Student Name

Mathematics Stage 4

LENGTH

Book 2 Describe the relationships between the features of circles

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA4-LEN-C-01 applies knowledge of the perimeter of plane shapes and the circumference of circles to solve problems

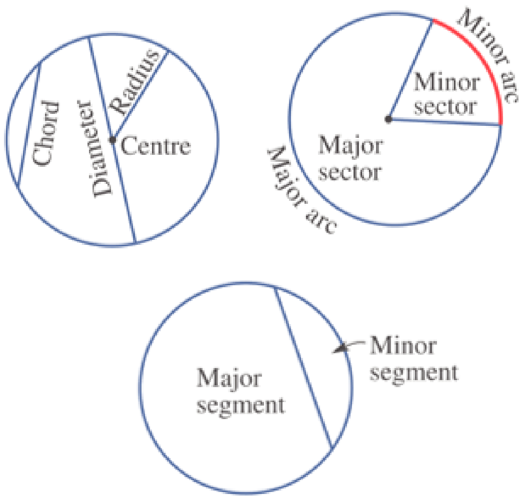
Describe the relationships between the features of circles

- Identify and describe the relationship between circle features, including the radius, diameter, arc, chord, sector and segment of a circle, and a tangent to a circle
- Define π as the ratio of the circumference to the diameter of any circle
- Verify that the number π is a constant and develop the formula for the circumference of a circle
- Apply the formula for the circumference of a circle in terms of the diameter d or radius r (circumference of a circle $C = \pi d$ or $= 2\pi r$) to solve related problems to solve related problems
- Establish the arc length formula $l = \frac{\theta}{360} \times 2\pi r$ where l is the arc length and θ is the angle subtended at the centre by the arc
- Solve problems by finding arc lengths and the perimeter of sectors, giving an exact answer in terms of π or an approximate answer
- Find the perimeter of quadrants, semicircles and simple composite figures consisting of 2 shapes in a variety of contexts, including using digital tools

SUMMARY

Circumference of Circles

Features of Circles



Pi π

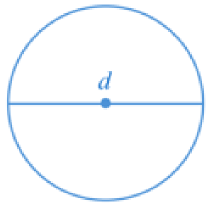
The ratio of $\frac{\text{Circumference}}{\text{Diameter}}$ for any circle is 3.1415159 ...

This number is called pi π

Circumference Formula

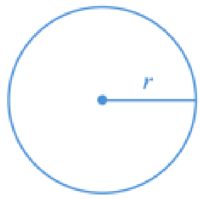
$$C = \pi d$$

$\uparrow$ diameter



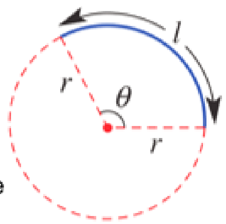
$$C = 2\pi r$$

$\uparrow$ radius



Arc Length Formula

$$\ell = \underbrace{\frac{\theta}{360}}_{\text{Fraction of circle}} \times \underbrace{2\pi r}_{\text{Circumference}}$$



Finding Radius Given Circumference

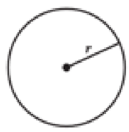
1. Write the formula.
2. Substitute given values.
3. Solve for the unknown using inverse operations

The circumference of a circle is 12.8 cm.
What is the radius? Round to 3 d.p.

1. Write the formula $\quad C = 2\pi r$
2. Substitute $\quad 12.8 = 2\pi r$
3. Solve $\quad \div 2\pi \quad \div 2\pi$

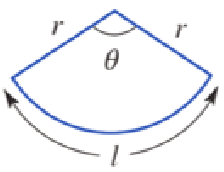
$$\frac{12.8}{2\pi} = r$$

$$2.037 \approx r$$

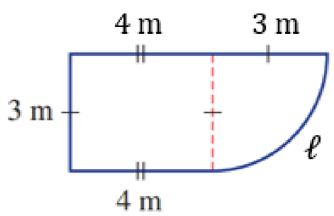


Perimeter of a Sector

$$P = l + 2r$$



Perimeter of a Composite Figure involving Arc Length



$$P = 3 + 4 + 3 + 4 + \ell$$

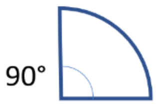
REVIEW OF PRIOR KNOWLEDGE

1 Round these numbers as indicated.

Number	Nearest whole	One decimal place	Two decimal places
31.2			
31.27			
31.472			
31.4128			
31.953			
31.992			

2 What fraction of a whole circle is shown? All angles show the interior angle.
Answer as a simplified fraction.

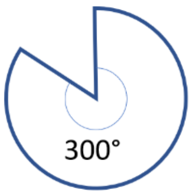
a



$$\frac{\square}{360} = \frac{\square}{\square}$$

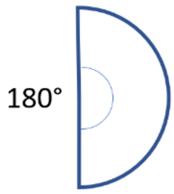
d

g

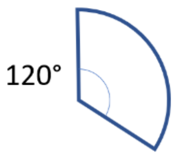


j

b



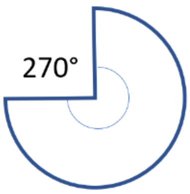
e



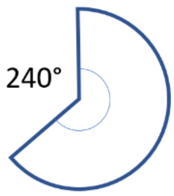
h

k

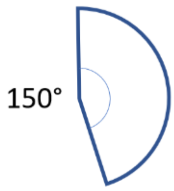
c



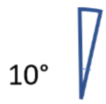
f



i



l



3 Simplify these expressions.

a $2 \times x$

b $2 \times 4 \times x$

c $2 \times 8 \times y$

d $2 \times 3x$

e $3 \times \pi$

f $4 \times 3 \times \pi$

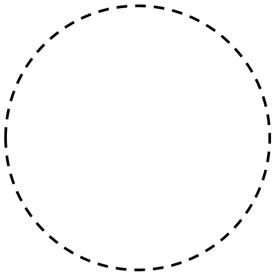
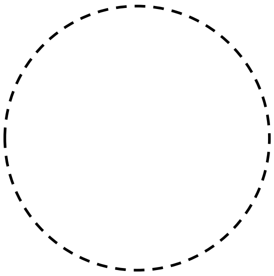
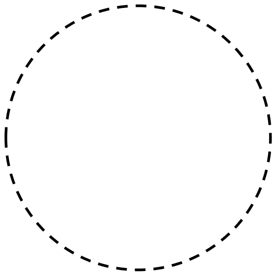
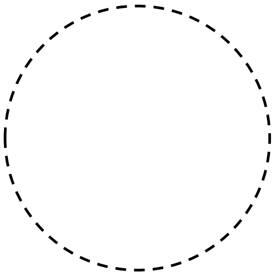
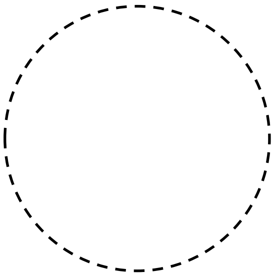
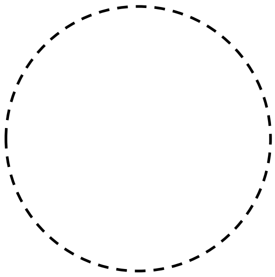
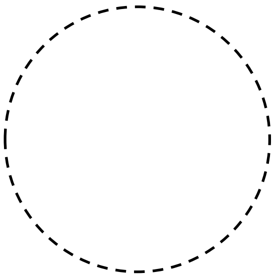
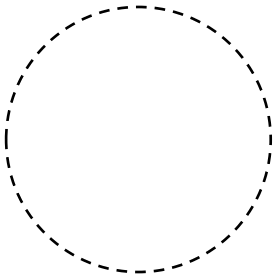
g $3 \times 5 \times \pi$

h $8 \times 5\pi$

FEATURES OF CIRCLES

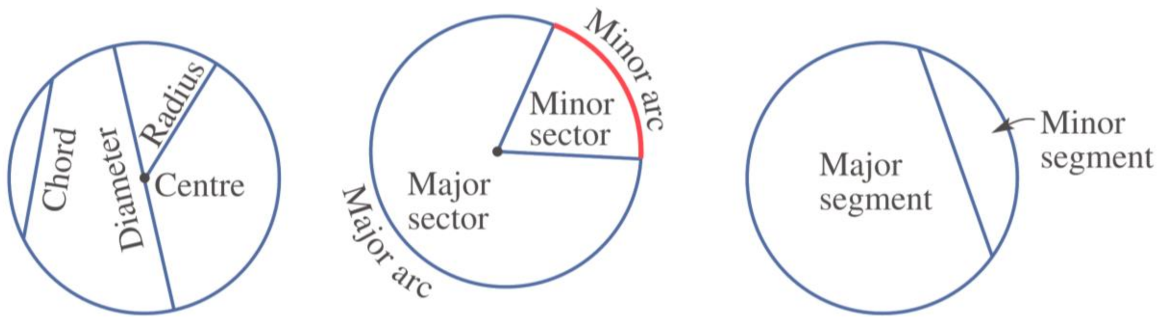
Parts of a Circle: https://www.geogebra.org/m/a9ahwj5m

Investigation Parts of a circle.

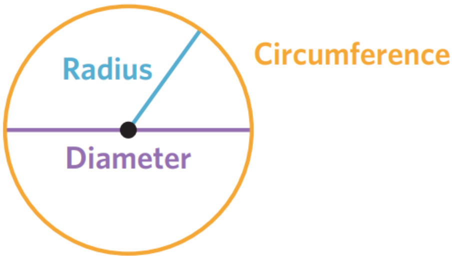
Feature	Definition	Example(s)
Circumference	The circumference of a circle is the perimeter, or distance around the outside of a circle.	
Arc	An arc is a part of the circumference.	 
Centre	All points on the circumference is the same distance from the centre. The real definition of a circle is: a shape made by connecting all points an equal distance from the centre.	
Tangent	A tangent is a straight line that touches the circumference at a single point without entering the circle.	 
Chord	A chord is a straight line that goes from one point on the circumference to another point	 

Diameter	The diameter is a special chord that goes through the centre of the circle.		
Radius	The radius is a line connecting the centre of a circle to the circumference. It is half the diameter.		
Segment	A segment is the area between a chord and the circumference.		
Sector	A sector is the area between two radii.	Quadrant:	Semicircle:

Summary of Circle Features



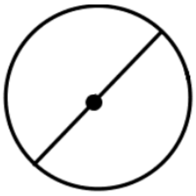
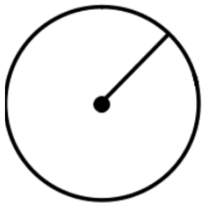
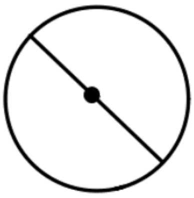
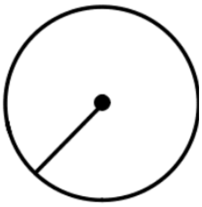
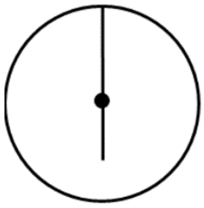
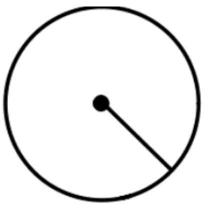
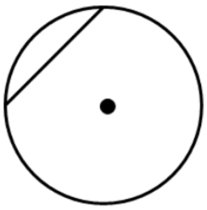
CIRCUMFERENCE, RADIUS, DIAMETER



- **Circumference** is a circle’s perimeter.
- A **radius** is a straight line from the centre to the circumference.
- A **diameter** is a straight line through the centre, connecting two points on the circumference.
- The radius is half the diameter; the diameter is double the radius.

Example Distinguish between radius and diameter.

For each circle, identify whether the straight lines show the radius, diameter, or neither.

a  Radius Diameter Neither	**b**  Radius Diameter Neither
c  Radius Diameter Neither	**d**  Radius Diameter Neither
e  Radius Diameter Neither	**f**  Radius Diameter Neither
g  Radius Diameter Neither	**h**  Radius Diameter Neither

1 Complete the sentences.

a The circumference is the ……………… of a circle.

perimeter

b The radius is a straight line connecting the ……………………… to the ……………………

Centre, circumference

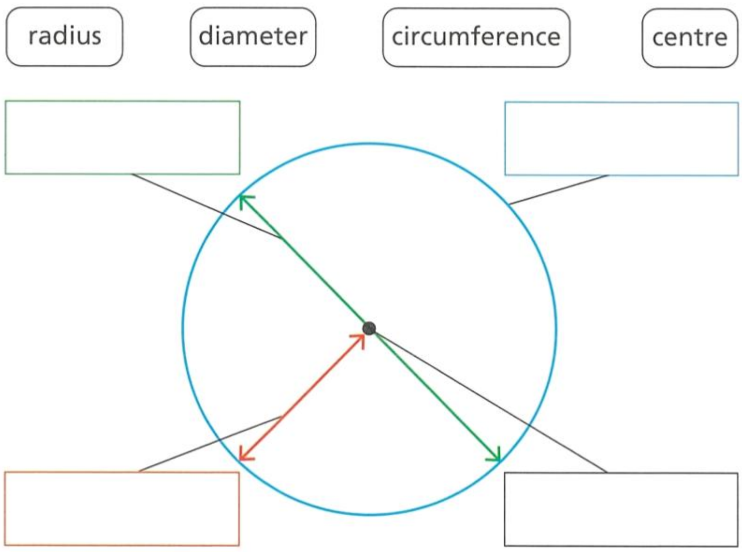
c The diameter is a straight line connecting two points on the ………………………, passing through the ……………………

circumference, centre

d The diameter is ………………… the length of the radius.

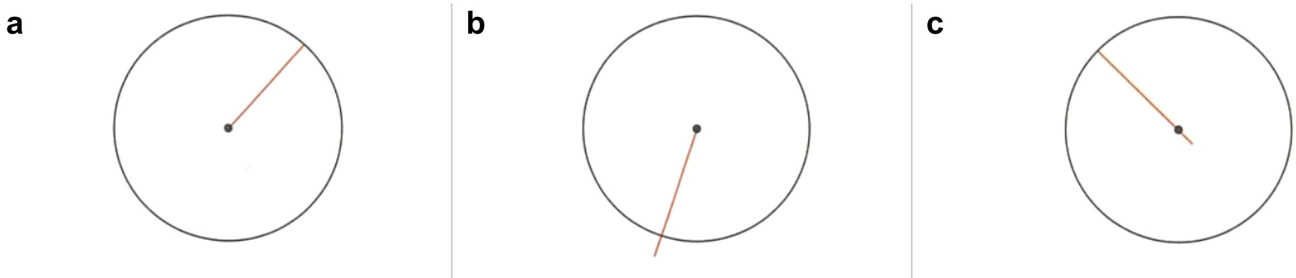
double

2 Label these parts of the circle.



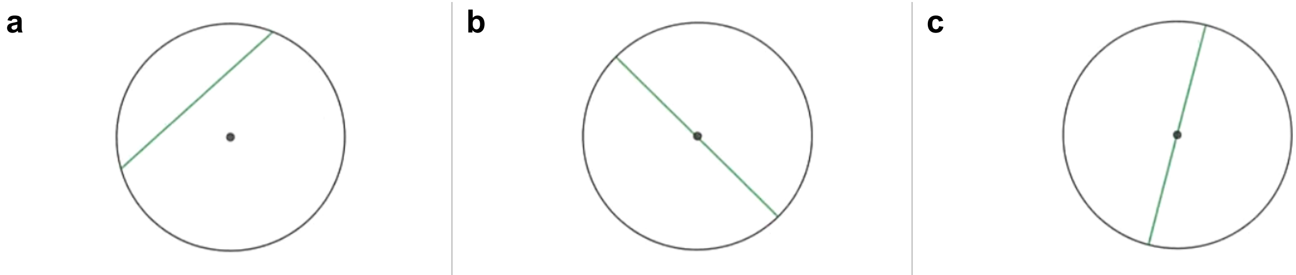
From top left: diameter, circumference, centre, radius

3 Circle which of these (a, b, or c) show a radius.



A

4 Circle which of these (a, b, or c) show a diameter.



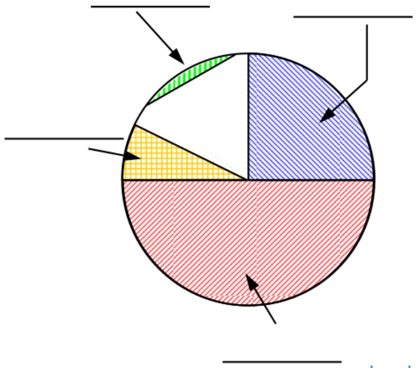
B

5 Complete the table.

Radius	Diameter
4 cm	8 cm
50 mm	100 mm
12.5 cm	25 cm
3.4 cm	6.8 cm
$\frac{3}{4}$ cm	$\frac{6}{4} = \frac{3}{2}$ cm

6 Name the parts of the circle, then complete the sentences.

Answer to diagram, from top left: segment, quadrant, semicircle, sector



a Pizzas are usually sliced into .... sectors

b A .... has a 90-degree angle. quadrant

c Two .... make a semicircle. quadrants

d An arc and two radii make a .... sector

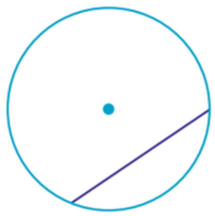
e The .... is a line through the circle that may not touch the centre. chord

f The area between a chord and the circumference is called the .... segment

g A .... has the circle's diameter as one of the sides. Semicircle

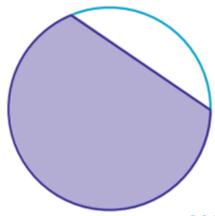
7 Name the highlighted parts of the circle.

a



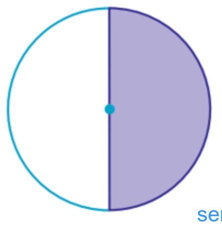
chord

b



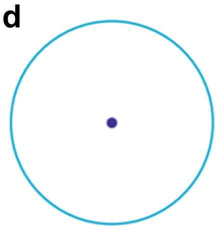
segment

c



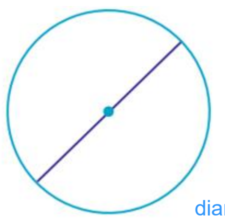
semicircle

d



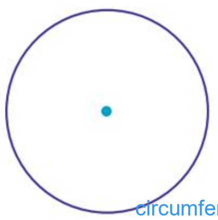
centre

e



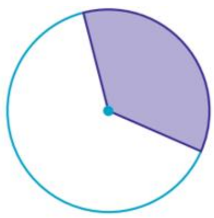
diameter

f



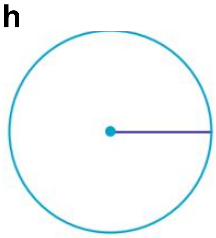
circumference

g



sector

h

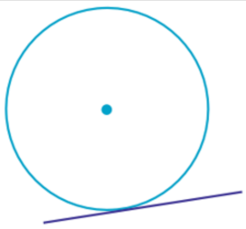


radius

i

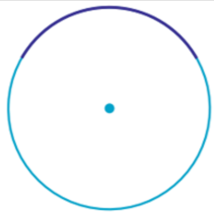
segment

j



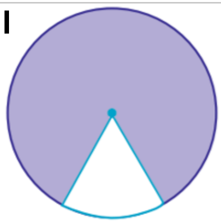
tangent

k



arc

l



sector

CIRCUMFERENCE OF A CIRCLE

Investigation Pi π

Pi: https://www.geogebra.org/m/jvhknzkw

Circumference	Diameter	$\frac{\text{Circumference}}{\text{Diameter}}$
	2	
	3	
	6	
	10	

For any circle, circumference ………………… ……… diameter is always 3.14159…

This means that the diameter of any circle will fit around the ………………… just over 3 times.

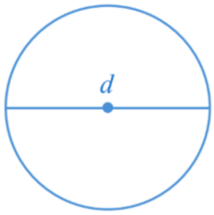
We represent this special number using the pronumeral π, and call it "………"

CALCULATING CIRCUMFERENCE USING DIAMETER

Pi as a Ratio: https://www.geogebra.org/m/wtxutssn

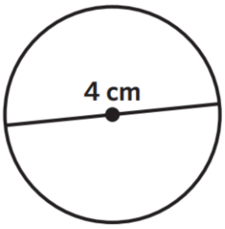
Circumference: Diameter Formula

$$C = \pi \underset{\substack{\uparrow \\ \text{diameter}}}{d}$$



Example Calculate circumference of a circle using diameter.

Calculate the circumference of this circle, round your answer to 3 decimal places.



$C = \pi d$

$= \pi \times 4$

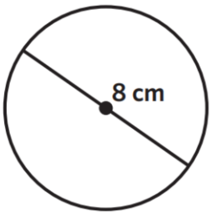
≈ 12.566 cm

Explain how $\pi = \frac{C}{d}$ can be used to get the formula $C = \pi d$

What does the $\approx$ symbol mean?

Calculate the circumference of these circles to 3 decimal places.

a



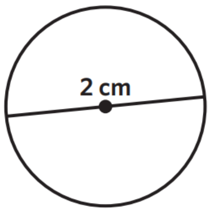
$C = \pi d$

$= \pi \times$ ....

$\approx$ ....

25.133 cm

b



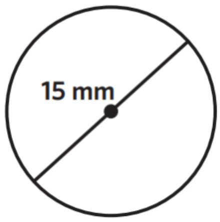
$C = \pi d$

$=$ ....

$=$ ....

6.283 cm

c



$C =$ ....

$=$ ....

$=$ ....

47.124 mm

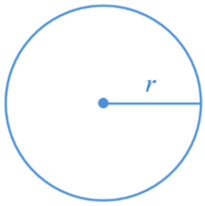
CALCULATING CIRCUMFERENCE USING RADIUS

Circumference: Radius Formula

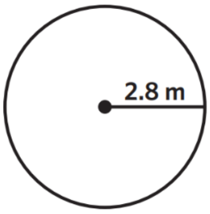
$$C = 2\pi r$$

↑

radius

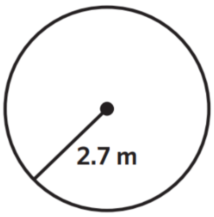


Example Calculate circumference of a circle using radius.

Calculate the circumference of this circle, round your answer to 3 decimal places.	Explain how $C = \pi d$ can be used to get the formula $C = 2\pi r$.
 $C = 2\pi r$ $= 2 \times \pi \times 2.8$ ≈ 17.593 m	

Calculate the circumference of these circles to 3 decimal places.

a



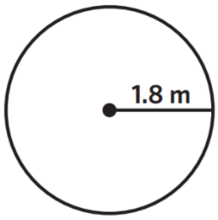
$C = 2\pi r$

$= 2\pi \times$ ....

$\approx$ ....

16.965 m

b



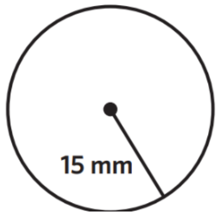
$C = 2\pi r$

$=$ ....

$=$ ....

11.310 m

c



$C =$ ....

$=$ ....

$=$ ....

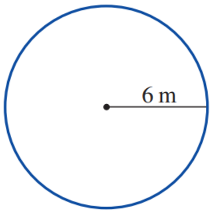
94.248 mm

Key ideas

1. The .... of the circumference to the diameter is always the same number; this number is called ....
2. There are two formulas you can use to calculate the circumference.
   a. .... if you know the diameter.
   b. .... if you know the radius.

1 Identify the radius and diameter of these circles.

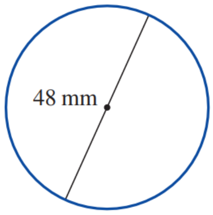
a



Radius: ……………
6 m

Diameter: ……………
12 m

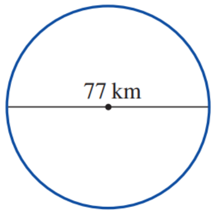
b



Radius: ……………
24 mm

Diameter: ……………
48 mm

c

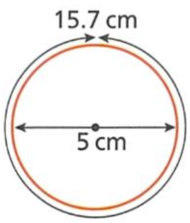


Radius: ……………
38.5 km

Diameter: ……………
77 km

2 Use these diagrams to find approximate values for π

a



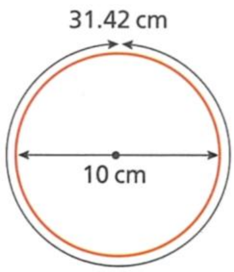
$\pi = \dfrac{C}{d}$

= ………………

= ………………

3.14

b



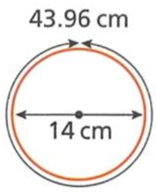
$\pi = \dfrac{C}{d}$

= ………………

= ………………

3.14

c



$\pi = \dfrac{C}{d}$

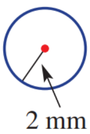
= ………………

= ………………

3.14

3 Calculate the circumference of these circles using a calculator. Choose the correct formula and follow the setting out. Round answers to 2 decimal places.

a



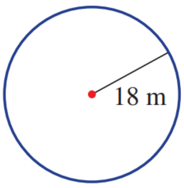
$C = 2\pi r$

= ………………

= ………………

12.57 mm

b



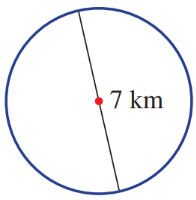
$C = 2\pi r$

= ………………

= ………………

113.10 m

c



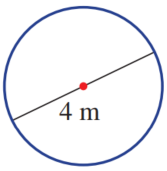
$C = \pi d$

= ………………

= ………………

21.99 km

d



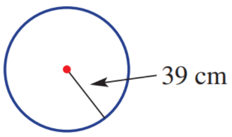
$C =$ ………………

= ………………

= ………………

12.57 m

e



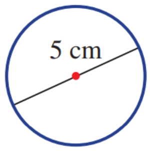
$C =$ ………………

= ………………

= ………………

245.04 km

f



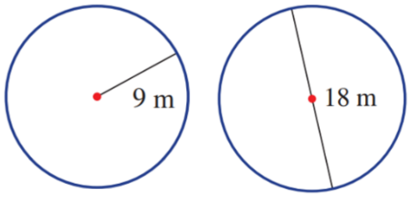
$C =$ ………………

= ………………

= ………………

15.71 cm

4 Calculate the circumferences of these two circles and show that they are the same circle. Explain why.

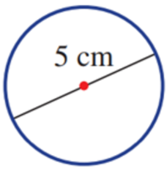


Circle 1: $C = 18\pi$

Circle 2: $C = 18\pi$

A radius of 9 m is the same as a diameter of 18 cm

5 Carefully look at this working out. Explain why it is incorrect.



$$C = 2 \times \pi \times 5$$

$$C = 10 \times \pi$$

$$C = 31.42\ (2\ d.p.)$$

They used $C = 2\pi r$ and incorrectly used 5 as the radius. The radius should be 2.5 cm

6 A water tank has a diameter of 3.5 m. Find its circumference, correct to 1 decimal place.

11.0 m

7 An athlete trains on a circular track of radius 40 m and jogs 10 laps each day, 5 days a week. How far does he jog each week? Round the answer to the nearest whole number of metres.

125 66 m

8 Which sushi roll has the larger circumference?

One with a radius 20 mm or one with a diameter 4.5 cm? Justify your answer.

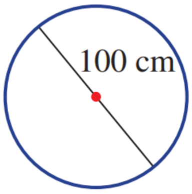
20 cm radius: $C = 125.66$ mm

4.5 cm diameter: $C = 141.37$ mm

The sushi with 4.5 cm diameter has larger circumference.

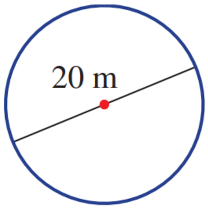
9 It is more precise to give circle calculations in terms of π. $C = 2 \times \pi \times 3$ gives $C = 6\pi$.
This gives the **exact** answer and not a rounded decimal.
Calculate the circumferences of these circles, giving exact answers in terms of π.

a



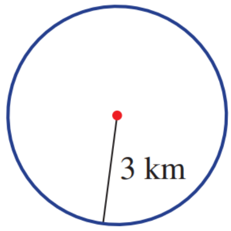
100π cm

b



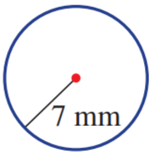
20π m

c



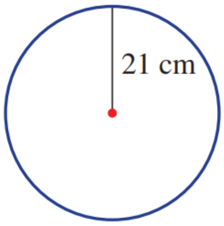
6π km

d



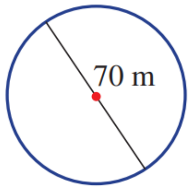
14π mm

e



42π cm

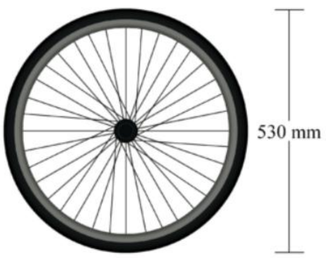
f



70π m

10 **NAPLAN A+**

A wheel on a bicycle has a diameter of 530 mm. The wheel turns 3 times.
Approximately how many metres has the bicycle travelled?



5 m

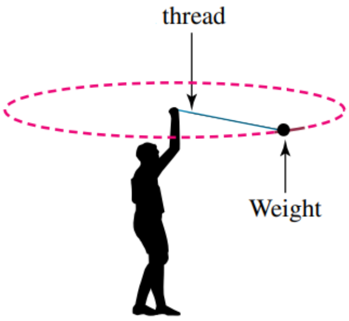
11 Faith goes on a 12 km bike ride. The wheels of her bike each have diameter 55 cm.

How many revolutions do her wheels make in the journey?

1 revolution is 55π cm

$12000 \div 55\pi = 6945$ revolutions

12 In a Physics experiment, students spin a mass around on the end of a thread.
The thread is 80 cm in length. The mass is 5 kg.
Calculate the distance the weight travels if it spins a total of 4230°.
Give your answer in metres rounded to two decimal places.



$4230 \div 360 = 11.75$ revolutions

$11.75 \times 2\pi(0.8) = 59.06$ m

13 Darling Harbour is planning to install a new Ferris wheel. The wheel has a diameter of 80 m.
The pods can carry 10 passengers each but need to be spaced out by at least 10 m around the circumference of the Ferris wheel.
What is the maximum number of passengers that can be on the Ferris wheel at one time?

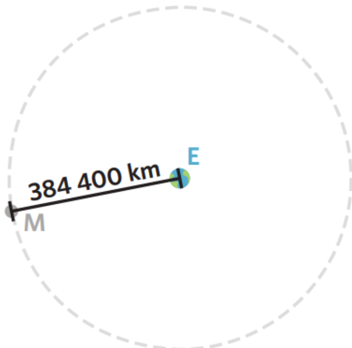
$C = 80\pi$ m

Number of pods = $80\pi \div 10 \approx 25$

Number of people $= 25 \times 10 = 250$ people

14 An object in space orbits another object on a path that is very close to a circle.
Answer these questions rounded to the nearest whole number.

a The Moon is approximately 384,400 km from the Earth.
Calculate the distance the Moon travels in one orbit of Earth.
Round to the nearest whole.



$C = 2\pi(384400) \approx 2\,415\,256$ km

b The Moon takes 27.32 days to orbit the Earth.
Calculate how many kilometres it travels per hour.
Round to the nearest whole.

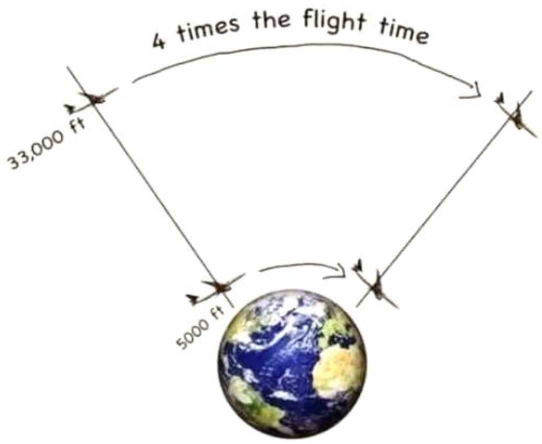
$$\frac{2415256 \text{ km}}{27.32 \text{ days}} = 88406.16 \text{ km/day}$$

$88406.16 \div 24 \approx 3684$ km/h

c The Earth is approximately 150 million km from the Sun. It orbits the Sun in 365.25 days.
Calculate how many kilometres it travels around the Sun each hour.
Round to the nearest whole.

$$\frac{2\pi(150\,000\,000) \text{ km}}{365.25 \times 24 \text{ hours}} = 107\,515 \text{ km/h}$$

15 Mr Ding is scrolling Instagram and comes across this post:



"If the Earth were round, then planes wouldn't fly so far up, as they would waste lots of fuel and money. This proves that the earth must be flat!"

Using mathematical reasoning, justify whether this post is factually correct.

The radius of the Earth is around 21 000 000 feet. Planes usually do fly at 33 000 feet.

Consider a plane flying around the Earth.

At 5000 feet, $C = 2\pi(21\ 005\ 000) = 42\ 010\ 000\pi$

At 33000 feet: $C = 2\pi(21\ 033\ 000) = 42\ 066\ 000\pi$

Comparing the two: $\frac{42\ 066\ 000\pi}{42\ 010\ 000\pi} = 1.001333016 \ldots$

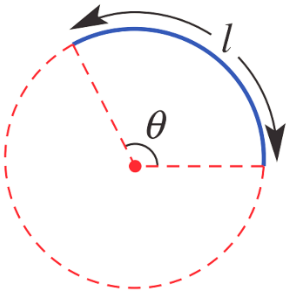
The increase in distance is only 0.13%, so the flight would not take 4 times as long.

The diagram is misleading, as the Earth is not drawn to the correct scale (planes don't fly in space).

ARC LENGTH OF A CIRCLE

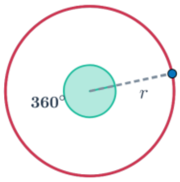
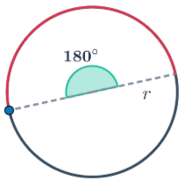
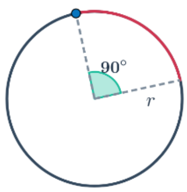
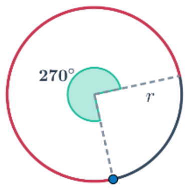
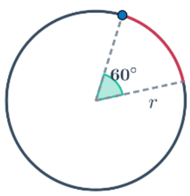
ARC

- An **arc** is a part of the circumference.
- Arcs **subtend**, or form, an angle θ at the centre of an imaginary circle.



Investigation Arc length.

Fill in the table below.

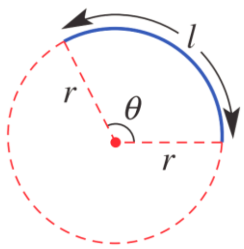
Angle	Diagram	Fraction of a whole circle	Arc length
$360°$		$\dfrac{360°}{360°} = 1$	$l = 2\pi r$
$180°$		$\dfrac{180°}{360°} =$	$l = \underline{\hspace{1.5em}} \times 2\pi r$
$90°$			
$270°$			
$60°$			

The arc length is a ……………… of the circumference. First we find the ……………… of the circumference that the arc length represents by looking at the angle …………………… at the centre, then ………………… the fraction by the circumference.

ARC LENGTH

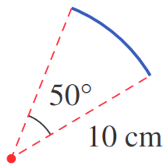
Arc Length of a Circle: https://www.geogebra.org/m/mbwgzpc8

Arc Length Formula
$\ell = \dfrac{\theta}{360} \times 2\pi r$ $\uparrow$ Fraction of circle $\quad \uparrow$ Circumference



Example Calculate arc length of a circle.

Calculate the arc length. Round your answer to 2 decimal places.



$$l = \frac{\theta}{360} 2\pi r$$

$$= \frac{50}{360} 2\pi(10)$$

$= 8.73$ cm

Why do we divide the angle at the centre by 360?

360° represents a whole circle.

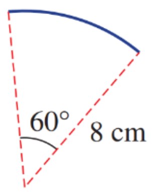
$\frac{\theta}{360}$ *represents the .... of the circle.*

Explain why we multiply the fraction we find by $2\pi r$.

To find a fraction of a whole, we .... the fraction by the whole.

Calculate the arc length. Round your answer to 2 decimal places.

a



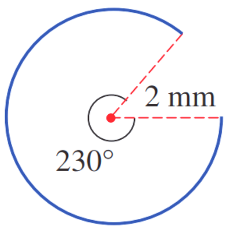
$$l = \frac{\theta}{360} 2\pi r$$

$$= \frac{\quad}{360} 2\pi(8)$$

= ....

8.38 cm

b



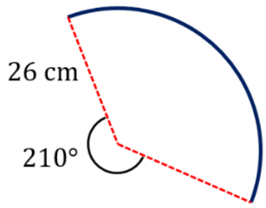
$$l = \frac{\theta}{360} 2\pi r$$

= ....

= ....

8.03 mm

c



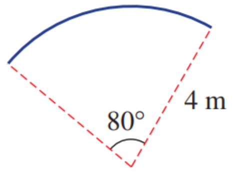
68.07 cm

Key ideas

1. An .... is a part of the circumference.
2. To calculate the length of an arc, we first find the .... of the circumference the arc represents by using the .... at the centre of the circle.

1 Calculate these arc lengths. Round your answer to 2 decimal places.

a



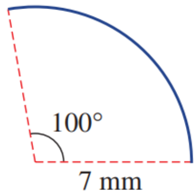
$$l = \frac{\theta}{360} \times 2\pi r$$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

5.59 m

b



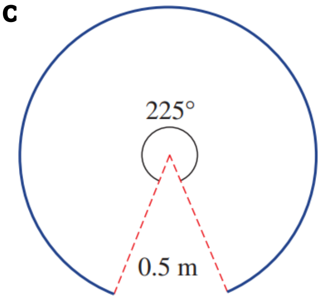
$l = \ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

6.98 mm

c



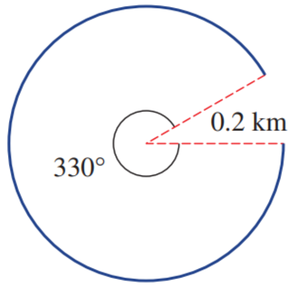
$l = \ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

1.96 m

d



$l = \ldots\ldots\ldots\ldots\ldots\ldots$

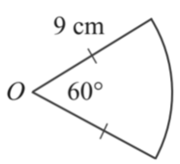
$= \ldots\ldots\ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

1.15 km

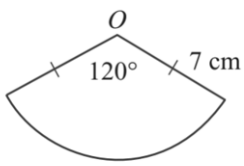
2 Calculate the length of these arcs. Give your answer in exact form by leaving it in terms of π.

a



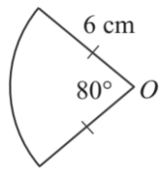
3π cm

b



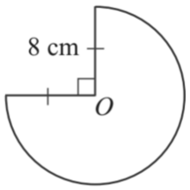
$\frac{14}{3}\pi$ cm

c



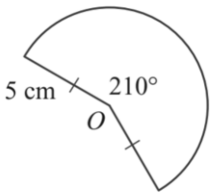
$\frac{8}{3}\pi$ cm

d



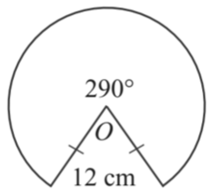
12π cm

e



$\frac{35}{6}\pi$ cm

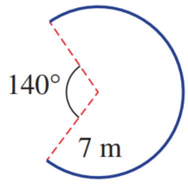
f



$\frac{58}{3}\pi$ cm

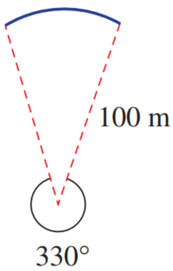
3 Calculate the arc length. The angle shown is the exterior angle. Answer to 2 decimal places.

a



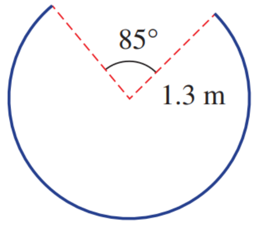
26.88 m

b



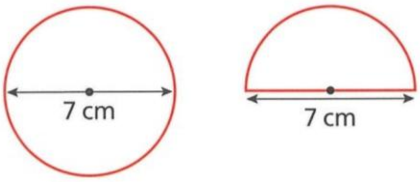
52.36 m

c

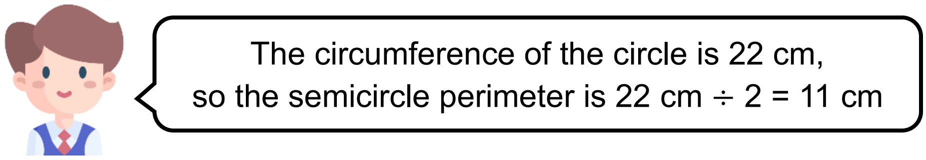


6.24 m

4 Peter works out the circumference of the circle correctly:



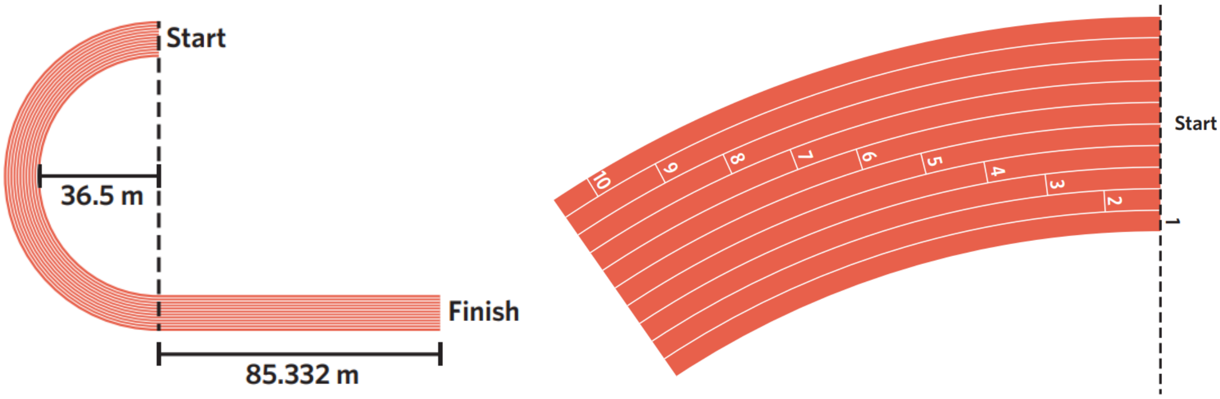
$C = \pi d$
When $d = 7$
$C = \pi \times 7$
$C = 21.99\ldots \text{ cm} \approx 22 \text{ cm}$



Explain why he is wrong and find the correct perimeter of the semicircle.

The perimeter includes the diameter, so it is 18 cm.

5 One lap of an athletics track is 400 m.
One of the races at the Olympics is the 200 m, which is half the track.



a Show that the arc length of a semicircle with a radius of 36.5 m and a straight line of 85.332 m has a total length of 200 m, correct to 3 decimal places.

Arc + straight distance

$$= \frac{1}{2}(2\pi)(36.5) + 85.332$$

$$= 200 \text{ m}$$

b If each lane is one metre wide, that means the second lane is a semicircle with a radius of 37.5 m and a straight line of 85.332 m. Calculate the distance of the second lane.

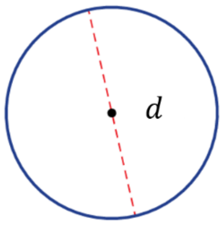
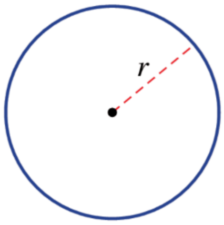
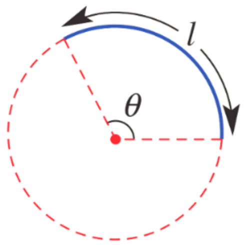
$$= \frac{1}{2}(2\pi)(37.5) + 85.332 = 203.14 \text{ m}$$

c Hence explain why the start of the lanes are slightly offset
(i.e., they don't all start at the same place).

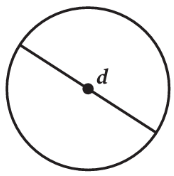
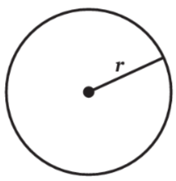
If the athletes all start at the same place, the runners at the outer lanes will run further, therefore the lanes are offset.

FINDING THE RADIUS GIVEN CIRCUMFERENCE

- If you are given the circumference and asked to find the radius:
  1. Write the formula.
  2. Substitute given values.
  3. Solve for the unknown value using inverse operations.

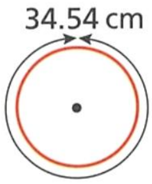
Circumference		**Arc Length**
$C = \pi d$	$C = 2\pi r$	$\ell = \frac{\theta}{360} 2\pi r$
		

Example Calculate the radius given the circumference.

Worked Example	**Your Turn**
The circumference of a circle is 9 cm. What is the diameter? Round to 3 d.p. 1. Write the formula $C = \pi d$ 2. Substitute $9 = \pi d$ 3. Solve $\div \pi \quad \div \pi$ $\frac{9}{\pi} = d$ $2.865 \approx d$ 	The circumference of a circle is 60 cm. What is its diameter? 1. 2. 3. 19.099 cm
The circumference of a circle is 12.8 cm. What is the radius? Round to 3 d.p. 1. Write the formula $C = 2\pi r$ 2. Substitute $12.8 = 2\pi r$ 3. Solve $\div 2\pi \quad \div 2\pi$ $\frac{12.8}{2\pi} = r$ $2.037 \approx r$ 	The circumference of a circle is 10.6 cm. What is its radius? 1. 2. 3. 1.687 cm

1 Calculate the diameter of these circles, given the circumference. Round to nearest whole.

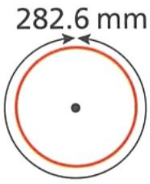
a



$C = \pi d$

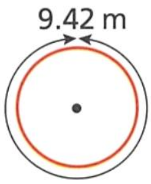
11 cm

b



90 cm

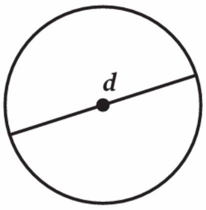
c



3 cm

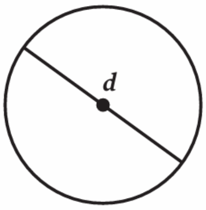
2 Determine the values of the unknowns, correct to 3 decimal places.

a $C = 5$ m



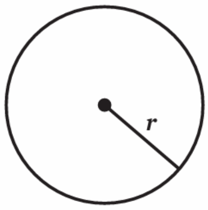
1.592 m

b $C = 15.3$ cm



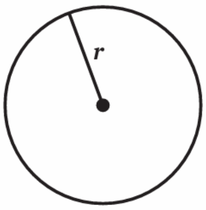
4.870 cm

c $C = 6$ km



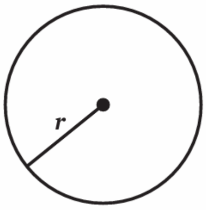
0.955 km

d $C = 7.6$ mm



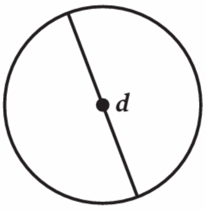
1.210 mm

e $C = 0.23$ km



0.037 km

f $C = \frac{3\pi}{2}$ cm



$\frac{3}{2}$ cm

3 **NAPLAN A**

A car tyre has a circumference of 120 cm. Which of these is closest to the radius of the tyre?

9.5 cm	19.1 cm	38.0 cm	753.6 cm
⬭	⬭	⬭	⬭

$120 = 2\pi r$

$r = \frac{120}{2\pi} = 19.09859\ldots$

B

4 The Earth is approximately a perfect circle at the equator.

The distance around the equator is 40 075 km.

Estimate the distance from the surface of the Earth to the centre, answer to the nearest km.

$40075 = 2\pi r$

$r = \frac{40075}{2\pi} \approx 6378 \text{ km}$

5 Flo uses a tape measure to find the circumference of a tree trunk.
She finds the circumference is 150 cm. She divided by 3 to estimate the diameter of the trunk.

a Explain why she divided by 3.

$\pi \approx 3$

b Would the actual diameter be more or less? Why?

$\pi \approx 3.14 > 3$. The actual diameter would be smaller, as dividing by a larger number gives a smaller result.

c What should she divide by to estimate the radius?

Divide diameter by 2, or divide circumference by 6.

6 Imagine that we wrapped a giant belt tightly around the Earth, along the Equator.
This belt would touch the Earth at all points on its circumference, assuming the Earth was a perfect sphere.

a If the diameter of the Earth is 12 755 metres,
calculate to 2 decimal places, the length of the tight belt.

$C = \pi(12755) \approx 40071.01 \text{ m}$

b Now suppose we added one extra metre to the length of the belt.
Then it would become loose and not touch the Earth anymore.
Add 1 metre to the length of the tight belt. What is its length?

$C = 40072.01 \text{ m}$

c Calculate the diameter of the longer, looser belt.

$d = \frac{C}{\pi} = \frac{40072.01}{\pi} = 12755.32 \text{ m}$

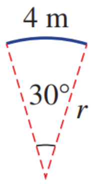
d What is the gap between the Earth and the loose belt?

The gap is half the difference of the two diameters.

$(12755.32 - 12755) \div 2 = 0.16 \text{ m}$

7 Calculate the radius of these shapes, given the arc length. Answer to 2 d.p.

a

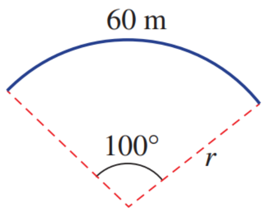


$$l = \frac{\theta}{360} \times 2\pi r$$

$$4 = \frac{30}{360} \times 2\pi r$$

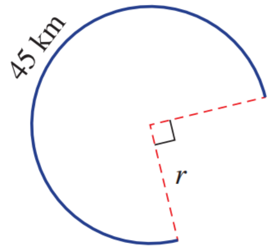
7.64 cm

b



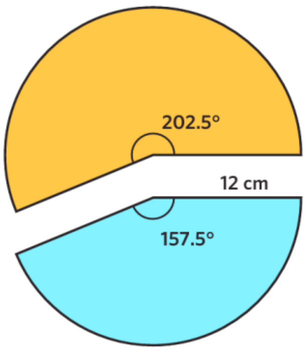
34.38 m

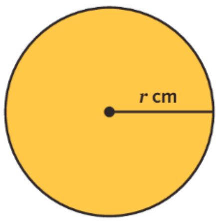
c



9.55 km

8 Sean's frisbee has a radius of 12 cm.
He cuts it into sectors of 202.5° and 157.5° and turns both into smaller frisbees.
What is the radius of the bigger of the two new frisbees? Answer to 2 d.p.





Let arc length of orange sector be l

$$l = \frac{202.5}{360}(2\pi)(12) = 42.41150082\ldots$$

This is the same as the circumference of the new frisbee

$$42.41150082 = 2\pi r$$

$$r = 6.75 \text{ cm}$$

9 Suppose a belt tightly stretched around the equator of the earth just fits.
The diameter of Earth is 12 755 km.

a How much longer should you make the belt so that the belt could encircle the earth at a distance of 1 m away at all points?

Tight belt length = $C = \pi(12755) \approx 40071.0143$ km

Loose belt length = $C = \pi(12756) = 40077.2974$ km

Difference = $40077.2974 - 40071.043 \approx 6.283 \approx 2\pi$

b Explain why the answer to "how much longer should you make a belt such that it encircles 1 m away from the original circumference" is *always* 2π.

Original diameter: d
Original circumference: πd

New diameter: $d + 2$
New circumference: $\pi(d + 2) = \pi d + 2\pi$

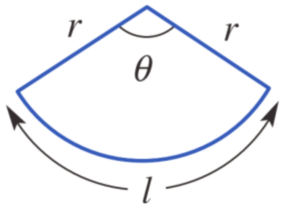
$\therefore$ new circumference is always exactly 2π more, doesn't matter if it's a balloon or the Earth.

PERIMETER OF A SECTOR

- A sector is the area enclosed by two radii and the arc between them.
- The perimeter of a sector is the sum of the arc length and the two radii.

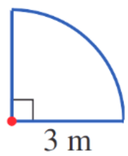
Perimeter of a Sector

$$P = l + 2r$$



Example Calculate perimeter of a sector.

Calculate the perimeter of the sector, round your answer to 1 decimal place.



$$P = l + 2r$$

$$= \left(\frac{90}{360} 2\pi(3)\right) + (2)(3)$$

$$= 10.7 \text{ m}$$

Where does the formula $P = l + 2r$ come from?

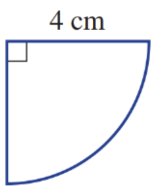
The total distance around a sector is the two .... plus the ....

Where did $\frac{90}{360} 2\pi(3)$ come from?

The formula for arc-length is ....

Calculate the perimeter of these sectors to one decimal place.

a



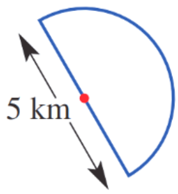
$P = l + 2r$

$= \frac{\quad}{360} 2\pi(\quad) + 2(\quad)$

$=$ ....

14.3 cm

b



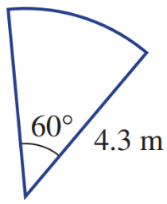
$P = l + 2r$

$=$ ....

$=$ ....

12.9 km

c



13.1 m

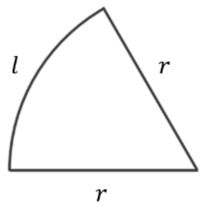
Key ideas

1. A sector is the area between two .... and an arc.
2. The perimeter of an object is the total distance .... a 2D shape.
3. To calculate the perimeter of a sector, we add the two .... and the length of the ....

FOUNDATION

1 Describe the difference between the arc length and the perimeter of a sector.

Refer to the diagram.

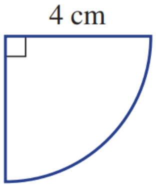


Arc length is just l

Perimeter is all sides on the outside of the shape $l + 2r$

2 Calculate the perimeter of these shapes. Answer to 2 d.p.

a



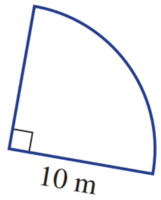
$P = l + 2r$

$= \frac{}{360} \times 2\pi(\quad) + 2(\quad)$

$=$ ....

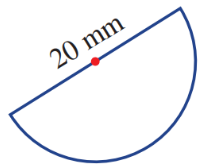
14.28 cm

b



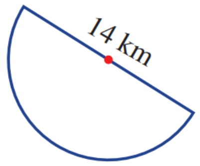
35.71 m

c



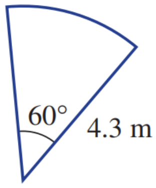
51.42 mm

d



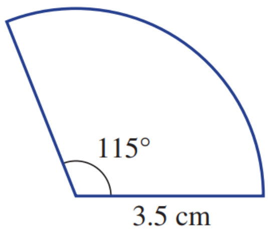
35.99 km

e



13.1 m

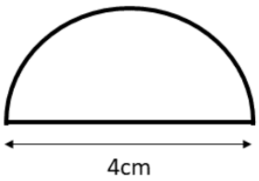
f



14.02 cm

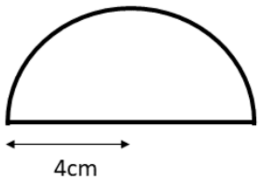
3 Calculate the perimeter of these shapes. Answer to 1 d.p.

a



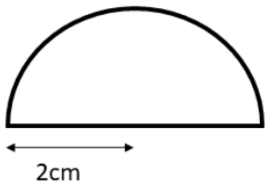
10.28 cm

b



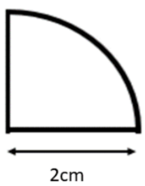
20.57 cm

c



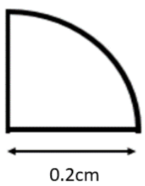
10.28 cm

d



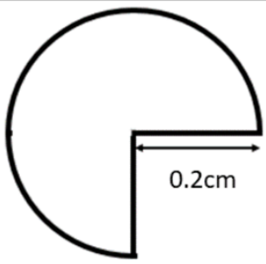
7.14 cm

e



0.71 cm

f

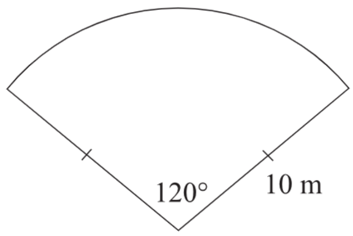


1.34 cm

DEVELOPMENT

4 2019 HSC Standard 2 Band 4

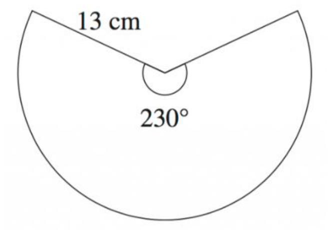
The diagram shows a sector with an angle of 120° cut from a circle with radius 10 m.
What is the perimeter of the sector? Write your answer correct to 1 decimal place.



$$P = \frac{120}{360} \times 2\pi(10) + 20$$
$$= 40.94 \text{ m}$$

5 2012 HSC Standard 2 Band 5

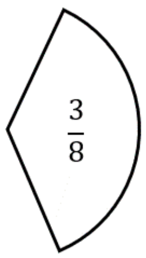
The sector shown has a radius of 13 cm and an angle of 230°.
What is the perimeter of the sector to the nearest centimetre?



$$P = \frac{230}{360} \times 2\pi(13) + 26$$
$$= 78.19 \text{ cm}$$

6 NAPLAN A+

Peter cuts a sector from a circle so that three eighths of the area remains.
If the circle's radius is 5 cm, what is the perimeter of the shape, to the nearest centimetre?



$$P = \frac{3}{8} \times 2\pi(5) + 10 = 22 \text{ cm}$$

7 The perimeter of a sector is 40 cm.
If the radius of the sector is 10 cm, what is the angle at the centre? Answer to 1 d.p.

$$P = l + 2r$$
$$40 = \frac{\theta}{360} \times 2\pi(10) + 2(10)$$
$$40 = \frac{\pi}{360} \times 20\pi + 20$$
$$20 = \frac{\theta}{360} \times 20\pi$$
$$7200 = 20\pi\theta$$
$$\theta = \frac{7200}{20\pi} \approx 114.6°$$

8 If the radius of a sector is 16 cm, is it possible to make a sector with perimeter 215 cm? Justify your answer.

$$P = \left(\frac{\theta}{360}\right)(2\pi r) + 2r$$

$$215 = \left(\frac{\theta}{360}\right)(2\pi)(16) + 2(16)$$

$$215 = \left(\frac{\theta}{360}\right)(32\pi) + 32$$

$$183 = \frac{32\pi\theta}{360}$$

$$\theta = \frac{183 \times 360}{32\pi}$$

$$\theta \approx 655.7°$$

No, it is not possible as a sector cannot have an angle of 655.7°

9 The radii of a sector subtend an angle of $40°$ at the centre. Its perimeter is 10 cm. What is the radius? Answer to 2 d.p.

$$P = l + 2r$$

$$10 = \frac{40}{360} \times 2\pi r + 2r$$

$$10 = \frac{2\pi r}{9} + 2r$$

$$10 = r\left(\frac{2\pi}{9} + 2\right)$$

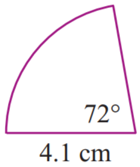
$$r = \frac{10}{\left(\frac{2\pi}{9} + 2\right)} \approx 3.71 \text{ cm}$$

PERIMETER OF SHAPES INVOLVING ARC LENGTH

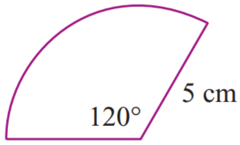
Prior knowledge check

1 Calculate the perimeter of these sectors.

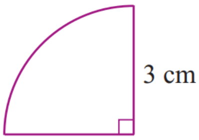
a



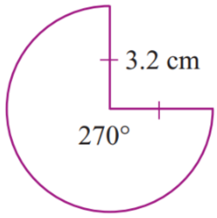
b



c



d



2 Simplify the following, leaving your answer in exact form.
Do not round or use a calculator. Remember to follow the order of operations.

a $2 \times 3 \times \pi = 6\pi$

b $6 \times 2\pi$

c $7 \times 2.5 \times \pi$

d $3 + \frac{1}{2} \times 4\pi$

e $2 \times 6 + \frac{1}{4} \times 12\pi$

f $2 \times 5 + \frac{2}{5} \times 2 \times \pi \times 5$

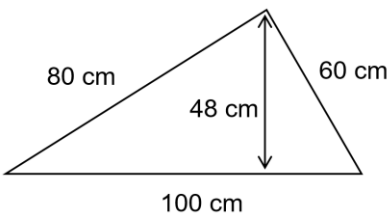
g $2 \times 4 + \frac{90}{360} \times 2 \times \pi \times 4$

h $3 + \frac{270}{360} \times \pi$

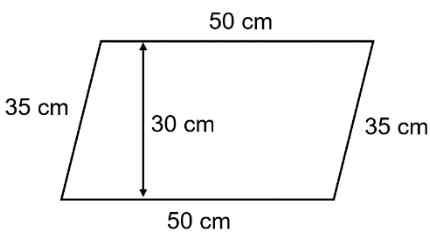
i $7 + \frac{30}{360} \times \pi$

3 Work out the perimeter of these shapes.

a



b

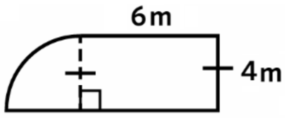


PERIMETER OF SHAPES INVOLVING ARC LENGTH

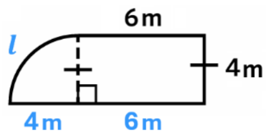
- To calculate the perimeter of a composite shape involving arc length:
  1. Label the diagram showing all lengths of the perimeter.
  2. Add the straight-line segments and the arc length.
  3. Calculate the answer.

Example Calculate perimeter of a shape involving an arc length.

Calculate the perimeter of this shape, round your answer to 2 d.p.



1.



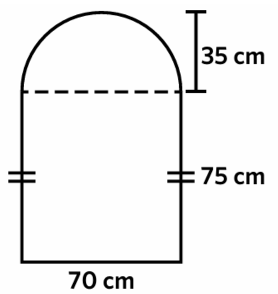
2. $P = 6 + 4 + 6 + 4 + l$

3. $P = 20 + \left(\frac{90}{360} \times 2\pi(4)\right)$

$P = 26.28$ m

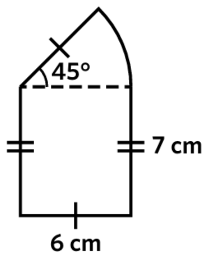
Calculate the perimeter of these sectors to one decimal place.

a



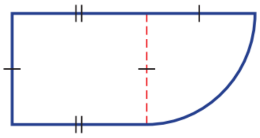
330.0 cm

b



30.7 cm

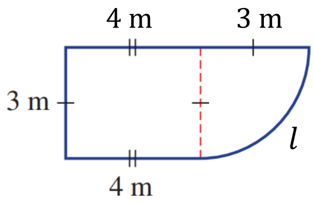
1 Explain why you cannot use the formula $P = l + 2r$ for this shape.



$l + 2r$ is the perimeter of a sector. This is not a sector.

2 Calculate the perimeter of these shapes. Answer to 1 d.p.

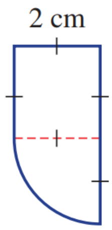
a



$P = 3 + 4 + 3 + 4 + l$

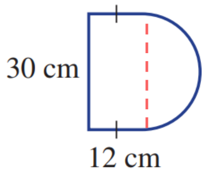
18.7 m

b



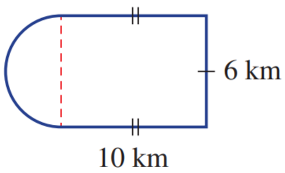
11.14 cm

c



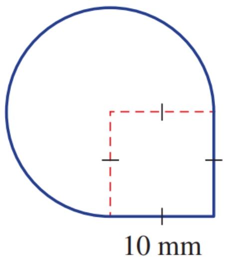
101.12 cm

d



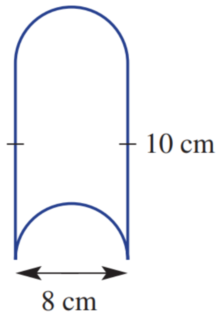
35.42 km

e



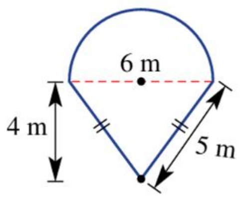
67.12 mm

f



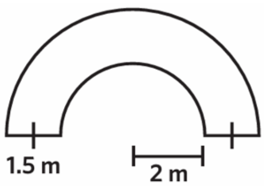
45.13 cm

g



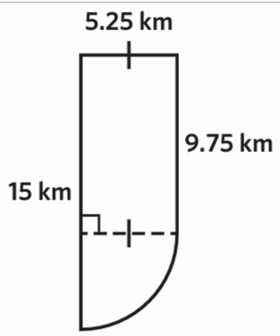
19.42 m

h



20.28 m

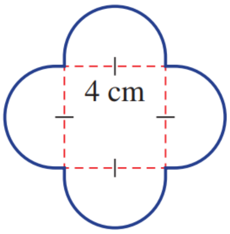
i



51.21 km

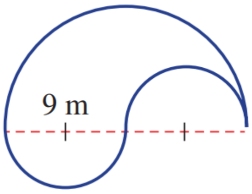
3 Calculate the perimeter of these shapes. Leave your answer in exact form in terms of π.

a



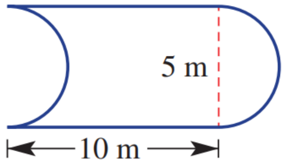
8π cm

b



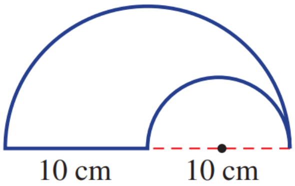
18π m

c



$20 + 5\pi$ m

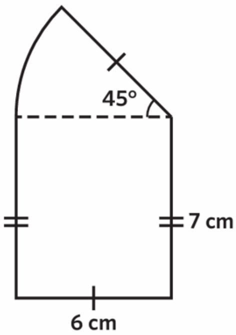
4 Find the exact perimeter of this shape. The curved sections are semicircles.



$15\pi + 10$ cm

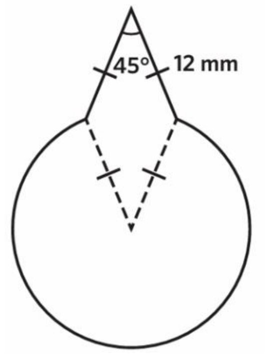
5 Find the perimeter of these shapes. Answer to 1 d.p.

a



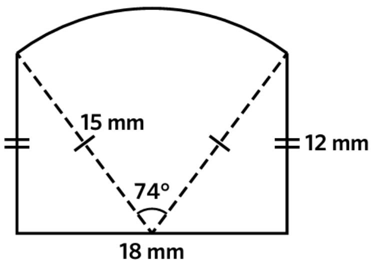
30.7 cm

b



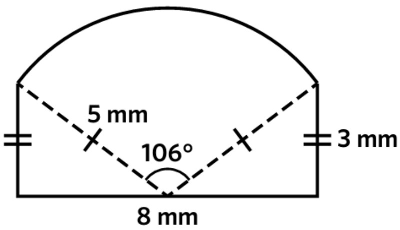
90.0 mm

c



61.37 mm

d

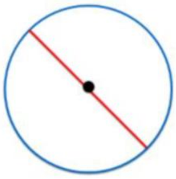


23.3 mm

CHECK YOUR UNDERSTANDING

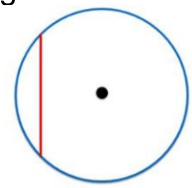
Features of circles

1 What is the correct name for the straight line shown?



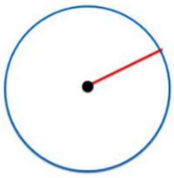
a Arc **b** Diameter **c** Radius **d** Tangent

2 What is the correct name for the straight line shown?



a Diameter **b** Chord **c** Tangent **d** Radius

3 What is the correct name for the straight line shown?

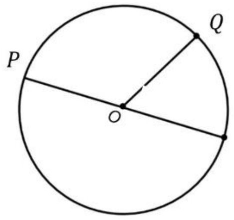


a Radius **b** Tangent **c** Chord **d** Diameter

4 What is the correct name for the perimeter of a circle?

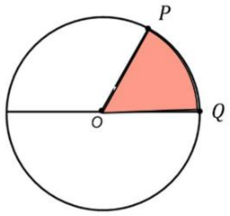
a Chord **b** Sector **c** Circumference **d** Tangent

5 What is the correct name for the curve that joins P and Q?



a Circumference **b** Segment **c** Sector **d** Arc

6 What is the correct name for the shaded part of the circle?



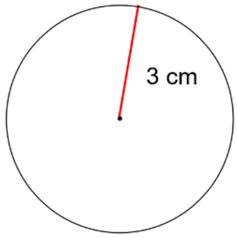
a Arc **b** Sector **c** Segment **d** Section

7 If the diameter of a circle is 3.6 cm, then the radius is:

a 1.8 cm **b** 3.6 cm **c** 1.9 cm **d** 7.2 cm

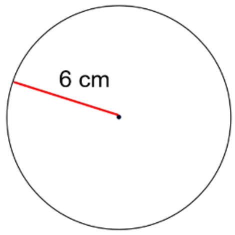
Circumference

1 Calculate the circumference of this circle.
Answer in terms of π



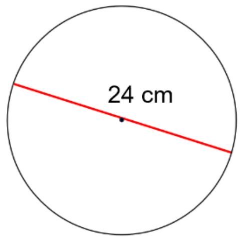
a 6π cm **b** 9π cm **c** 1.5π cm **d** 3π cm

2 Calculate the circumference of this circle.
Answer in terms of π



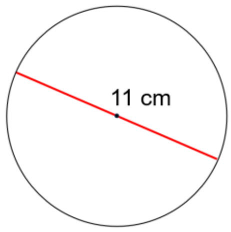
a 3π cm **b** 36π cm **c** 12π cm **d** 9π cm

3 Calculate the circumference of this circle.
Answer in terms of π



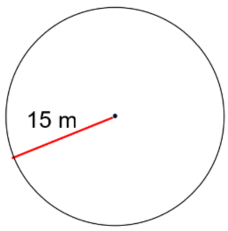
a 6π cm **b** 12π cm **c** 24π cm **d** 144π cm

4 Calculate the circumference of this circle.
Round your answer to 1 d.p.



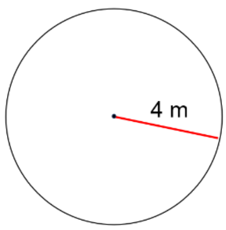
a 95.0 cm **b** 380.1 cm **c** 17.3 cm **d** 34.6 cm

5 Calculate the circumference of this circle
Round your answer to 1 d.p.



a 2828.4 m **b** 47.1 m **c** 706.9 m **d** 94.2 m

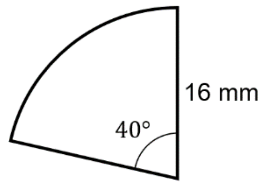
6 Calculate the circumference of this circle
Round your answer to 2 d.p.



a 12.57 m **b** 25.13 m **c** 50.27 m **d** 39.48 m

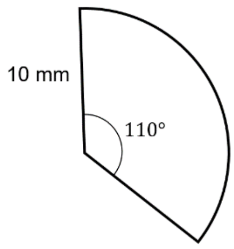
Arc length

1 Which expression represents the arc length?



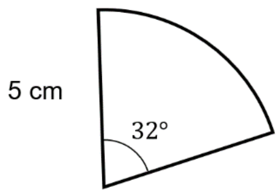
a $\frac{360}{40} \times \pi \times 32$ **b** $\frac{360}{40} \times \pi \times 16$ **c** $\frac{30}{360} \times \pi \times 32$ **d** $\frac{40}{360} \times \pi \times 16$

2 Which expression represents the arc length?



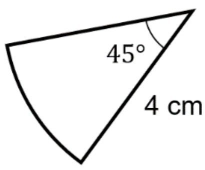
a $\frac{110}{360} \times \pi \times 10$ **b** $\frac{360}{110} \times \pi \times 10^2$ **c** $\frac{110}{360} \times \pi \times 20$ **d** $\frac{360}{110} \times \pi \times 20$

3 Which expression represents the arc length AB?



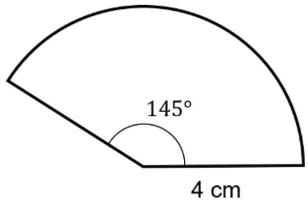
a $\frac{32}{360} \times \pi \times 25$ **b** $\frac{180}{32} \times \pi \times 5$ **c** $\frac{32}{360} \times \pi \times 10$ **d** $\frac{32}{180} \times \pi \times 10$

4 Which expression represents the arc length?



a 4π cm **b** 2π cm **c** $\frac{8\pi}{45}$ cm **d** π cm

5 What is the arc length, rounded to 1 d.p.?



a 62.4 cm **b** 5.1 cm **c** 10.1 cm **d** 20.2 cm