

Student Name

Mathematics Stage 5 Paths

INDICES C

Book 1 Describe surds

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OVERVIEW

SYLLABUS CONTENT

MA5-IND-P-02 describes and performs operations with surds and fractional indices

Describe surds

- Describe a real number as a number that can be represented by a point on the number line
- Examine the differences between rational and irrational numbers and recognise that all rational and irrational numbers are real
- Convert between recurring decimals and their fractional form using digital tools
- Describe the term *surd* as referring to irrational expressions of the form $\sqrt[n]{x}$ where x is a rational number and n is an integer such that $n \geq 2$, and $x > 0$ when n is even
- Recognise that a surd is an exact value that can be approximated by a rounded decimal
- Demonstrate that $\sqrt{x}$ is undefined for $x < 0$ and that $\sqrt{x} = 0$ when $x = 0$ using digital tools
- Describe $\sqrt{x}$ as the positive square root of x for $x > 0$ and $\sqrt{0} = 0$

REVIEW OF PRIOR KNOWLEDGE

1 Express these recurring decimals using dot or bar notation.

a $0.777777777777 \dots$

b $0.377777777777 \dots$

c $0.828282828282 \dots$

d $0.872872872872 \dots$

e $0.702170217021 \dots$

f $0.825555555555 \dots$

2 Using a calculator, express these fractions as decimals using division.

a $\frac{1}{3}$

b $\frac{1}{6}$

c $\frac{10}{12}$

d $\frac{17}{18}$

e $\frac{293}{300}$

f $\frac{4}{9}$

3 Round these decimals to two decimal places.

a 3.4567

b 2.3479

c 0.7

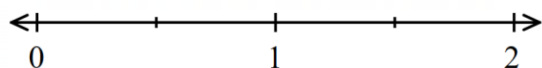
d 89.231

e 9.499

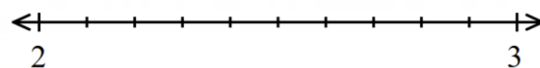
f 5.5851

4 Place a dot to represent these decimals on the number lines.

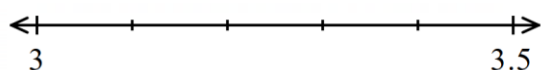
a 1.7



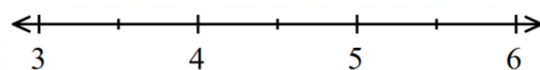
b 2.35



c 3.15



d 4.75



TERMINATING AND RECURRING DECIMALS AS FRACTIONS

TERMINATING AND RECURRING DECIMALS

- **Terminating** decimals **end**, for example: 0.25
- **Recurring** decimals **repeat forever**, for example: 0.333333333333333333333333333333...
- Recurring digits are shown as:
 - a dot for a single recurring digit. $0.666666 \dots = 0.\dot{6}$
 - a bar for many recurring digits. $0.686868 \dots = 0.\overline{68}$

TERMINATING AND RECURRING DECIMALS ARE RATIONAL

- Every terminating decimal can be written as a fraction using knowledge of place value:

$$\begin{aligned} 0.827 &= 827 \text{ thousandths} \\ &= \frac{827}{1000} \end{aligned}$$

- Every recurring decimal can also be written as a fraction by looking at patterns.
You will learn a formal proof after learning algebraic equations.

$$0.\dot{3} \rightarrow \frac{1}{3}$$

- 1
- $\frac{1}{3}$ as a decimal is 0.333333333333333...

a

Express this recurring decimal using the correct notation.

0.3

b

Without using a calculator, what would $\frac{2}{3}$ be?

0.6

c

Check $2 \div 3$ on a calculator. Explain why it is different to your answer to part b.

The calculator ran out of digits in its display, so it rounded the last digit up to 7.

2

Convert to a decimal:

a	$\frac{1}{3} = 0.3333333333333333 \dots = 0.\dot{3}$	h	$\frac{1}{7} =$	0.142857
b	$\frac{2}{3} =$	i	$\frac{2}{7} =$	0.285714
c	$\frac{2}{6} =$	j	$\frac{3}{7} =$	0.428571
d	$\frac{3}{6} =$	k	$\frac{4}{7} =$	0.571428
e	$\frac{4}{6} =$	l	$\frac{5}{7} =$	0.714285
f	$\frac{6}{6} =$	m	$\frac{6}{7} =$	0.857142
g	$\frac{7}{6} =$			

3

Using a calculator, convert these to decimals.

a	$\frac{1}{9}$	b	$\frac{2}{9}$	c	$\frac{3}{9}$
d	$\frac{4}{9}$	e	$\frac{7}{9}$	f	$\frac{8}{9}$

Using the pattern above, express these as fractions:

g	0.4	h	0.8	i	0.6
	$\frac{4}{9}$		$\frac{8}{9}$		$\frac{6}{9} = \frac{2}{3}$

4 Using a calculator, convert these to decimals.

a $\frac{1}{99}$	b $\frac{8}{99}$	c $\frac{14}{99}$
$0.\overline{01}$	$0.\overline{08}$	$0.\overline{14}$
d $\frac{48}{99}$	e $\frac{72}{99}$	f $\frac{87}{99}$
$0.\overline{48}$	$0.\overline{72}$	$0.\overline{87}$

Using the pattern above, express these as fractions:

g $0.\overline{64}$	h $0.\overline{05}$	i $0.\overline{93}$
$\frac{64}{99}$	$\frac{5}{99}$	$\frac{93}{99} = \frac{31}{33}$

5 Using a calculator, convert these to decimals.

a $\frac{1}{999}$	b $\frac{12}{999}$	c $\frac{412}{999}$
$0.\overline{001}$	$0.\overline{012}$	$0.\overline{412}$

Using the pattern above, express these as fractions:

d $0.\overline{942}$	e $0.\overline{023}$	f $0.\overline{138}$
$\frac{942}{999} = \frac{314}{333}$	$\frac{23}{999}$	$\frac{138}{999}$

6 Convert these decimals to fractions.

a $0.\dot{3}$	g $0.1\dot{4}$	m $1.0\overline{14}$
$\frac{1}{3}$	$\frac{1}{10} + \frac{4}{90} = \frac{13}{90}$	$1\frac{144}{990} = 1\frac{7}{445}$
b $0.\overline{34}$	h $0.01\dot{4}$	n $2.\dot{4}$
$\frac{34}{99}$	$\frac{1}{100} + \frac{4}{900} = \frac{13}{900}$	$2\frac{4}{9}$
c $0.\overline{35}$	i $0.\overline{014}$	o $2.\overline{43}$
$\frac{35}{99}$	$\frac{14}{999}$	$2\frac{43}{99}$
d $0.\overline{357}$	j $1.\dot{4}$	p $3.\overline{43}$
$\frac{357}{999}$	$1\frac{4}{9}$	$3\frac{43}{99}$
e $0.\dot{4}$	k $1.0\dot{4}$	q $3.0\overline{43}$
$\frac{4}{9}$	$1\frac{13}{90}$	$3\frac{43}{990}$
f $0.0\dot{4}$	l $1.\overline{14}$	
$\frac{4}{90} = \frac{2}{45}$	$1\frac{14}{99}$	

- 7 Here is an example of converting recurring decimals to fractions without using a calculator. Read the example carefully and answer the questions.

Worked Solution	Explanation
Convert $0.\dot{6}$ to a fraction. Let $x = 0.6666666 \dots$ (1) $10x = 6.6666666 \dots$ (2) Using subtraction: (2) – (1): $\begin{array}{r} 9x = 6 \\ x = \frac{6}{9} \\ \therefore x = 0.6666666 \dots = \frac{6}{9} \end{array}$	Why is $10x = 6.6666666 \dots$? How did we get the equation $9x = 6$? How did we determine that $0.6666666 \dots = \frac{6}{9}$?

Convert these recurring decimals to fractions using the above method.

a $0.\dot{8}$ $x = 0.8888888 \dots$ $10x = 8.8888888 \dots$ $9x = \dots\dots\dots$ $\therefore x = \dots\dots\dots$ $\frac{8}{9}$	b $0.\dot{4}$ $x = 0.4444444 \dots$ $10x = \dots\dots\dots$ $9x = \dots\dots\dots$ $\therefore x = \dots\dots\dots$ $\frac{4}{9}$	c $0.\dot{7}$ $\frac{7}{9}$
d $1.\dot{2}$ $\frac{2}{9}$	e $5.\dot{7}$ $\frac{7}{9}$	f $8.\dot{2}$ $\frac{2}{9}$
g $0.\overline{81}$ $x = 0.81818181 \dots$ $100x = 81.81818181 \dots$ $99x = \dots\dots\dots$ $\therefore x = \dots\dots\dots$ $\frac{9}{11}$	h $0.\overline{69}$ $\frac{23}{33}$	i $1.\overline{43}$ $1\frac{43}{99}$

9

a	0.1	b	0.01	c	0.81	d	0.18	e	0.118
	$\frac{1}{9}$		$\frac{1}{90}$		$\frac{73}{90}$		$\frac{18}{99} = \frac{2}{11}$		$\frac{13}{110}$
f	0.2	g	0.02	h	0.72	i	0.27	j	0.527
	$\frac{2}{9}$		$\frac{2}{90} = \frac{1}{45}$		$\frac{65}{90} = \frac{13}{18}$		$\frac{27}{99} = \frac{3}{11}$		$\frac{29}{55}$
k	0.3	l	0.03	m	0.63	n	0.36	o	0.436
	$\frac{3}{9} = \frac{1}{3}$		$\frac{3}{90} = \frac{1}{30}$		$\frac{55}{90} = \frac{11}{18}$		$\frac{36}{99} = \frac{4}{11}$		$\frac{24}{55}$
p	0.4	q	0.04	r	0.54	s	0.45	t	0.345
	$\frac{4}{9}$		$\frac{4}{90} = \frac{2}{45}$		$\frac{49}{90}$		$\frac{45}{99} = \frac{5}{11}$		$\frac{19}{55}$
u	0.5	v	0.05	w	0.45	x	0.54	y	0.954
	$\frac{5}{9}$		$\frac{5}{90} = \frac{1}{18}$		$\frac{41}{90}$		$\frac{54}{99} = \frac{6}{11}$		$\frac{21}{22}$
z	0.6	aa	0.06	bb	0.36	cc	0.63	dd	0.263
	$\frac{6}{9} = \frac{2}{3}$		$\frac{6}{90} = \frac{1}{15}$		$\frac{33}{90} = \frac{11}{30}$		$\frac{63}{99} = \frac{7}{11}$		$\frac{29}{110}$
ee	0.7	ff	0.07	gg	0.27	hh	0.72	ii	0.772
	$\frac{7}{9}$		$\frac{7}{90}$		$\frac{25}{90} = \frac{5}{18}$		$\frac{72}{99} = \frac{8}{11}$		$\frac{17}{22}$
jj	0.8	kk	0.08	ll	0.18	mm	0.81	nn	0.681
	$\frac{8}{9}$		$\frac{4}{45}$		$\frac{17}{90}$		$\frac{81}{99} = \frac{9}{11}$		$\frac{15}{22}$

IRRATIONAL NUMBERS

RATIONAL NUMBERS

- **Rational numbers** are a way to represent values between integers.
- A rational number can be written as a ratio (or fraction):

$$\frac{a}{b}$$

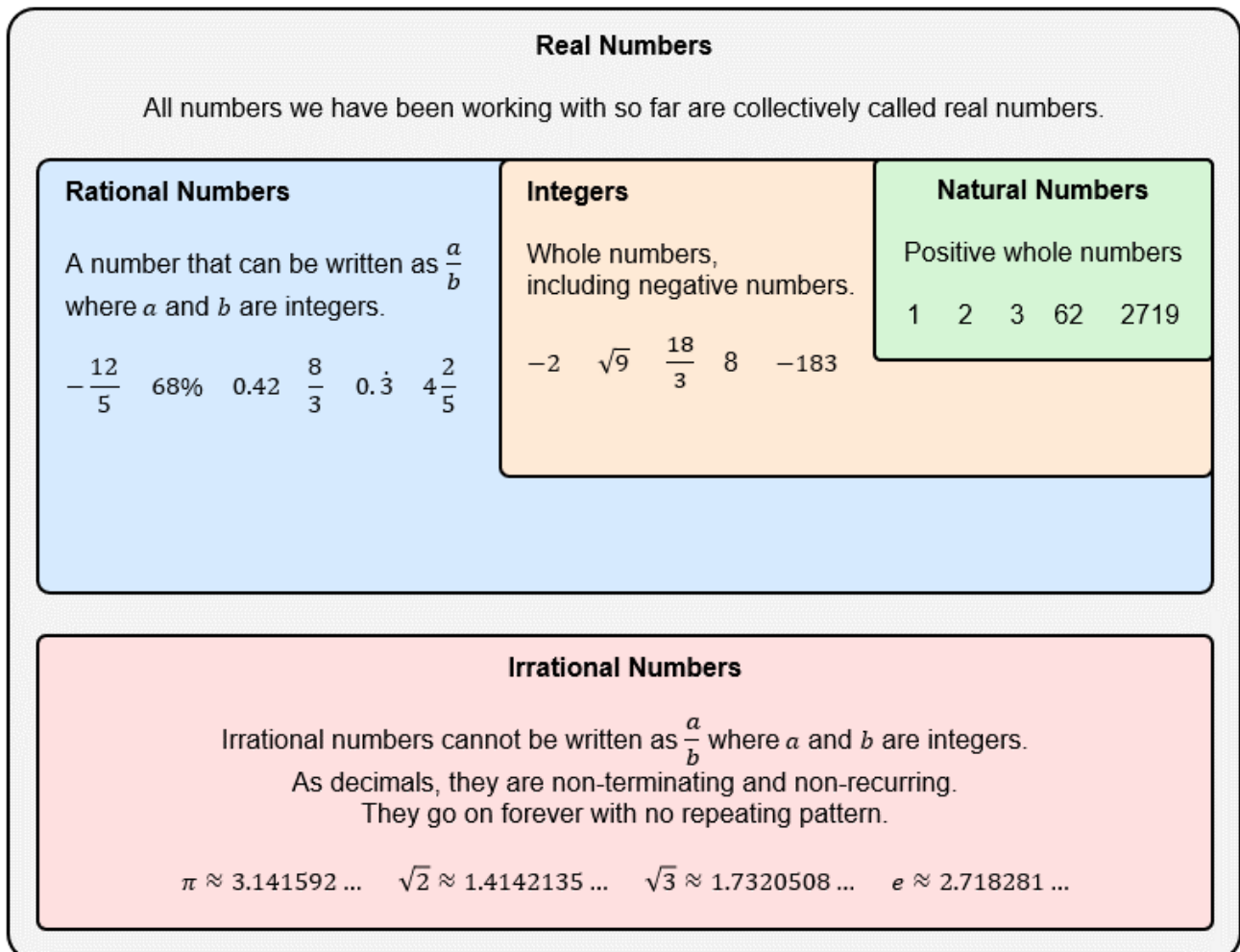
where a and b are integers, and $b \neq 0$

- Rational numbers include:

Fractions	Integers	Percentages	Terminating Decimals	Recurring Decimals
$\frac{2}{3}$	$2 \rightarrow \frac{2}{1}$	$47\% \rightarrow \frac{47}{100}$	$0.125 \rightarrow \frac{125}{1000}$	$0.333333 \rightarrow \frac{1}{3}$

IRRATIONAL NUMBERS

- **Irrational numbers** are **non-terminating non-recurring** decimals, such as π and $\sqrt{2}$.
- All numbers are collectively known as **real numbers**.



Example Classify numbers as rational or irrational.

Using a calculator, determine whether these numbers are irrational or rational.

a $\frac{1}{3}$ Rational Irrational

b 3 Rational Irrational

c 0.3 Rational Irrational

d $0.\dot{3}$ Rational Irrational

e $\sqrt{3}$ Rational Irrational

f $\sqrt{9}$ Rational Irrational

g 0.25 Rational Irrational

h 25% Rational Irrational

i $\sqrt{4}$ Rational Irrational

j $\sqrt{5}$ Rational Irrational

k $\sqrt{5} + 1$ Rational Irrational

l $\frac{\sqrt{2}}{2}$ Rational Irrational

m $2\sqrt{2}$ Rational Irrational

n π Rational Irrational

o $-\frac{22.5}{7}$ Rational Irrational

p $3.\overline{14}$ Rational Irrational

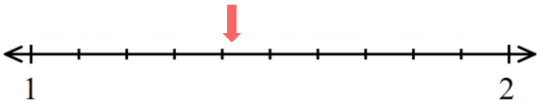
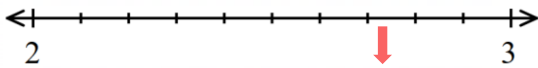
Example Approximate irrational numbers.

Approximate $\sqrt{2}$ to 2 decimal places. $\sqrt{2} = 1.414213562 \dots$ ≈ 1.41	When rounding to 2 decimal places, which is the critical digit that we look at?
Approximate e to 2 decimal places. $e = 2.718281828 \dots$ ≈ 2.72	How do we know whether to round up to down?

Approximate these numbers to 2 decimal places.

a $\sqrt{3}$	b π	c $\sqrt[3]{25}$
d $\sqrt{18}$	e $\frac{1}{\pi}$	f $2\sqrt{2}$

Example Locate the approximate position of irrational numbers on a number line.

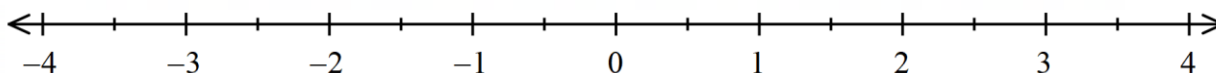
$\sqrt{2} \approx 1.41$ 	$e \approx 2.72$ 
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Using a calculator, locate these numbers on the number line.

$\sqrt{3}$

$\sqrt[3]{7}$

$-\pi$



Key ideas

1. Rational numbers can be written as a fraction $\frac{a}{b}$, where a and b are and b 0.
2. If a number can be written as a terminating or decimal, then it is rational.
3. Irrational numbers cannot be written as a fraction with integers, when written as a decimal, they are non-..... and non-.....

Check your understanding

1 Which of these two numbers are rational?

7 and 7.27629845

a 7 **b** 7.27629845 **c** Neither **d** Both

2 Which of these two numbers are rational?

π and 3.142

a π **b** 3.142 **c** Neither **d** Both

3 Which of these two numbers are rational?

3.5 and -3.5

a 3.5 **b** -3.5 **c** Neither **d** Both

4 Which of these two numbers are rational?

$\frac{1}{3}$ and $\frac{4}{1}$

a $\frac{1}{3}$ **b** $\frac{1}{4}$ **c** Neither **d** Both

5 Which of these two numbers are rational?

$\sqrt{36}$ and $\sqrt{12}$

a $\sqrt{36}$ **b** $\sqrt{12}$ **c** Neither **d** Both

- 1 Circle which of these numbers are integers.

10 9.6 $\frac{3}{4}$ -3 117

10, -3, 117

Explain why all integers are rational numbers.

Integers can be written with a denominator of 1.

- 2 Circle which of these numbers are rational.

$\frac{7}{2}$ $\sqrt{3}$ 1.5 11

$\frac{7}{2}$, 1.5, 11

Explain how you know whether a number is rational.

The number can be written as a fraction with whole number numerator and denominator.

- 3 Explain why $\sqrt{4}$ is a rational number.

$\sqrt{4} = 2$, which is an integer, so it's rational.

- 4 Look at these numbers.

$\sqrt{2}$ $\sqrt{3}$ $\sqrt{4}$ $\sqrt{5}$ $3\sqrt{7}$ $2\sqrt{100}$ $-\sqrt{3}$ $-\sqrt{25}$

- a Which numbers are irrational?

$\sqrt{3}$, $\sqrt{5}$, $3\sqrt{7}$, $-\sqrt{3}$

- b Which numbers are rational?

$\sqrt{4}$, $2\sqrt{100}$, $-\sqrt{25}$

- 5 Emily says:



0.7777... recurring is irrational as you cannot write it as a fraction.

Explain why she is incorrect.

0.777... can be written as $\frac{7}{9}$, so it's rational.

6 Darius says:



π is a rational number, since it is roughly equal to $\frac{22}{7}$

Explain why Darius is incorrect.

π is roughly equal to $\frac{22}{7}$ but not exactly. π is a non-terminating, non-recurring decimal, so it is irrational.

7 Becky says:



$\sqrt{25}$ is not rational, because it is a square root.

Explain why Becky is incorrect.

$\sqrt{25}$ simplifies to 5, which is rational.

8 Using a calculator, approximate these numbers to 2 decimal places.

a $\sqrt{5}$

b $2\sqrt{5}$

c 1.61803399

2.24

4.47

1.62

d $\frac{22}{7}$

e $\frac{\pi}{2}$

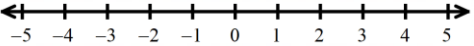
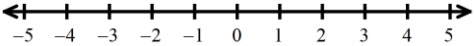
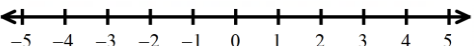
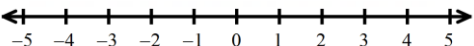
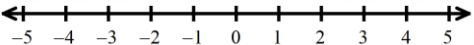
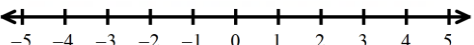
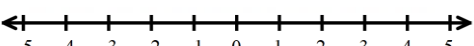
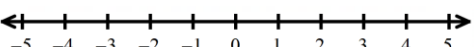
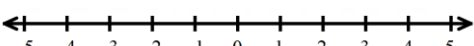
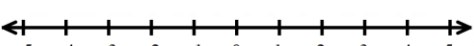
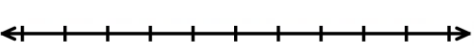
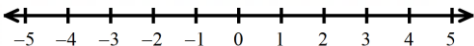
f e^2

3.14

1.57

7.39

9 Complete the table.

Number	Rational/ Irrational?	Approximation (2 d.p.)	Approximation (3 d.p.)	Approximate location on number line
e	Irrational	≈ 2.72	≈ 2.718	
$\sqrt{6}$	Irrational	2.45	2.449	
$2\sqrt{6}$	Irrational	4.90	4.899	
$\sqrt{24}$	Irrational	4.90	4.899	
$\sqrt{13}$	Irrational	3.61	3.606	
$\sqrt{16}$	Rational	4.00	4.000	
π	Irrational	3.14	3.142	
3π	Irrational	9.42	9.425	
$0.\dot{3}$	Rational	0.33	0.333	
$\frac{7}{3}$	Rational	2.33	2.333	
-5	Rational	-5.00	-5.000	
$-\sqrt{3}$	Irrational	-1.73	-1.732	

Prior knowledge check

1 Complete the sentences:

- a A rational number can be written as $\frac{a}{b}$ where a and b are
- b A number is irrational if its decimal representation is non-..... and non-.....
- c All recurring decimals are, since they can be written as a fraction.
- d All numbers are collectively called numbers; they can be represented on a number line.

2 Determine whether these numbers are rational or irrational by entering each number in the calculator.

a $\sqrt{5}$	b 18%	c $\frac{2}{5}$	d -124%
e $1\frac{5}{7}$	f $-\sqrt{4}$	g $2\sqrt{3}$	h π
i 0.8	j 0.18	k $0.\overline{18}$	l $0.41\dot{6}$

3 Solve these equations. Leave your answer in exact form.

a $x^2 = 4$	b $x^2 = 5$	c $x^2 = -1$	d $x^2 + 3 = 12$
e $x^2 - 5 = 18$	f $x^3 = 27$	g $x^3 = -8$	h $4x^3 = 12$

ROOTS AND SURDS

- Square rooting $\sqrt{x}$ is the inverse operation of squaring x^2 .
- We call a root a **surd** if it is irrational: E.g., $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt[3]{7}$
- It is best to leave surds in exact form, rather than a rounded decimal.
- It is impossible (for now) to square root a negative number $\sqrt{-4}$, we say it is **not real**.

Example Identify surds.

Determine whether these roots are surds				
a	$\sqrt{3}$	Surd	Rational	Not real
b	$2\sqrt{3}$	Surd	Rational	Not real
c	$\sqrt{4}$	Surd	Rational	Not real
d	$\sqrt{8}$	Surd	Rational	Not real
e	$\sqrt{16}$	Surd	Rational	Not real
f	$2\sqrt{16}$	Surd	Rational	Not real
g	$3 + \sqrt{16}$	Surd	Rational	Not real
h	$\sqrt[3]{16}$	Surd	Rational	Not real
i	$\sqrt[3]{8}$	Surd	Rational	Not real
j	$\sqrt[3]{6}$	Surd	Rational	Not real
k	$\sqrt{6}$	Surd	Rational	Not real
l	$\sqrt{9}$	Surd	Rational	Not real
m	$-\sqrt{9}$	Surd	Rational	Not real
n	$\sqrt[3]{9}$	Surd	Rational	Not real
o	$\sqrt{0}$	Surd	Rational	Not real
p	$\sqrt{-0}$	Surd	Rational	Not real
q	$\sqrt{-1}$	Surd	Rational	Not real
r	$-\sqrt{1}$	Surd	Rational	Not real
s	$-\sqrt{-1}$	Surd	Rational	Not real
t	$\sqrt[3]{-8}$	Surd	Rational	Not real
u	$\sqrt{-(-25)}$	Surd	Rational	Not real

Key ideas

1. A number that cannot be expressed as a fraction with integer numerator and denominator is called
2. A is an irrational root, for example $\sqrt{2}$ or $\sqrt[3]{5}$
3. The decimal representation of a surd is a decimal that goes on and never
4. $\sqrt{7}$ is a surd because its representation doesn't repeat and goes on forever.
5. $\sqrt{25}$ is a surd because its decimal representation is

Check your understanding

- | | | | | | | | |
|----------|---|----------|-------------|----------|-------------|----------|-------------|
| 1 | Tom says $\sqrt{1}$ is a surd
Katie says $\sqrt{2}$ is a surd
Who is correct? | | | | | | |
| a | Only Tom | b | Only Katie | c | Both | d | Neither |
| 2 | Tom says $\sqrt{9}$ is not a surd
Katie says $\sqrt{3}$ is not a surd
Who is correct? | | | | | | |
| a | Only Tom | b | Only Katie | c | Both | d | Neither |
| 3 | Which of the following is not a surd? | | | | | | |
| a | $\sqrt{45}$ | b | $\sqrt{47}$ | c | $\sqrt{49}$ | d | $\sqrt{51}$ |
| 4 | Which of the following is not a surd? | | | | | | |
| a | $\sqrt{84}$ | b | $\sqrt{83}$ | c | $\sqrt{82}$ | d | $\sqrt{81}$ |

- 1 Jono says $\sqrt{18}$ is a surd. Explain why he is correct.

$\sqrt{18}$ cannot be expressed as a fraction with integers. It is a non-terminating, non-recurring decimal.

- 2 Becky says $\sqrt{25}$ is not a surd. Explain why she is correct.

$\sqrt{25}$ can be simplified to 5

- 3 Circle the surds.

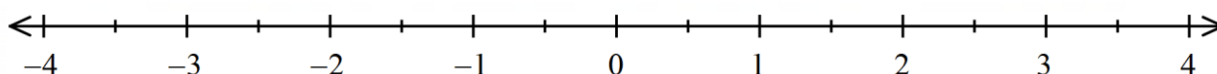


$\sqrt{24}, \sqrt{2}, \sqrt{8}, \sqrt{130}$

- 4 Express each number as a decimal using a calculator and then decide whether they are irrational or rational. Round the decimals to 2 d.p.

a $\sqrt{5}$ Irrational, 2.24	b 18% Rational, 0.18	c $\frac{2}{5}$ Rational, 0.4	d -124% Rational, -1.24
e $1\frac{5}{7}$ Rational, $1.\overline{714285}$	f $-\sqrt{2}$ Irrational, -1.41	g $2\sqrt{3}$ Irrational, 3.46	h π Irrational, 3.14

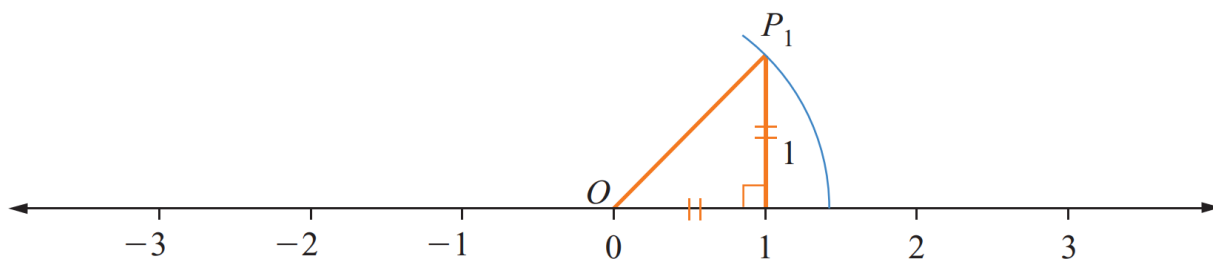
Show each number's approximate location on the number line.



- 5 Using a calculator, or otherwise, classify these numbers as either surds, rational, or not real.

a $\sqrt{7}$ surd	b $2\sqrt{11}$ surd	c $2\sqrt{25}$ rational	d $-5\sqrt{144}$ rational
e $\frac{3\sqrt{9}}{2}$ rational	f $\frac{-5\sqrt{3}}{2}$ surd	g $1 - \sqrt{3}$ surd	h $2 + \sqrt{4}$ rational
i $\sqrt{\frac{4}{25}}$ rational	j $\sqrt{\frac{5}{16}}$ surd	k $\sqrt{-11}$ Not real	l $\sqrt{99}$ surd

10 Consider the number line:



a Use Pythagoras' Theorem to calculate the length of OP_1

$\sqrt{2}$

b Using a pair of compasses, with point at O , the location of $\sqrt{2}$ on the number line has been accurately marked. What is the approximate value of $\sqrt{2}$?

1.41

c $\sqrt{2}$ can be placed on a number line. Is it a real number?

yes

d Use the following diagram to accurately place $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$ on the number line.

