6, 5)$ and $D\ (6, 8)$.
Write the coordinates of square $A'B'C'D'$ after a translation by vector $\binom{3}{7}$

………………………………………………………………………………………………………

7 To get the coordinates of the image, we can add the x- and y-components of the vector to the coordinates of the original.
Write the coordinates of the image of the point $A(13, -1)$ after translation by the given vectors.

a $\binom{2}{3}$	**b** $\binom{8}{0}$	**c** $\binom{0}{7}$	**d** $\binom{-4}{3}$
e $\binom{-2}{-1}$	**f** $\binom{-10}{-5}$	**g** $\binom{-2}{-8}$	**h** $\binom{6}{-9}$

8 State a single translation vector that can replace these vectors.

a $\binom{0}{3}\binom{7}{0}$	**b** $\binom{-1}{3}\binom{0}{3}\binom{7}{0}$	**c** $\binom{-1}{-4}\binom{4}{0}\binom{0}{3}$	**d** $\binom{2}{3}\binom{-5}{1}\binom{3}{-3}\binom{-2}{-4}$

9 A point undergoes the following multiple translations with these given vectors.
State the value of x and y of the vector that would take the image back to its *original* position.

a $\binom{3}{4}\binom{-1}{-2}\binom{x}{y}$	**b** $\binom{2}{5}\binom{-7}{2}\binom{-1}{-3}\binom{x}{y}$
c $\binom{0}{4}\binom{7}{0}\binom{-4}{-6}\binom{x}{y}$	**d** $\binom{-4}{20}\binom{12}{0}\binom{-36}{40}\binom{x}{y}$

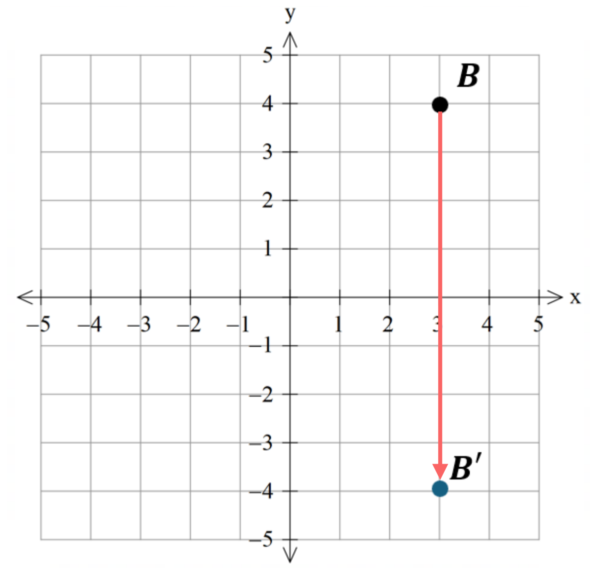
REFLECTIONS USING COORDINATES

Reflection: https://www.geogebra.org/m/rzx3gegq

Reflection across x-axis	Reflection across y-axis
$A(x, y)$ $\downarrow$ $A'(x, -y)$	$A(x, y)$ $\downarrow$ $A'(-x, y)$

Example Draw images after reflection on a Cartesian plane.

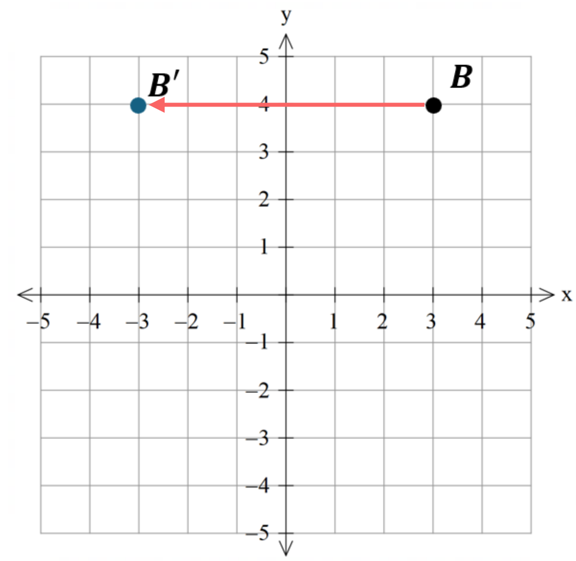
Reflect B across the x-axis.



$B(x, y)$ $\quad \therefore$ $B(3,4)$

$\rightarrow B'(x, -y)$ $\quad \rightarrow B'(3, -4)$

Reflect B across the y-axis.

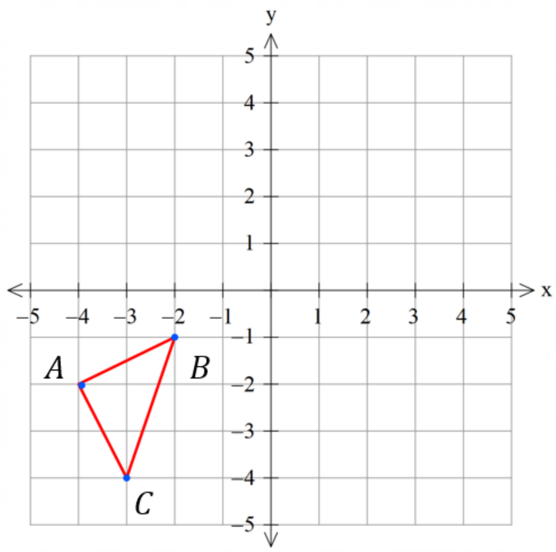


$B(x, y)$ $\quad \therefore$ $B(3,4)$

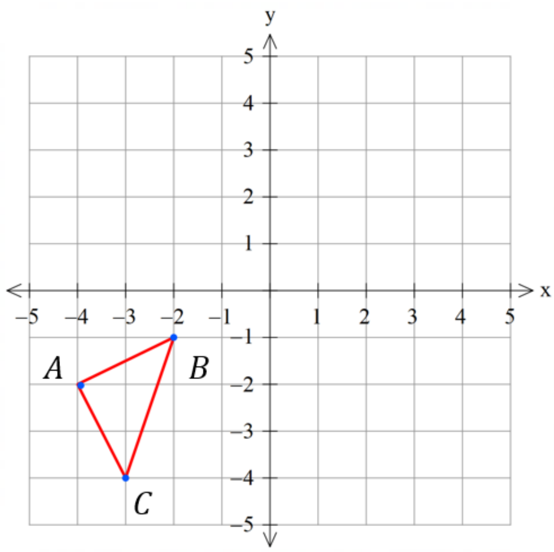
$\rightarrow B'(-x, y)$ $\quad \rightarrow B'(-3,4)$

Draw the image after reflection across the given axes and write the coordinates of the image.

a x-axis



b y-axis



1 State the coordinates of A′, B′, C′, and D’ after the rectangle is reflected in the given axes.

a x-axis

$A'($, $)$

$B'($, $)$

$C'($, $)$

$D'($, $)$

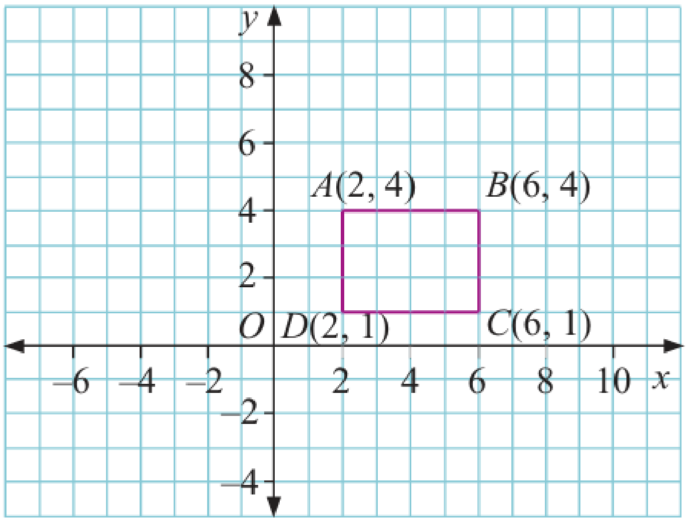
b y-axis

$A'($, $)$

$B'($, $)$

$C'($, $)$

$D'($, $)$



2 State the coordinates of the image after reflection:

a $A(-2,3)$ Reflect across x axis	**b** $B(1,4)$ Reflect across y axis	**c** $C(-2,-3)$ Reflect across x axis
d $D(-1,-4)$ Reflect across x axis	**e** $E(4,2)$ Reflect across y axis	**f** $F(-3,1)$ Reflect across y axis

3 A point is reflected in the x-axis, then in the y-axis and finally in the x-axis again.
What single reflection could replace all three reflections?

………………………………………………………………………………………………………

………………………………………………………………………………………………………

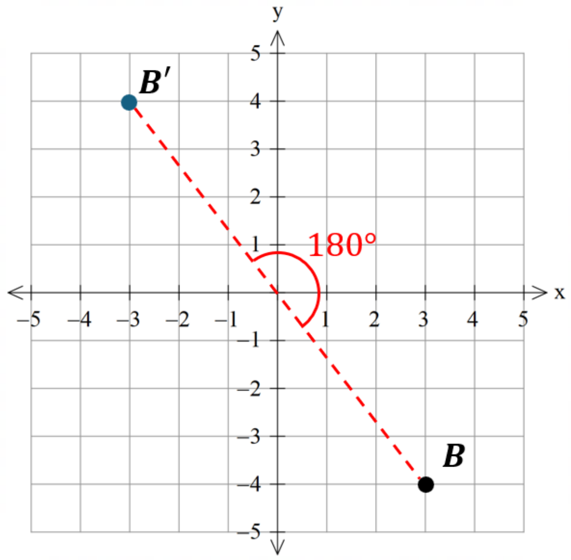
ROTATIONS BY 90° ABOUT THE ORIGIN

Rotation: https://www.geogebra.org/m/yzpna3hj

90° Anticlockwise	90° Clockwise	180°
$A\,(x\,,y)$ $\downarrow$ $A'(-y,x)$	$A\,(x,y)$ $\downarrow$ $A'(y,-x)$	$A\,(x,\ y)$ $\downarrow$ $A'(-x,-y)$

Example Draw images after rotation on a Cartesian plane.

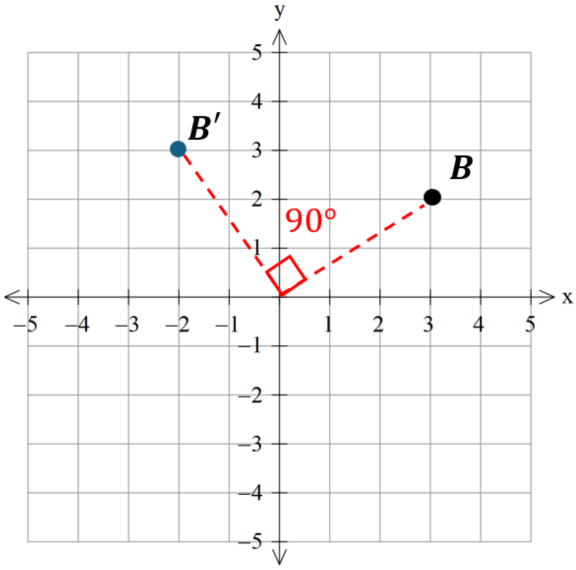
Draw the image of $B(3,-4)$ after rotation by 180° clockwise.



$B(x,y)$ $\quad\therefore\ B(3,-4)$

$\rightarrow B'(-x,-y)$ $\quad\rightarrow B'(-3,4)$

Draw the image of $B(3,2)$ after rotation by 270° clockwise.



$B(x,y)$ $\quad\therefore\ B(3,2)$

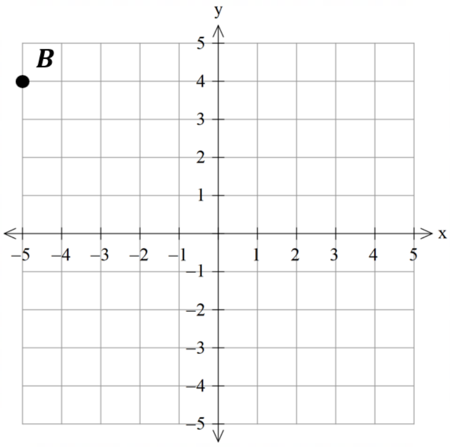
$\rightarrow B'(-y,x)$ $\quad\rightarrow B'(-2,3)$

In the second example, explain why we rotated by 90° instead of 270°

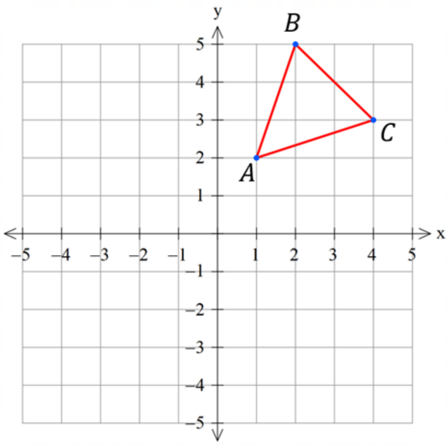
It is faster to rotate 90° ……………… once than 270° clockwise.

Draw the image after rotation about the origin and write the coordinates of the image.

a Clockwise 90°



b Anticlockwise 270°



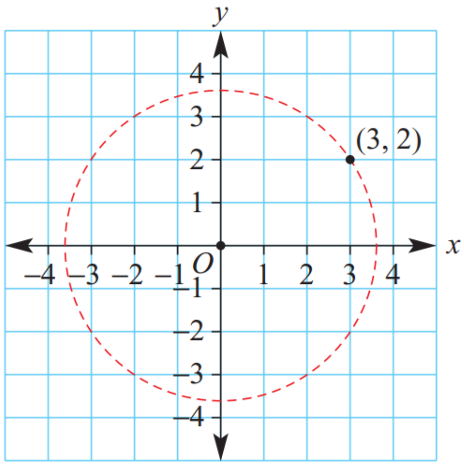
1 Give the coordinates of each of the following points after rotation about the origin (0, 0) by:

a

90° clockwise

90° antic'wise

180°

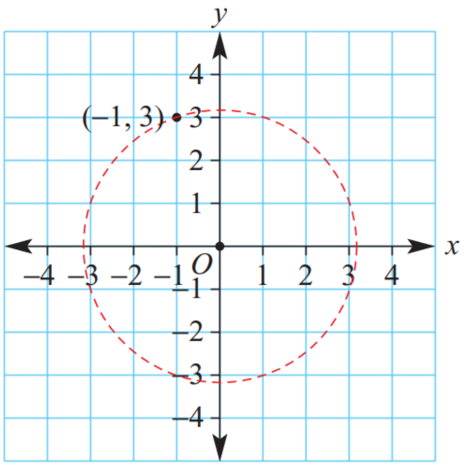


b

90° clockwise

90° antic'wise

180°



2 The point A(4, 3) is rotated about the origin C(0, 0).

Give the coordinates of A′ after rotating by:

a 180° clockwise ....

b 180° anticlockwise ....

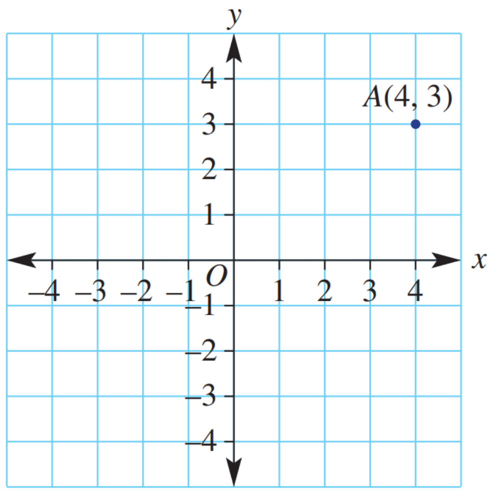
c 90° clockwise ....

d 90° anticlockwise ....

e 270° clockwise ....

f 270° anticlockwise ....

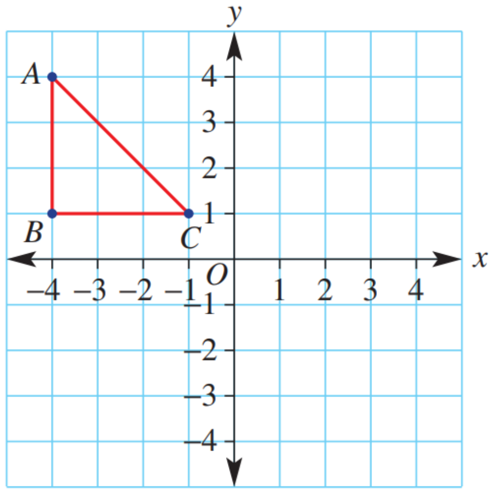
g 360° clockwise ....



3 Write the coordinates of the image for the given rotations around the origin.

a $A(-2,3)$ 90° clockwise	**b** $B(1,4)$ 180° anticlockwise	**c** $C(-2,-3)$ 270° clockwise
d $D(-1,-4)$ 90° clockwise	**e** $E(4,2)$ 270° anticlockwise	**f** $F(-3,1)$ 450° clockwise

4 The triangle shown here is rotated about (0, 0) by the given angle and direction.
Give the coordinates of the image points A′, B′ and C′



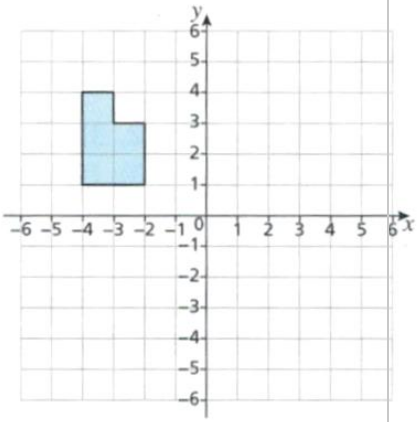
a 180° clockwise

b 90° clockwise

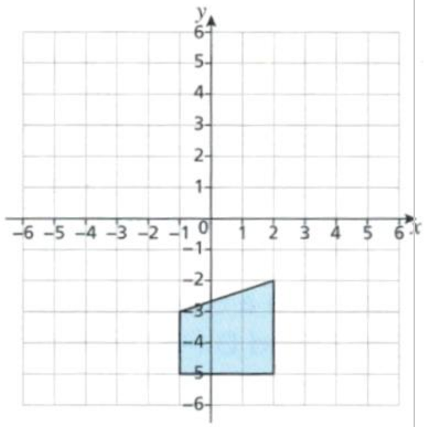
c 90° anticlockwise

5 Rotate each shape as instructed using the origin as the centre of rotation.

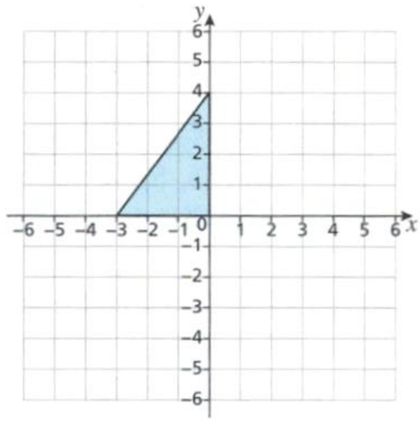
a 180°



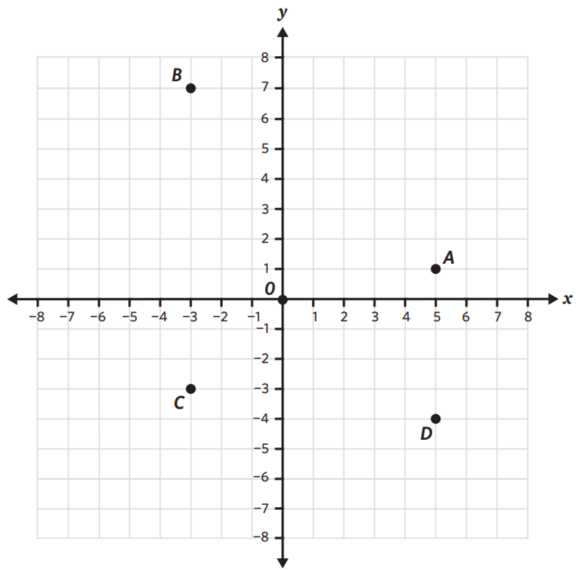
b 270° clockwise



c 90° anticlockwise



6



a Rotate A, B, C, D $180°$ around the origin.

b Reflect A, B, C, D across the x-axis.

c Reflect your images in part **b** across the y-axis.

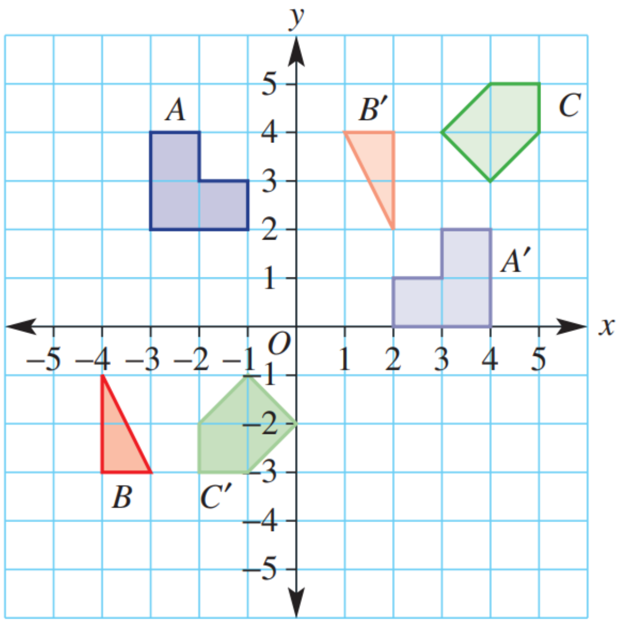
d Compare your answers to part **a** and part **c**.
What do you notice?

7 Explain why a rotation of $180°$ is the same as reflection across both x and y axes.

………

………

8 Describe the combination of transformations (reflections, translations and/or rotations) that map each shape to its image under the given conditions. The reflections that are allowed include only those in the x- and y-axes and rotations will use (0, 0) as its centre.



a A to A′ with a reflection and then a translation

b A to A′ with a rotation and then a translation

c B to B′ with a rotation and then a translation

d B to B′ with two reflections and then a translation

e C to C′ with two reflections and then a translation

f C to C′ with a rotation and then a translation