Student Name

Mathematics Stage 5 Paths

INDICES C

Book 3 Describe and use fractional indices

Version: 260207

Report mistakes & suggest improvements:
https://MrDingMaths.com

CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-IND-P-02 describes and performs operations with surds and fractional indices

Describe and use fractional indices

- Apply index laws to describe fractional indices as: $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- Translate expressions in surd form to expressions in index form and vice versa
- Evaluate numerical expressions involving fractional indices including using digital tools

REVIEW OF PRIOR KNOWLEDGE

1 Work out the following without a calculator.

a 2^3	**b** 2^4	**c** 2^5	**d** 2^6
e 3^2	**f** 3^3	**g** 5^2	**h** 5^3

2 Write each of these as a power of 2.

a 32	**b** 8	**c** 128	**d** 256

3 Work these out using a calculator, if needed.

a $\sqrt{100}$	**b** $\sqrt{49}$	**c** $\sqrt{4}$	**d** $\sqrt{0.36}$
e $\sqrt[3]{27}$	**f** $\sqrt[3]{125}$	**g** $\sqrt[4]{10\,000}$	**h** $\sqrt[5]{32}$

4 Write the following expressions without brackets.

a $(2x)^3$	**b** $(-3r)^2$	**c** $(5cd^3)^4$	**d** $(-2s^3)^3$
e $\left(\frac{p}{q}\right)^3$	**f** $\left(\frac{2}{r^3}\right)^2$	**g** $\left(\frac{2a^2}{3}\right)^3$	**h** $\left(\frac{3n^3}{2m^4}\right)^3$

5 Write the following expressions with positive indices.

a x^{-1}	**b** a^{-4}	**c** p^7q^{-2}	**d** $x^{-4}y^{-3}$
e $\frac{1}{y^{-1}}$	**f** $\frac{3}{b^{-2}}$	**g** $\frac{7}{x^{-4}y^3}$	**h** $\frac{1}{u^{-3}b^2}$
i $\left(\frac{4}{1}\right)^{-1}$	**j** $\left(\frac{3}{4}\right)^{-1}$	**k** $\left(\frac{2}{3}\right)^{-2}$	**l** $\left(\frac{2x^2}{5y}\right)^{-3}$

FRACTIONAL INDICES

Investigation Fractional Indices.

1

a Use the index laws to simplify $\left(a^{\frac{1}{2}}\right)^2$ ……

b Simplify $\left(\sqrt{a}\right)^2$ ……

c Therefore, what can you conclude about $a^{\frac{1}{2}}$ and $\sqrt{a}$?

……

2

a Use the index laws to simplify $\left(a^{\frac{1}{3}}\right)^3$ ……

b Simplify $\left(\sqrt[3]{a}\right)^3$ ……

c Therefore, what can you conclude about $a^{\frac{1}{3}}$ and $\sqrt[3]{a}$?

……

- For a fractional index, the denominator is the index of the surd.

Fractional Indices
$a^{\frac{1}{2}} = \sqrt{a}$
$a^{\frac{1}{3}} = \sqrt[3]{a}$
$a^{\frac{1}{n}} = \sqrt[n]{a}$

Example Express numbers in index form as surds.

$7^{\frac{1}{2}}$	$2^{\frac{1}{5}}$	$8^{\frac{1}{3}}$
$= \sqrt{7}$	$= \sqrt[5]{2}$	$= \sqrt[3]{8}$
		$= 2$

Express these numbers in surd form.

a $m^{\frac{1}{2}}$ $\sqrt{m}$	**b** $y^{\frac{1}{4}}$ $\sqrt[4]{y}$	**c** $p^{\frac{1}{3}}$ $\sqrt[3]{p}$	**d** $62^{\frac{1}{n}}$ $\sqrt[n]{62}$
e $49^{\frac{1}{2}}$ 7	**f** $11^{0.5}$ $\sqrt{11}$	**g** $25^{0.5}$ 5	**h** $z^{\frac{1}{x}}$ $\sqrt[x]{z}$

Express these numbers in index form.

a $\sqrt{10}$ $10^{\frac{1}{2}}$	**b** $\sqrt[3]{7}$ $7^{\frac{1}{3}}$	**c** $\sqrt[5]{k}$ $k^{\frac{1}{5}}$	**d** $\sqrt[n]{x}$ $x^{\frac{1}{n}}$
e $\sqrt[3]{y}$ $y^{\frac{1}{3}}$	**f** $\sqrt[4]{z}$ $z^{\frac{1}{4}}$	**g** $\sqrt[n]{5}$ $5^{\frac{1}{n}}$	**h** $\sqrt{5}$ $5^{\frac{1}{2}}$

Key ideas

1. Since $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a$, we can conclude that $a^{\frac{1}{2}}$ is the same as .....
2. Since $a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}} = a$, we can conclude that $a^{\frac{1}{3}}$ is the same as .....
3. In general, we can convert between surd form and fractional form using the rule .....$= \sqrt[n]{x}$

1 Complete the table to evaluate these fractional indices.

Expression with powers	Working
$16^{\frac{1}{2}}$	$\sqrt{16} = \square$
$25^{\frac{1}{2}}$	$\sqrt{\square} = \square$
$36^{\frac{1}{2}}$	$\sqrt{\square} = \square$
$64^{\frac{1}{2}}$	$\sqrt{\square} = \square$
$100^{\frac{1}{2}}$	$\sqrt{\square} = \square$

4

$\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{64} = 8$

$\sqrt{100} = 10$

2 Match each expression with a power to its correct value.

$64^{\frac{1}{2}}$ $64^{\frac{1}{3}}$ $64^{\frac{1}{6}}$ 64^{0}

4 8 1 2

$64^{\frac{1}{2}} = 8,$ $64^{\frac{1}{3}} = 4,$ $64^{\frac{1}{6}} = 2,$ $64^{0} = 1$

3 Complete the table.

Index Form	As a root	Decimal	Surd? (yes or no)
$2^{\frac{1}{2}}$	$\sqrt{2}$	1.414213562…	yes
$4^{\frac{1}{2}}$	$\sqrt{4}$	2	No
$11^{\frac{1}{2}}$	$\sqrt{11}$	3.31662479…	Yes
$36^{\frac{1}{2}}$	$\sqrt{36}$	6	No
$\left(\frac{1}{9}\right)^{\frac{1}{2}}$	$\sqrt{\frac{1}{9}}$	$0.\dot{3}$	No
$(0.1)^{\frac{1}{2}}$	$\sqrt{0.1}$	0.316227766…	Yes
$3^{\frac{1}{3}}$	$\sqrt[3]{3}$	1.44224957	Yes
$8^{\frac{1}{3}}$	$\sqrt[3]{8}$	2	No
$15^{\frac{1}{3}}$	$\sqrt[3]{15}$	2.466212074…	Yes
$\left(\frac{1}{27}\right)^{\frac{1}{3}}$	$\sqrt[3]{\frac{1}{27}}$	$0.\dot{3}$	No
$5^{\frac{1}{4}}$	$\sqrt[4]{5}$	1.495348781…	Yes

4 Write these numbers as a root.

a $3^{\frac{1}{2}}$ $\sqrt{3}$	**b** $7^{\frac{1}{2}}$ $\sqrt{7}$	**c** $5^{\frac{1}{3}}$ $\sqrt[3]{5}$	**d** $12^{\frac{1}{3}}$ $\sqrt[3]{12}$
e $31^{\frac{1}{5}}$ $\sqrt[5]{31}$	**f** $18^{\frac{1}{7}}$ $\sqrt[7]{18}$	**g** $9^{\frac{1}{10}}$ $\sqrt[10]{9}$	**h** $324^{\frac{1}{8}}$ $\sqrt[8]{324}$

5 Write these numbers in index form.

a $\sqrt{8}$ $8^{\frac{1}{2}}$	**b** $\sqrt{19}$ $19^{\frac{1}{2}}$	**c** $\sqrt[3]{10}$ $10^{\frac{1}{3}}$	**d** $\sqrt[3]{31}$ $31^{\frac{1}{3}}$
e $\sqrt[4]{5}$ $5^{\frac{1}{4}}$	**f** $\sqrt[5]{95}$ $95^{\frac{1}{5}}$	**g** $\sqrt[8]{111}$ $111^{\frac{1}{8}}$	**h** $\sqrt[524]{20}$ $20^{\frac{1}{524}}$

6 Evaluate these numbers, rounding to 2 d.p. where necessary.

a $4^{\frac{1}{2}}$ and $\sqrt{4}$ 2	**b** $8^{\frac{1}{3}}$ and $\sqrt[3]{8}$ 2	**c** $9^{0.5}$ and $\sqrt{9}$ 3
d $12^{\frac{1}{2}}$ and $\sqrt{12}$ 3.46	**e** $\sqrt[3]{1000}$ and $1000^{\frac{1}{3}}$ 10	**f** $\sqrt[5]{32}$ and $32^{\frac{1}{5}}$ 2

7 Evaluate these numbers, leaving your answer as a whole number.

a $25^{\frac{1}{2}}$ 5	**b** $49^{\frac{1}{2}}$ 7	**c** $81^{\frac{1}{2}}$ 9	**d** $169^{\frac{1}{2}}$ 13
e $8^{\frac{1}{3}}$ 2	**f** $64^{\frac{1}{3}}$ 4	**g** $125^{\frac{1}{3}}$ 5	**h** $1000^{\frac{1}{3}}$ 10
i $16^{\frac{1}{4}}$ 2	**j** $81^{\frac{1}{4}}$ 3	**k** $625^{\frac{1}{4}}$ 5	**l** $32^{\frac{1}{5}}$ 2

8 Work out the missing powers,

a $49^{\square} = 7$ $\frac{1}{2}$	**b** $125^{\square} = 5$ $\frac{1}{3}$	**c** $81^{\square} = 3$ $\frac{1}{4}$	**d** $100\,000^{\square} = 10$ $\frac{1}{5}$
e $1000^{\square} = 10$ $\frac{1}{3}$	**f** $81^{\square} = 9$ $\frac{1}{2}$	**g** $128^{\square} = 2$ $\frac{1}{7}$	**h** $(-8)^{\square} = -2$ $\frac{1}{3}$

9 Explain why it is not possible to evaluate $(-9)^{\frac{1}{2}}$

This is $\sqrt{-9}$, it is not possible to evaluate the square root of a negative (yet)

10 State whether these statements are true or false.

a $\sqrt[3]{8} = 8^{\frac{1}{3}}$ T	**b** $\sqrt{3} = 3^{0.5}$ T	**c** $5^{\frac{1}{3}} = \sqrt{5}$ F	**d** $10^{\frac{1}{6}} = \sqrt[10]{6}$ F
e $8^{\frac{1}{2}} = 4$ F	**f** $16^{\frac{1}{2}} = 4$ T	**g** $27^{\frac{1}{3}} = 9$ F	**h** $100^{0.5} = 10$ T

11 Evaluate these expressions.

a $8^{\frac{1}{3}}$ 2	**b** $16^{\frac{1}{4}}$ 2	**c** $32^{\frac{1}{5}}$ 2	**d** $100^{\frac{1}{2}}$ 10
e $1000^{\frac{1}{3}}$ 10	**f** $343^{\frac{1}{3}}$ 7	**g** $125^{\frac{1}{3}}$ 5	**h** $625^{\frac{1}{4}}$ 5
i $0.25^{\frac{1}{2}}$ 0.5	**j** $(-125)^{\frac{1}{3}}$ −5	**k** $(-32)^{\frac{1}{5}}$ −2	

12 Evaluate these without a calculator. First rewrite each question using positive indices.

a $4^{-\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} = \frac{1^{\frac{1}{2}}}{4^{\frac{1}{2}}} = \ldots\ldots$ $\frac{1}{2}$	**b** $8^{-\frac{1}{3}}$ $\frac{1}{2}$	**c** $32^{-\frac{1}{5}}$ $\frac{1}{2}$	**d** $81^{-\frac{1}{4}}$ $\frac{1}{3}$
e $25^{-\frac{1}{2}}$ $\frac{1}{5}$	**f** $27^{-\frac{1}{3}}$ $\frac{1}{3}$	**g** $1000^{-\frac{1}{3}}$ $\frac{1}{10}$	**h** $256^{-\frac{1}{4}}$ $\frac{1}{4}$

13 Use index laws to simplify these expressions. Leave your answer in index form.

a $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$ a	**b** $a^{\frac{1}{3}} \times a^{\frac{1}{3}}$ $a^{\frac{2}{3}}$	**c** $a^{\frac{2}{3}} \times a^{\frac{4}{3}}$ a^2	**d** $a^2 \times a^{\frac{1}{2}}$ $a^{\frac{5}{2}}$
e $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}}$ $x^{\frac{1}{3}}$	**f** $\dfrac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$ x	**g** $\dfrac{x^{\frac{7}{6}}}{x^{\frac{2}{6}}}$ $x^{\frac{5}{6}}$	**h** $\dfrac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$ x
i $(y^2)^{\frac{1}{2}}$ y	**j** $(y^3)^{\frac{2}{3}}$ y^2	**k** $(y^{\frac{1}{2}})^3$ $y^{\frac{3}{2}}$	**l** $(x^{\frac{1}{2}})^{\frac{1}{2}}$ $x^{\frac{1}{4}}$
m $(x^{\frac{2}{3}})^4$ $x^{\frac{8}{3}}$	**n** $(a^{\frac{2}{5}})^{\frac{1}{3}}$ $a^{\frac{2}{15}}$	**o** $(a^{\frac{3}{4}})^{\frac{1}{2}}$ $a^{\frac{3}{8}}$	**p** $(n^{\frac{2}{5}})^{\frac{10}{3}}$ $n^{\frac{4}{3}}$
q $a \times a^{\frac{1}{3}}$ $a^{\frac{4}{3}}$	**r** $a^{\frac{1}{2}} \times a^{\frac{1}{5}}$ $a^{\frac{7}{10}}$	**s** $a^{\frac{2}{3}} \times a^{\frac{3}{7}}$ $a^{\frac{23}{21}}$	**t** $a^5 \div a^{\frac{7}{3}}$ $a^{\frac{8}{3}}$

14 Simplify the following.

a $a^{\frac{1}{2}} \times a^{-\frac{1}{2}}$ 1	**b** $a^{\frac{2}{3}} \times a^{-\frac{2}{3}}$ 1	**c** $a^{\frac{4}{7}} \times a^{-\frac{4}{7}}$ 1
d $a^{\frac{5}{6}} \div a^{\frac{5}{6}}$ 1	**e** $\left(a^{\frac{1}{2}}\right)^{\frac{1}{2}} \div a^{\frac{1}{4}}$ 1	**f** $a^2 \div (a^3)^{\frac{2}{3}}$ 1

What do you notice about your answers?

They all evaluate to 1

15 Evaluate without a calculator.

a $36^{\frac{1}{2}}$ 6	**b** $27^{\frac{1}{3}}$ 3	**c** $64^{\frac{1}{3}}$ 4
d $49^{\frac{1}{2}}$ 7	**e** $16^{\frac{1}{4}}$ 2	**f** $125^{\frac{1}{3}}$ 5
g $9^{-\frac{1}{2}}$ $\frac{1}{3}$	**h** $32^{-\frac{1}{5}}$ $\frac{1}{2}$	**i** $81^{-\frac{1}{4}}$ $\frac{1}{3}$
j $1000^{-\frac{1}{3}}$ $\frac{1}{10}$	**k** $400^{-\frac{1}{2}}$ $\frac{1}{20}$	**l** $10000^{-\frac{1}{4}}$ $\frac{1}{10}$

16 Use the fact that $\left(\frac{4}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$ to evaluate the following:

a $\left(\frac{16}{25}\right)^{\frac{1}{2}}$ $\frac{4}{5}$	**b** $\left(\frac{9}{49}\right)^{\frac{1}{2}}$ $\frac{3}{7}$	**c** $\left(\frac{4}{81}\right)^{\frac{1}{2}}$ $\frac{2}{9}$	**d** $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ $\frac{2}{3}$
e $\left(\frac{64}{125}\right)^{\frac{1}{3}}$ $\frac{4}{5}$	**f** $\left(\frac{16}{81}\right)^{\frac{1}{4}}$ $\frac{2}{3}$	**g** $\left(\frac{256}{625}\right)^{\frac{1}{4}}$ $\frac{4}{5}$	**h** $\left(\frac{1000}{343}\right)^{\frac{1}{3}}$ $\frac{10}{7}$

17 Use the fact that $\left(\frac{4}{9}\right)^{-\frac{1}{2}} = \left(\frac{9}{4}\right)^{\frac{1}{2}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$ to evaluate the following:

a $\left(\frac{9}{4}\right)^{-\frac{1}{2}}$ $\frac{2}{3}$	**b** $\left(\frac{49}{144}\right)^{-\frac{1}{2}}$ $\frac{12}{7}$	**c** $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$ $\frac{5}{2}$	**d** $\left(\frac{1296}{625}\right)^{-\frac{1}{4}}$ $\frac{5}{6}$

FURTHER FRACTIONAL INDICES

- The denominator is the index of the surd.
- The numerator is the power.

Fractional Indices
$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Example Express numbers in index form as surds.

$x^{\frac{2}{3}}$ $= \sqrt[3]{x^2}$	How do we know $x^{\frac{2}{3}} = \sqrt[3]{x^2}$ and not $\sqrt{x^3}$? *The numerator is the .....,* *the denominator is the .....*

Example Express surds in index form.

$\sqrt[3]{a^2}$ $= a^{\frac{2}{3}}$	How do we know $\sqrt[3]{a^2} = a^{\frac{2}{3}}$ and not $a^{\frac{3}{2}}$? *The index of the surd is the .....,* *the power of the radicand is the .....*
$y\sqrt[4]{x^7}$ $= yx^{\frac{7}{4}}$	Why does the y stay the same? *The y is not inside the .....*

Express in index form.

a $\sqrt{29}$ $29^{\frac{1}{2}}$	**b** $\sqrt[5]{x^2}$ $x^{\frac{2}{5}}$	**c** $\sqrt{2a}$ $(2a)^{\frac{1}{2}}$
d $7\sqrt{x^5}$ $7x^{\frac{5}{2}}$	**e** $6\sqrt[3]{x^2}$ $6x^{\frac{2}{3}}$	**f** $\frac{1}{\sqrt{y}}$ $y^{-\frac{1}{2}}$

Express in surd form.

a $x^{\frac{1}{5}}$ $\sqrt[5]{x}$	**b** $x^{\frac{7}{2}}$ $\sqrt{x^7}$	**c** $y^{\frac{2}{7}}$ $\sqrt[7]{y^2}$
d $36^{-\frac{1}{2}}$ $\frac{1}{6}$	**e** $y^{-\frac{2}{3}}$ $\frac{1}{\sqrt[3]{y^2}}$	**f** $\frac{a}{100^{\frac{3}{2}}}$ $\frac{a}{1000}$

1 Write the following with fractional indices.

a	$\sqrt{8}$	**b**	$\sqrt[3]{7}$	**c**	$\sqrt[5]{10}$
	$8^{\frac{1}{2}}$		$7^{\frac{1}{3}}$		$10^{\frac{1}{5}}$
d	$\sqrt[13]{100}$	**e**	$\sqrt[3]{5}$	**f**	$\sqrt{5}$
	$100^{\frac{1}{13}}$		$5^{\frac{1}{3}}$		$5^{\frac{1}{2}}$
g	$\sqrt{29}$	**h**	$\sqrt[3]{35}$	**i**	$\sqrt[5]{x^2}$
	$29^{\frac{1}{2}}$		$35^{\frac{1}{3}}$		$x^{\frac{2}{5}}$
j	$(\sqrt{m})^5$	**k**	$\sqrt{m^5}$	**l**	$(\sqrt{a})^5$
	$m^{\frac{5}{2}}$		$m^{\frac{5}{2}}$		$a^{\frac{5}{2}}$
m	$\sqrt[4]{10^3}$	**n**	$(\sqrt[4]{10})^3$	**o**	$(\sqrt[7]{x})^n$
	$10^{\frac{3}{4}}$		$10^{\frac{3}{4}}$		$x^{\frac{n}{7}}$
p	$\sqrt[4]{b^3}$	**q**	$\sqrt{a}$	**r**	$\sqrt[3]{t^2}$
	$b^{\frac{3}{4}}$		$a^{\frac{1}{2}}$		$t^{\frac{2}{3}}$

2 Express the following in surd form.

a	$2^{\frac{1}{5}}$	**b**	$8^{\frac{1}{7}}$	**c**	$6^{\frac{1}{3}}$
	$\sqrt[5]{2}$		$\sqrt[7]{8}$		$\sqrt[3]{6}$
d	$3^{\frac{3}{2}}$	**e**	$7^{\frac{2}{3}}$	**f**	$2^{\frac{3}{5}}$
	$\sqrt[2]{3^3}$		$\sqrt[3]{7^2}$		$\sqrt[5]{2^3}$
g	$25^{\frac{3}{4}}$	**h**	$x^{\frac{n}{5}}$	**i**	$x^{\frac{5}{n}}$
	$\sqrt[4]{25^3}$		$\sqrt[5]{x^n}$		$\sqrt[n]{x^5}$

3 Look at the examples below for evaluating $16^{\frac{5}{4}}$

Method 1

$$16^{\frac{5}{4}} = (16^5)^{\frac{1}{4}}$$
$$= (1\,048\,576)^{\frac{1}{4}}$$
$$= \sqrt[4]{1\,048\,576}$$
$$= 32$$

Method 2

$$16^{\frac{5}{4}} = \left(16^{\frac{1}{4}}\right)^5$$
$$= (\sqrt[4]{16})^5$$
$$= 2^5$$
$$= 32$$

Why is method 2, rooting before raising to the power, easier?

You work with smaller numbers.

4 Evaluate the following without a calculator.

a	$27^{\frac{2}{3}}$	**b**	$64^{\frac{2}{3}}$	**c**	$9^{\frac{3}{2}}$
	9		16		27
d	$16^{\frac{5}{4}}$	**e**	$4^{\frac{5}{2}}$	**f**	$81^{\frac{3}{2}}$
	32		32		729
g	$\left(\frac{1}{9}\right)^{\frac{3}{2}}$	**h**	$\left(\frac{4}{25}\right)^{\frac{4}{2}}$	**i**	$\left(\frac{27}{1000}\right)^{\frac{4}{3}}$
	$\frac{1}{27}$		$\frac{16}{625}$		$\frac{81}{10000}$

5 Express in surd form, or as a rational number.

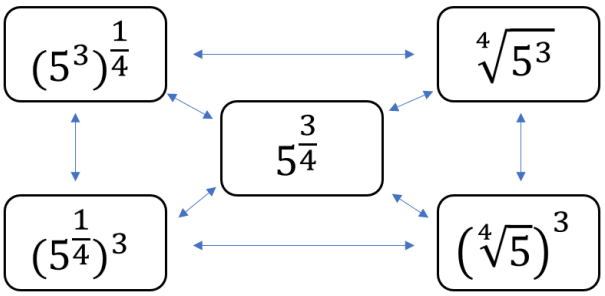
a	$100^{\frac{1}{2}}$	**k**	$27^{\frac{1}{3}}$	**u**	$9^{\frac{1}{3}}$
	10		3		$\sqrt[3]{9}$
b	$200^{\frac{1}{2}}$	**l**	$27000^{\frac{1}{3}}$	**v**	$16^{\frac{1}{4}}$
	$\sqrt{200}$		30		2
c	$400^{\frac{1}{2}}$	**m**	$27^{\frac{2}{3}}$	**w**	$9^{\frac{1}{4}}$
	20		9		$\sqrt[4]{9}$
d	$4^{\frac{1}{2}}$	**n**	$27000^{\frac{2}{3}}$	**x**	$9^{\frac{1}{2}}$
	2		900		3
e	$2^{\frac{1}{2}}$	**o**	$27^{\frac{4}{3}}$	**y**	$9^{\frac{3}{4}}$
	$\sqrt{2}$		81		$\sqrt[4]{729}$
f	$1^{\frac{1}{2}}$	**p**	$27^{\frac{40}{3}}$	**z**	$\left(\frac{1}{9}\right)^{\frac{3}{4}}$
	1		3^{40}		$\frac{1}{\sqrt[4]{729}}$
g	$8^{\frac{1}{2}}$	**q**	$\left(\frac{8}{27}\right)^{\frac{1}{3}}$	**aa**	$\left(\frac{16}{9}\right)^{\frac{1}{4}}$
	$\sqrt{8}$		$\frac{2}{3}$		$\frac{2}{\sqrt[4]{9}}$
h	$8^{\frac{1}{3}}$	**r**	$\left(\frac{8}{27}\right)^{-\frac{1}{3}}$	**bb**	$\frac{16^{\frac{1}{4}}}{9}$
	2		$\frac{3}{2}$		$\frac{2}{9}$
i	$8^{\frac{2}{3}}$	**s**	$\left(\frac{8}{27}\right)^{\frac{4}{3}}$	**cc**	$\frac{16}{9^{\frac{1}{4}}}$
	4		$\frac{16}{81}$		$\frac{16}{\sqrt[4]{9}}$
j	$8^{\frac{4}{3}}$	**t**	$\left(\frac{27}{8}\right)^{-\frac{4}{3}}$	**dd**	$0.16^{\frac{1}{2}}$
	16		$\frac{16}{81}$		0.4

6 Evaluate without a calculator.

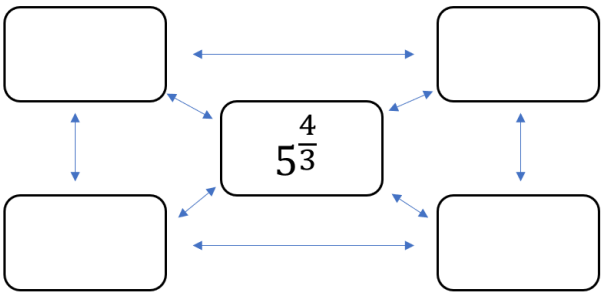
a $8^{\frac{2}{3}}$ 4	**b** $32^{\frac{3}{5}}$ 8	**c** $36^{\frac{3}{2}}$ 216
d $16^{\frac{5}{4}}$ 32	**e** $27^{-\frac{2}{3}}$ $\frac{1}{9}$	**f** $25^{-\frac{3}{2}}$ $\frac{1}{125}$
g $64^{-\frac{2}{3}}$ $\frac{1}{16}$	**h** $625^{-\frac{3}{4}}$ $\frac{1}{125}$	**i** $\frac{2}{4^{\frac{5}{2}}}$ $\frac{1}{16}$

7 Fill in the missing expressions. The first is done for you.

a



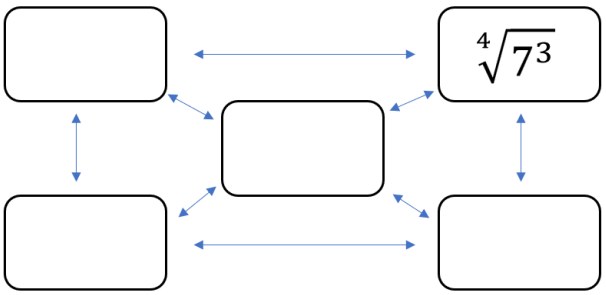
b



Left to right, top to bottom

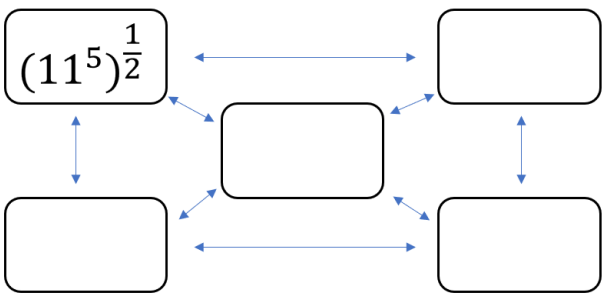
$(5^4)^{\frac{1}{3}}$, $(\sqrt[3]{5})^4$, $(5^{\frac{1}{3}})^4$, $\sqrt[3]{5^4}$,

c



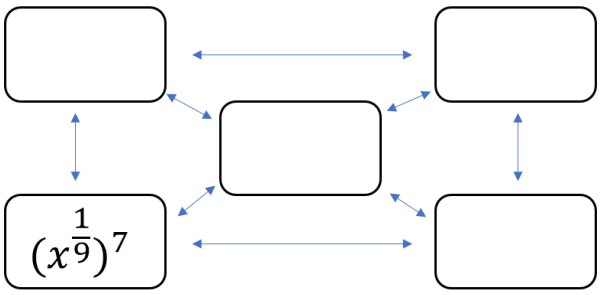
$(7^3)^{\frac{1}{4}}$, $7^{\frac{3}{4}}$, $(7^{\frac{1}{4}})^3$, $(\sqrt[4]{7})^3$

d



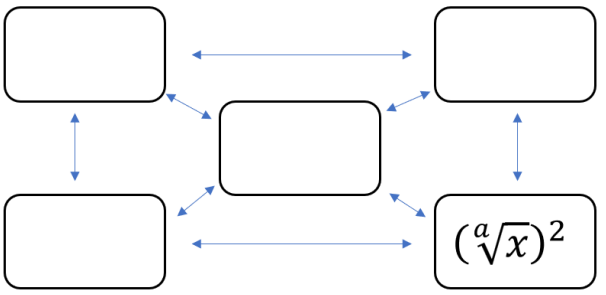
$\sqrt{11^5}$, $11^{\frac{5}{2}}$, $(11^{\frac{1}{2}})^5$, $(\sqrt{11})^5$

e



$(x^7)^{\frac{1}{9}}$, $\sqrt[9]{x^7}$, $x^{\frac{7}{9}}$, $(x^7)^{\frac{1}{9}}$

f



$(x^2)^{\frac{1}{a}}$, $\sqrt[9]{x^2}$, $x^{\frac{2}{9}}$, $(x^{\frac{1}{9}})^2$

INDEX LAWS AND SURD LAWS

- As surds can be written with fractional indices, all index laws apply when working with surds.
- When changing numbers from index form to surd form, deal with negative indices first.

Example Apply index laws to surds.

$\sqrt{x^5y^3}$ $= (x^5y^3)^{\frac{1}{2}}$ $= x^{\frac{5}{2}}y^{\frac{3}{2}}$	Which index law did we use to simplify $(x^5y^3)^{\frac{1}{2}}$ to $x^{\frac{5}{2}}y^{\frac{3}{2}}$? *Index law of .....*
$\frac{1}{\sqrt[3]{x^2}}$ $= \frac{1}{x^{\frac{2}{3}}}$ $= x^{-\frac{2}{3}}$	What does a negative index mean? *Negative index means to take the ..... of the base.*

Write in index form.

a $\sqrt[5]{3t^2}$

$3^{\frac{1}{5}}t^{\frac{2}{5}}$

b $5\sqrt[3]{4}$

$5 \times 4^{\frac{1}{3}}$

c $2\sqrt[4]{y^{12}}$

$2y^3$

d $\sqrt{4x^2}$

$2x$

e $\sqrt[3]{27t^{12}}$

$3t^4$

f $\frac{2}{3\sqrt{x^5}}$

$\frac{2}{3}x^{-\frac{5}{2}}$

1 Write the following with fractional indices.

a $\sqrt[5]{10t^2}$

$10^{\frac{1}{5}}t^{\frac{2}{5}}$

b $\sqrt[8]{8m^4}$

$8^{\frac{1}{8}}m^{\frac{1}{2}}$

c $\sqrt[4]{7x^7}$

$7^{\frac{1}{4}}x^{\frac{7}{4}}$

d $6\sqrt{5}$

$6 \times 5^{\frac{1}{2}}$

e $7\sqrt{6}$

$7 \times 6^{\frac{1}{2}}$

f $5\sqrt[3]{4}$

$5 \times 4^{\frac{1}{3}}$

g $7\sqrt{x^5}$

$7x^{\frac{5}{2}}$

h $6\sqrt[3]{n^7}$

$6n^{\frac{7}{3}}$

i $3\sqrt[4]{y^{12}}$

$3y^3$

j $5\sqrt[3]{p^2r}$

$5p^{\frac{2}{3}}r^{\frac{1}{3}}$

k $2\sqrt[3]{a^4b^2}$

$2a^{\frac{4}{3}}b^{\frac{2}{3}}$

l $2\sqrt[4]{g^3h^5}$

$2g^{\frac{3}{4}}h^{\frac{5}{4}}$

2 Write the following in surd form.

a $x^{\frac{3}{4}}$

$\sqrt[4]{x^3}$

b $x^{\frac{4}{3}}$

$\sqrt[3]{x^4}$

c $x^{-\frac{4}{3}}$

$\frac{1}{\sqrt[3]{x^4}}$

d $2x^{\frac{1}{5}}$

$2\sqrt[5]{x}$

e $(2x)^{\frac{1}{5}}$

$\sqrt[5]{2x}$

f $x^{-\frac{7}{5}}y^{\frac{1}{5}}$

$\frac{\sqrt[5]{y}}{\sqrt[5]{x^7}}$

3 Express in surd form.

a $(64x^3)^{\frac{1}{3}}$

$4x$

b $(64x)^{\frac{1}{3}}$

$4\sqrt[3]{x}$

c $64x^{\frac{1}{3}}$

$64\sqrt[3]{x}$

d $64x^{-\frac{1}{3}}$

$\frac{64}{\sqrt[3]{x}}$

e $(64x)^{-\frac{1}{3}}$

$\frac{1}{4\sqrt[3]{x}}$

f $(64x)^{-\frac{1}{2}}$

$\frac{1}{8\sqrt{x}}$

g $64x^{-\frac{1}{2}}$

$\frac{64}{\sqrt{x}}$

h $64x^{\frac{1}{2}}$

$64\sqrt{x}$

i $\left(64x^{\frac{1}{2}}\right)^3$

$64^3\sqrt{x^3}$

j $(64x^3)^{\frac{1}{2}}$

$8\sqrt{x^3}$

k $\left(64x^{\frac{1}{3}}\right)^{\frac{1}{2}}$

$8\sqrt[6]{x}$

l $\left(64x^{\frac{1}{2}}\right)^{\frac{1}{3}}$

$4\sqrt[6]{x}$

m $(64x^3)^{\frac{1}{6}}$

$2\sqrt{x}$

n $64(x^2)^{\frac{1}{6}}$

$64\sqrt[3]{x}$

o $\left(64x^{\frac{1}{3}}\right)^6$

64^6x^2

4 Fully simplify the following.

a $\sqrt{25s^4}$ $5s^2$	**b** $\sqrt[3]{27t^6}$ $3t^2$	**c** $\sqrt[4]{16t^8}$ $2t^2$
d $\sqrt[3]{125t^{12}}$ $5t^4$	**e** $(x^3)^{\frac{1}{3}}$ x	**f** $(b^{12})^{\frac{1}{3}}$ b^4
g $(16a^2b^8)^{\frac{1}{2}}$ $4ab^4$	**h** $(32x^{10}y^{15})^{\frac{1}{5}}$ $2x^2y^3$	**i** $\sqrt[3]{\frac{8x^3}{27}}$ $\frac{2x}{3}$

5 Write without negative or fraction powers.

a x^{-3} $\frac{1}{x^3}$	**b** $3x^{-1}$ $\frac{3}{x}$	**c** $(3x)^{-1}$ $\frac{1}{3x}$	**d** $5x^{-2}$ $\frac{5}{x^2}$
e $\frac{2}{3}x^{-4}$ $\frac{2}{3x^4}$	**f** $\frac{4}{5}x^{-3}$ $\frac{4}{5x^3}$	**g** $\frac{4x^{-3}}{5}$ $\frac{4}{5x^3}$	**h** $\frac{1}{5}x^{-3}$ $\frac{1}{5x^3}$
i $\frac{x^{-3}}{5}$ $\frac{1}{5x^3}$	**j** $x^{\frac{1}{2}}$ $\sqrt{x}$	**k** $x^{\frac{3}{2}}$ $\sqrt{x^3}$	**l** $3x^{\frac{5}{2}}$ $3\sqrt{x^5}$
m $x^{-\frac{1}{2}}$ $\frac{1}{\sqrt{x}}$	**n** $x^{-\frac{3}{4}}$ $\frac{1}{\sqrt[4]{x^3}}$	**o** $3x^{-\frac{2}{3}}$ $\frac{3}{\sqrt[3]{x^2}}$	**p** $\frac{1}{6}x^{-\frac{3}{4}}$ $\frac{1}{6\sqrt[4]{x^3}}$

6 Simplify the expression $(3a)^{-1} \times 3a^{-1}$

$\frac{1}{a^2}$

7 **NAPLAN A**

What is the value of p so that $\frac{a^2a^{-3}}{\sqrt{a}} = a^p$

$p = -1.5$

8 Write without roots, and if x is in the denominator, as a negative power.

a $\frac{4}{x}$ $4x^{-1}$	**b** $\frac{4}{x^2}$ $4x^{-2}$	**c** $\frac{3}{4x^2}$ $\frac{3x^{-2}}{4}$	**d** $\frac{1}{4x^2}$ $\frac{x^{-2}}{4}$
e $\frac{5}{x^3}$ $5x^{-3}$	**f** $\sqrt{x}$ $x^{\frac{1}{2}}$	**g** $\sqrt[3]{x}$ $x^{\frac{1}{3}}$	**h** $\sqrt[3]{x^2}$ $x^{\frac{2}{3}}$
i $5\sqrt{x^3}$ $5x^{\frac{3}{2}}$	**j** $\sqrt[5]{x^3}$ $x^{\frac{3}{5}}$	**k** $\frac{1}{\sqrt{x}}$ $x^{-\frac{1}{2}}$	**l** $\frac{1}{\sqrt[4]{x}}$ $x^{-\frac{1}{4}}$
m $\frac{4}{\sqrt[3]{x^2}}$ $4x^{-\frac{2}{3}}$	**n** $\frac{2}{3\sqrt{x}}$ $\frac{2x^{-\frac{1}{2}}}{3}$	**o** $\frac{1}{4\sqrt[3]{x^2}}$ $\frac{x^{-\frac{2}{3}}}{4}$	**p** $\frac{4}{5\sqrt{x^3}}$ $\frac{4x^{-\frac{3}{2}}}{5}$

9 Evaluate without a calculator.

a 5^0 1	**b** $49^{\frac{1}{2}}$ 7	**c** $125^{\frac{1}{3}}$ 5	**d** $8^{\frac{2}{3}}$ 4
e $(9^3)^{\frac{1}{2}}$ 27	**f** $\left(\frac{125}{64}\right)^{\frac{2}{3}}$ $\frac{25}{16}$	**g** $(-8)^{\frac{2}{3}}$ 4	**h** $9^{-\frac{1}{2}}$ $\frac{1}{3}$
i 5^{-2} $\frac{1}{25}$	**j** $\frac{1}{3^{-3}}$ 27	**k** $\left(\frac{2}{5}\right)^{-2}$ $\frac{25}{4}$	**l** $\left(\frac{1}{25}\right)^{-\frac{3}{2}}$ 125
m $4^{-\frac{1}{2}}$ $\frac{1}{2}$	**n** $125^{-\frac{2}{3}}$ $\frac{1}{25}$	**o** $8^{-3} \times 2^8$ $\frac{1}{2}$	

10 Write in index form with a single base.

a $x^2\sqrt{x}$ $x^{\frac{5}{2}}$	**b** $\frac{\sqrt{x}}{x^3}$ $x^{-\frac{5}{2}}$	**c** $x^3\sqrt[3]{x^2}$ $x^{\frac{11}{3}}$	**d** $\frac{x}{\sqrt[3]{x^2}}$ $x^{\frac{1}{3}}$
e $2\sqrt{2}$ $2^{\frac{3}{2}}$	**f** $3\sqrt{3}$ $3^{\frac{3}{2}}$	**g** $25\sqrt{5}$ $5^{\frac{5}{2}}$	**h** $27\sqrt{3}$ $3^{\frac{7}{2}}$

18 Use index laws to simplify these expressions. Answer in surd form.

a $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$ a	**b** $a^{\frac{1}{3}} \times a^{\frac{1}{3}}$ $\sqrt[3]{a^2}$	**c** $a^{\frac{2}{3}} \times a^{\frac{4}{3}}$ a^2	**d** $a^2 \times a^{\frac{1}{2}}$ $\sqrt{a^5}$
e $\frac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}}$ $\sqrt[3]{x}$	**f** $\frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$ x	**g** $\frac{x^{\frac{7}{6}}}{x^{\frac{2}{6}}}$ $\sqrt[6]{x^5}$	**h** $\frac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$ x
i $(y^2)^{\frac{1}{2}}$ y	**j** $(y^3)^{\frac{2}{3}}$ y^2	**k** $(y^{\frac{1}{2}})^3$ $\sqrt{y^3}$	**l** $(x^{\frac{1}{2}})^{\frac{1}{2}}$ $\sqrt[4]{x}$
m $(x^{\frac{2}{3}})^4$ $\sqrt[3]{x^8}$	**n** $(a^{\frac{2}{5}})^{\frac{1}{3}}$ $\sqrt{a^{15}}$	**o** $(a^{\frac{3}{4}})^{\frac{1}{2}}$ $\sqrt[8]{a^3}$	**p** $(n^{\frac{2}{5}})^{\frac{10}{3}}$ $\sqrt[3]{n^4}$
q $a \times a^{\frac{1}{3}}$ $\sqrt[3]{a^4}$	**r** $a^{\frac{1}{2}} \times a^{\frac{1}{5}}$ $\sqrt[10]{a^7}$	**s** $a^{\frac{2}{3}} \times a^{\frac{3}{7}}$ $\sqrt[21]{a^{23}}$	**t** $a^5 \div a^{\frac{7}{3}}$ $\sqrt[3]{a^8}$

11 Simplify the following using index laws. Write your answer without fractional or negative indices.

a $x^3 \times x$

x^4

b $\sqrt{x} \times \sqrt{x}$

x

c $x^2 \times \sqrt{x}$

$x^{\frac{5}{2}} = \sqrt{x^5}$

d $x^3 \times \sqrt{x}$

$x^{\frac{7}{2}} = \sqrt{x^7}$

e $x^2 \times \sqrt[3]{x}$

$x^{\frac{7}{3}} = \sqrt[3]{x^7}$

f $\sqrt{x} \times \sqrt[3]{x}$

$x^{\frac{5}{6}} = \sqrt[6]{x^5}$

g $x\sqrt{x} \times x$

$x^{\frac{5}{2}} = \sqrt{x^5}$

h $x\sqrt{x} \times x\sqrt{x}$

x^3

i $\frac{1}{x} \times \frac{1}{x}$

$x^{-2} = \frac{1}{x^2}$

j $\frac{1}{x} \times x$

1

k $\frac{3}{x} \times x$

3

l $\frac{1}{\sqrt{x}} \times \frac{1}{\sqrt{x}}$

$x^{-1} = \frac{1}{x}$

m $\frac{2}{\sqrt{x}} \times \sqrt{x}$

2

n $\frac{1}{\sqrt{x}} \times x$

$\sqrt{x}$

o $\frac{1}{\sqrt{x}} \times \frac{x}{\sqrt{x}}$

1

p $\frac{x^2}{\sqrt{x}} \times \frac{\sqrt{x}}{x\sqrt{x}}$

$\sqrt{x}$

12 Expand these expressions. Write your final answer without indices.

a $\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2$

$x + 2 + \frac{1}{x}$

b $\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)^2$

$x - 2 + \frac{1}{x}$

c $\left(x^{\frac{5}{2}} - x^{-\frac{5}{2}}\right)^2$

$x^5 - 2 + \frac{1}{x^5}$

d $(x + 5x^{-1})^2$

$x^2 + 10 + \frac{25}{x^2}$

e $(x^2 - 7x^{-2})^2$

$x^4 + 14 + \frac{49}{x^4}$

f $\left(3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}\right)^2$

$9x - 12 + \frac{4}{x}$

13 Write the following in index form.

a $\sqrt{2\sqrt{2}}$

$= (2\sqrt{2})^{\frac{1}{2}}$

$= \left(2^1 \times 2^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$=$

$2^{\frac{3}{4}}$

b $\sqrt{2\sqrt{2\sqrt{2}}}$

$2^{\frac{7}{8}}$

c $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$

$2^{\frac{15}{16}}$

d What do you notice?

The fractional index has denominator of 2^n where n is the number of twos, and numerator is $2^n - 1$

e What would be $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$... with 6 twos?

Answer in index form and check your answer using a calculator.

$2^{\frac{63}{64}}$

f What would be $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$... with n twos? Answer in index form.

$2^{\frac{2^{n-1}}{2^n}}$

EXPONENTIAL EQUATIONS

- Exponential equations involve finding the value of a pronumeral in the index.
- For example: $2^x = 8$ and $3^{2x} - 3^x - 6 = 0$.
- To solve exponential equations:
  1. Rewrite all terms with the smallest possible base number.
  2. Write an equation using the powers.

Example Solve exponential equations.

$25^x = 125$ 1. $5^{2x} = 5^3$ 2. $2x = 3$ $x = \frac{3}{2}$	What is the first step when solving exponential equations? *Change all ..... numbers to be the same.* How do we know what to change the base to (why didn't we try to change the base to 25)? *We change the base to the ..... possible number.*
$3^x = \frac{1}{9}$ 1. $3^x = \frac{1}{3^2}$ $3^x = 3^{-2}$ 2. $x = -2$	Why did we change $\frac{1}{3^2} = 3^{-2}$? *We wanted the base to not be in the .....*

a $9^x = 27$

$3 = 3$

$x = \frac{3}{2}$

b $2^x = \frac{1}{8}$

$x = -3$

c $16^x = 64$

$x = \frac{3}{2}$

Example Solve exponential equations with binomial powers.

$9^{2x-1} = 27$	Why is $9^{2x-1} = (3^2)^{2x-1}$?
1. $(3^2)^{2x-1} = 3^3$	*9 can be written as .....*
$3^{4x-2} = 3^3$	Why does $(3^2)^{2x-1}$ simplify to 3^{4x-2}?
2. $4x - 2 = 3$	*Since the power is a ....., we need to .....*
$4x = 5$	*every term.*
$x = \frac{5}{4}$	Using a calculator, check the solution by substituting it back into the original equation.

a $25^{x+3} = 125$

$x = -\frac{3}{2}$

b $27^{x+3} = 9^{2x}$

$x = 9$

1 Solve for x.

a $3^x = 27$ — 3

b $2^x = 32$ — 5

c $4^x = 64$ — 3

d $10^x = 10\,000$ — 4

2 Solve for x.

a $7^x = \frac{1}{49}$ — -2

b $11^x = \frac{1}{121}$ — -2

c $3^x = \frac{1}{243}$ — -5

d $3^{-x} = \frac{1}{3}$ — 1

e $2^{-x} = 64$ — -6

f $7^{-x} = \frac{1}{343}$ — 3

3 Solve for x.

a $8^x = 16$ — $\frac{4}{3}$

b $25^x = 125$ — $\frac{3}{2}$

c $81^x = 9$ — $\frac{1}{2}$

d $32^x = 2$ — $\frac{1}{5}$

e $10000^x = 10$ — $\frac{1}{4}$

f $7^{-x} = 49$ — -2

g $4^{-x} = 256$ — -4

h $16^{-x} = 64$ — $-\frac{3}{2}$

i $25^{-x} = 125$ — $-\frac{3}{2}$

4 Solve for x.

a $\left(\frac{3}{4}\right)^x = \frac{4}{3}$ — -1

b $\left(\frac{2}{3}\right)^x = \frac{9}{4}$ — -2

c $\left(\frac{2}{5}\right)^x = \frac{125}{8}$ — -3

d $\frac{1}{2^x} = 8$ — -3

e $\frac{1}{3^x} = 81$ — -4

f $\frac{1}{2^x} = 1$ — 0

5 Solve for x.

a $7^{x+9} = 49$

-7

b $2^{x+1} = 8^{x}$

$\frac{1}{2}$

c $9^{x+12} = 81^{x+5}$

2

d $27^{x+3} = 9^{2x}$

9

e $32^{2x+3} = 128^{2x}$

$\frac{15}{4}$

f $25^{x+3} = 125^{3x}$

$\frac{6}{7}$

g $27^{2x+3} = 9^{2x-1}$

$-\frac{11}{2}$

h $9^{x-1} = 27^{2x-6}$

4

i $49^{2x-3} = 343^{2x-1}$

$-\frac{3}{2}$

j $6^{2n-6} = 1$

3

k $11^{3n-1} = 11$

$\frac{2}{3}$

l $8^{5n-1} = \frac{1}{2}$

$\frac{2}{15}$

6 Solve for x.

a $3^{x} = \sqrt{81}$

2

b $6^{x} = \sqrt[3]{36}$

$\frac{2}{3}$

c $4^{x} = \sqrt[4]{64}$

$\frac{3}{4}$

d $3^{x} = \sqrt[9]{27}$

$\frac{1}{3}$

e $25^{x} = \sqrt[5]{125}$

$\frac{3}{10}$

f $9^{x} = \frac{1}{\sqrt[3]{27}}$

$-\frac{1}{2}$

7 Find the value of x in these equations.

a $\left(\frac{1}{3}\right)^x = \frac{1}{81}$ 4

b $\left(\frac{1}{3}\right)^x = 81$ −4

c $\left(\frac{1}{3}\right)^x = 27$ −3

d $\left(\frac{1}{3}\right)^x = \frac{1}{27}$ 3

e $\left(\frac{1}{4}\right)^x = 64$ −3

f $\left(\frac{1}{4}\right)^x = 256$ −4

g $\left(\frac{1}{4}\right)^x = \frac{1}{256}$ 4

h $\left(\frac{5}{25}\right)^x = 625$ −4

8 Solve for x

a $5^{x-1} = \frac{1}{25}$ −1

b $3^{x-3} = \frac{1}{9}$ 1

c $10^{x-5} = \frac{1}{1000}$ 2

d $\left(\frac{3}{4}\right)^{2x+1} = \frac{64}{27}$ −2

e $\left(\frac{2}{5}\right)^{3x-5} = \frac{25}{4}$ 1

f $\left(\frac{3}{2}\right)^{3x+2} = \frac{16}{81}$ −2

g $\left(\frac{7}{4}\right)^{1-x} = \frac{4}{7}$ 2

h $3^{2x-1} = \frac{1}{81}$ $-\frac{3}{2}$

i $7^{2x+3} = \frac{1}{49}$ $-\frac{5}{2}$

9 Solve $2^{3x-3} = 8^{2-x}$. $\frac{3}{2}$

10 Solve $3^{-4x} = 9^{6-x}$. −6

11 Solve for x. Rewrite the decimals as fractions.

a $10^x = 0.0001$

−4

b $2^x = 0.125$

−3

c $5^x = 0.00032$

−5

d $(0.25)^x = 0.5$

0.5

e $(0.04)^x = 125$

−1.5

f $(0.0625)^{x+1} = \frac{1}{2}$

$-\frac{3}{4}$

12 Solve for x.

a $3^x \times 9^x = 27$

1

b $5^{3x} \times 25^{-2x+1} = 125$

−1

c $2^{-3x} \times 4^{2x-2} = 16$

8

d $16^{-x} = \frac{1}{32}$

$\frac{5}{4}$

e $8^x = 4^{x+1}$

2

f $16^{2-x} = \frac{1}{8^x}$

8

g $\frac{3^{n-2}}{9^{1-n}} = 9$

2

h $\frac{5^{3n-3}}{25^{n-3}} = 125$

0

i $\frac{36^{3+2n}}{6^n} = 1$

−2

13 Solve for x

a $\left(\frac{1}{5}\right)^{x+1} = \left(\frac{1}{125}\right)^{x-1}$

2

b $\sqrt{2^{4x} \times 4^{2x} \times 8^{4x}} = 256^x$

0

14 Solve these exponential equations.

a $2^x = 128$ — 7

b $3^x = \frac{1}{3}$ — -1

c $15^y = \sqrt{15}$ — $\frac{1}{2}$

d $15^y = \frac{1}{\sqrt{15}}$ — $-\frac{1}{2}$

e $7^k = 7\sqrt{7}$ — $\frac{3}{2}$

f $5^x = 0.2$ — -1

g $49^k = 7$ — $\frac{1}{2}$

h $3^x = \sqrt[3]{9}$ — $\frac{2}{3}$

i $25^k = 125$ — $\frac{3}{2}$

j $7^x = \frac{1}{\sqrt{7}}$ — $-\frac{1}{2}$

k $6^k = 6\sqrt{6}$ — $\frac{3}{2}$

l $121^y \times \sqrt[3]{11} = 1331$ — $\frac{4}{3}$

m $(3^8)^{\frac{1}{2}} = 3^m$ — 4

n $2^{k-4} = 1$ — 4

o $2^{5-2x} = \frac{1}{4}$ — $\frac{7}{2}$

p $\frac{\sqrt[3]{5}}{5} = 5^m$ — $-\frac{2}{3}$

q $\left(\frac{1}{3}\right)^{x+1} = 27$ — -4

r $\left(\frac{1}{36}\right)^{5k+8} = 216$ — $-\frac{19}{10}$

s $10^y \times 100^y = 0.001$ — -1

t $\left(\frac{1}{8}\right)^{1-2k} = 128^k$ — -3

u $\left(\frac{25}{\sqrt[3]{5}}\right)^{\frac{x}{2}} = 5$ — $\frac{6}{5}$

v $1000 = \frac{100^n}{\sqrt[5]{10^n}}$ — $\frac{5}{3}$

w $\frac{9^y}{27^{5-y}} = \sqrt{243}$ — $\frac{7}{2}$

x $\frac{\sqrt[3]{5^m}}{\sqrt{5}} = 25^4$ — $\frac{51}{2}$

15 Solve simultaneously:

a $7^{2x-y} = 49$

$2^{x+y} = 128$

$x = 3, y = 4$

b $8^x \div 4^y = 4$

$11^{y-x} = \frac{1}{11}$

$x = 0, y = -1$

c $13^{x+4y} = 1$

$25^{x+5y} = 5$

$x = -2, y = \frac{1}{2}$

d $2^a + 3^b = 17$

$2^{a+1} - 3^{b+1} = -11$

$a = 3, b = 2$

FACTORISING EXPRESSIONS WITH INDICES

- We can split a base based on prime factors.
  This allows us to simplify when the bases aren't the same.

Example Simplify expressions with indices by factorisation.

$12^n \times 9 = (4 \times 3)^n \times 3^2$ $= 4^n \times 3^n \times 3^2$ $= 2^{2n} \times 3^{n+2}$	Which index law says $(4 \times 3)^n = 4^n \times 3^n$?

Write these expressions with prime bases and simplify.

a $2^n \times 4^n \times 8^n$

2^{6n}

b $100^{2n-1} \times 25^{-1} \times 8^{-1}$

$2^{4n-5} \times 5^{4n-4}$

c $\frac{9^{n+2} \times 3^{n+1}}{3 \times 27^n}$

3^4

- We can factorise expressions involving addition and subtraction by taking out a common factor. This is useful when simplifying fractions.

Example Simplify expressions with indices by factorisation.

$3^n + 3^{n+1}$ $= 3^n + 3 \times 3^n$ $= 3^n(1 + 3)$ $= 4 \times 3^n$	Why aren't we able to simplify this expression like we would $3^n \times 3^{n+1}$? *There is no index law for .....* Why are we able to rewrite 3^{n+1} as 3×3^n 3×3^n *is the same as* $3^{\square} \times 3^n$ How is this similar to simplifying $y + 3y$? *The common factor of* $3^n + 3 \times 3^n$ *is .....*

Simplify by factorising.

a $7^{n+2} + 7^n$

50×7^n

b $2^{2n+2} - 2^{2n-1}$

$7 \times 2^{2n-1}$ or 3.5×2^{2n}

c $\frac{3^{n+3} - 3^n}{3^n}$

26

1 Simplify by making all the bases the same.

a 3×3^x

3^{x+1}

b 32×2^x

2^{x+5}

c $\frac{1}{2} \times 2^x$

2^{x-1}

d $5^{2x} \times 5^{3x} \div 25^x$

5^{3x}

e $7^{3x+4} \div 49^{9-3x}$

7^{9x-14}

f $(9^x)^2 \times (3^{3x})^3$

3^{13x}

g $8^{2x} \div 2^{6x} \times 4^x$

2^{2x}

h $9^{3m+1} \times 3^{4m-1}$

3^{10m+1}

i $32^n \div 8^{2n} \div 4^{3n}$

2^{-7n}

j $6^x \times 4^x \div 3^x$

2^{3x}

k $12^x \times 18^x$

$2^{3x} \times 3^{3x}$

l $24^{x+1} \times 8^{-1}$

$3^{x+1} \times 2^{3x}$

2 Simplify by factorising each base to the smallest prime factors.

a $\frac{2^n \times 9^{2n+1}}{6^{n-2}}$

$= \frac{2^n \times 3^{4n+2}}{2^{n-2} \times 3^{n-2}}$

$=$

$2^2 \times 3^{3n+4}$

b $\frac{25^{3n} \times 5^{n-3}}{5^{4n+3}}$

5^{3n-6}

c $\frac{12^{x-2} \times 4^x}{6^{x-2}}$

2^{3x-2}

d $\frac{12^{n-3} \times 27^{1-n}}{9^{2n} \times 8^{n-1} \times 16^n}$

$2^{-5n-3} \times 3^{-6n}$

e $\frac{4^n \times 7^{n-3} \times 49^{3n+1}}{14^{n+2}}$

$2^{n-2} \times 7^{6n-3}$

f $\frac{35^2 \times 5^2 \times 7^6}{25^4 \times 49^3}$

$\frac{49}{625}$

3

a Simplify 2×2^x

b Hence explain why $2^x + 2^x$ can be written as 2^{x+1}

$2^x + 2^x = 2 \times 2^x = 2^{x+1}$

c What is the highest common factor of these two terms: 2^{x+1} and 2^x

2^x

d Therefore, factorise $2^{x+1} + 2^x$ by taking out the highest common factor.

$2^x(2 + 1) = 3(2^x)$

Factorise the following by factoring out a common factor.

e $3^x + 3^x + 3^x$

3^{x+1}

f $3^8 - 3^7$

2×3^7

g $3^n + 3^{n+1}$

4×3^n

h $\dfrac{7^n + 7^{n+2}}{7^{n-1} + 7^{n+1}}$

7

i $\dfrac{2^{2n} - 2^{n-1}}{2^n - 2^{-1}}$

2^n

j $\dfrac{5^n - 5^{n+1}}{5^{n+1} + 5^n}$

$-\frac{2}{3}$

k $5^n - 5^{n-3}$

$124 \times 5^{n-3}$

l $\dfrac{7^n + 7^{n+2}}{7^{n-1} + 7^{n+1}}$

7

m $\dfrac{2^{2n} - 2^{n-1}}{2^n - 2^{-1}}$

2^n

n $\dfrac{2^{n+3} - 4 \times 2^n}{2^{2n} - 4^{n-1}}$

$\frac{16}{3 \times 2^n}$

o $\dfrac{2^{n+1}}{(2^n)^{n+1}} \div \dfrac{\left(2^{2n+1}\right)^{n-1}}{4^n}$

2^{-3n^2+3n+2}

p $\dfrac{a^2x^m - b^2x^{m+4}}{a - bx^2}$

$x^m(a + bx^2)$

4 Simplify by a combination of prime factorisation and taking out a common factor.

a $6^n - 3^n$

$3^n(2^n - 1)$

b $\frac{6^n+3^n}{2^{n+1}+2}$

$\frac{3^n}{2}$

c $\frac{12^x+1}{6^{2x}+3^x}$

$\frac{1}{3^x}$

d $\frac{12^n-18^n}{3^n-2^n}$

-2^n3^n

5

a Factorise $5^{n+1} + 5^n$

b Hence simplify $\frac{5^{101}+5^{100}}{6}$

5^{100}

6 Find the value of x if 2^x is half of 2^{200}

199

7 Factorise and hence simplify the following. (hint: difference of two squares)

a $\dfrac{9^n-1}{3^n-1}$

(3^n+1)

b $\dfrac{5^{2n}-5^n}{5^n-1}$

5^n

8 Factorise: (hint: difference of two squares)

a $5^{2x}-2^{2y}$

$(5^x+2^y)(5^x-2^y)$

b $2^{4x}-2^{6x}$

$2^{4x}(1+2^x)(1-2^x)$

9 Solve

a $8^x=2^{x+3}+2^{x+3}$

2

b $32^x=2^{x+5}-2^{x+4}$

1

c $4^x-2^{x+1}-8=0$

2

10 Simplify $6^6+6^6+6^6+6^6+6^6+6^6$ and express your answer in index form.

6^7

11 Show that $2^{2m+3}+4^m$ is a multiple of 3 for all positive integers m.

$2^{2m+3}+4^m=9(2^m)$; ∴ multiple of 3

CHECK YOUR UNDERSTANDING

Fractional indices

1 Evaluate $25^{\frac{1}{2}}$

a 12.5 **b** 5 **c** 25.5 **d** $\frac{1}{25}$

2 Evaluate $64^{\frac{1}{2}}$

a 64.5 **b** 8 **c** 32 **d** $\frac{1}{64}$

3 Evaluate $64^{\frac{1}{3}}$

a 4 **b** 64.3333333… **c** 8 **d** $\frac{1}{64^3}$

Further fractional indices

1 Evaluate $\left(\frac{9}{4}\right)^{\frac{1}{2}}$

a $\frac{3}{2}$ **b** $\frac{4}{9}$ **c** $\frac{81}{16}$ **d** $\frac{4.5}{2}$

2 What is equivalent to $\sqrt[3]{r^5}$?

a $5r^3$ **b** $r^{\frac{3}{5}}$ **c** $r^{\frac{5}{3}}$ **d** $3r^5$

3 Evaluate $27^{\frac{2}{3}}$

a 18 **b** 9 **c** 243 **d** 3

4 Evaluate $16^{\frac{3}{2}}$

a 12 **b** 24 **c** 64 **d** 512

5 Evaluate $\left(\frac{27}{64}\right)^{\frac{1}{3}}$

a $\frac{4}{3}$ **b** $\frac{9}{21}$ **c** $\frac{64}{27}$ **d** $\frac{3}{4}$

6 Evaluate $(8)^{-\frac{1}{3}}$

a $7\frac{2}{3}$ **b** $\frac{1}{2}$ **c** 2 **d** $\frac{1}{8^3}$

7 Evaluate $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

a $\frac{2}{3}$ **b** $-\frac{8}{9}$ **c** $\frac{81}{24}$ **d** $-\frac{2}{3}$

8 Evaluate $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

a $-\frac{4}{9}$ **b** $-\frac{4}{6}$ **c** $\frac{4}{9}$ **d** $\frac{9}{4}$