Student Name

Mathematics Stage 5 Paths

FUNCTIONS AND OTHER GRAPHS

Book 1 Define relations and functions and use function notation.

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-FNC-P-01 uses function notation to describe and graph functions of one variable and graphs inequalities in one and 2 variables

Define relations and functions, and use function notation

- Define a relation over 2 sets as an association between the elements of one set to the elements of another set
- Notate relations as a set of ordered pairs (x, y) of real numbers
- Define a function as a relation over 2 sets where each element of the first set is associated with exactly one element in the second set
- Apply the vertical line test on a graph to decide whether it represents a function
- Use the notation $f(x)$ when expressing a function
- Use the notation $f(c)$ to determine the value of $f(x)$ when $x = c$

REVIEW OF PRIOR KNOWLEDGE

RELATIONS AND FUNCTIONS

Investigation Relations and functions.

A set is a collection of objects.

We use { } to represent a set.

The statement "x is the set of numbers 7, 3, 2, 14, and 6" is written as:

$x = \{\ldots, \ldots, \ldots, \ldots, \ldots, \ldots\}$

The statement "y is the set of numbers 1, 2, 3, and 7" is written as:

…………………………………………………

A **relation** associates (or maps) elements of one set to another.

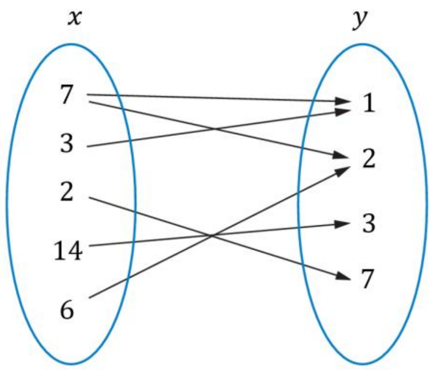
In simple terms, there is a relationship between x and y.

We can write associated elements using ordered pairs: (x, y)

The diagram to the right shows 6 ordered pairs:

(7, 1) (7, 2) (3, …)

(…, …) (…, …) (…, …)

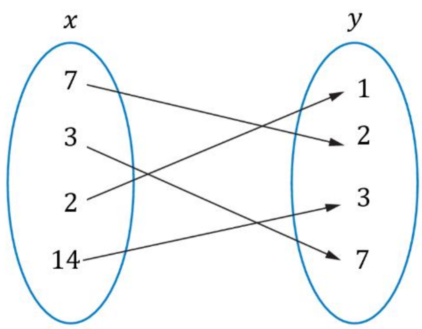


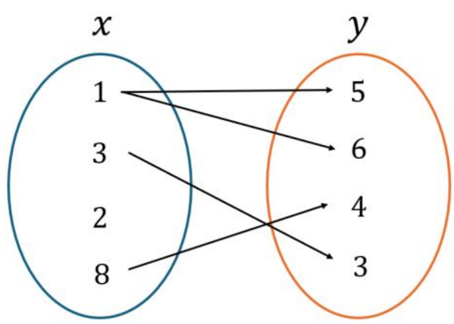
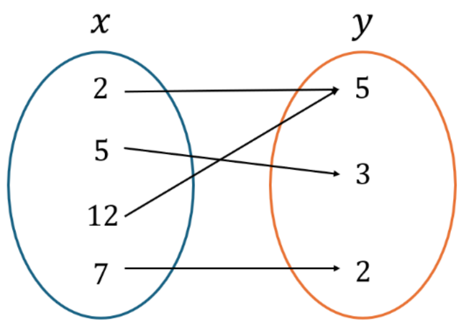
A **function** is a special type of relation such that for each x value, there is only one y-value.

Write the 4 ordered pairs shown:

(…, …) (…, …)

(…, …) (…, …)

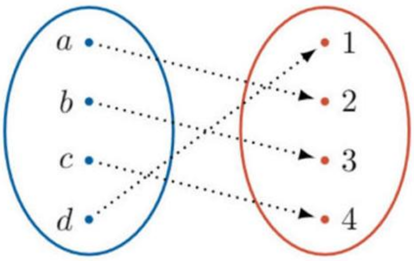


Relation	Function
A relation connects elements from two sets.  $(1,5), (1,6), (3,3), (8,4)$	A function is a special relation where each x-value has only one y-value. (only one arrow leaving each x value)  $(2,5), (5,3), (12,5), (7,2)$

Example Identify whether an association is a function.

Identify which of these relations are functions.

a



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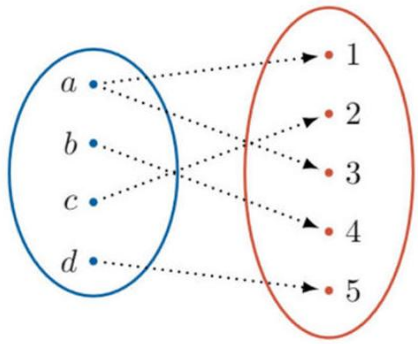

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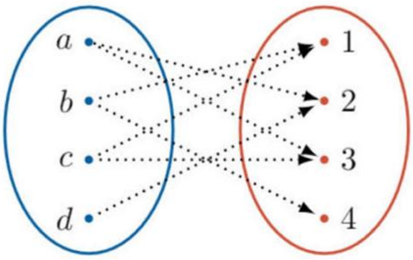

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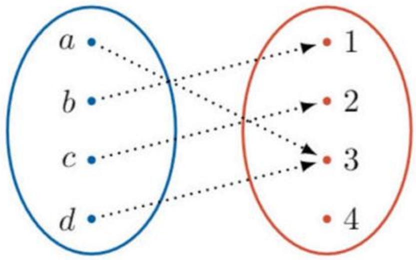
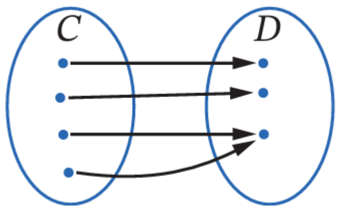


e $(1,2), (4,5), (2,3)$

f $(2,5), (2,7), (3,5)$

1 Identify which of the following relations are functions.
Remember, a function has only one value of y for each value of x.

a



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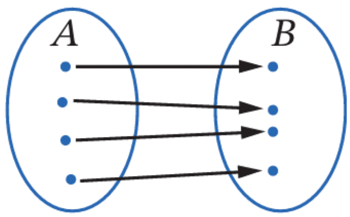

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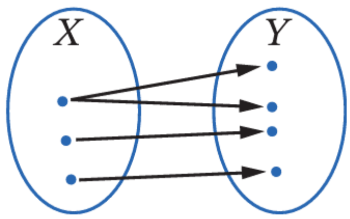

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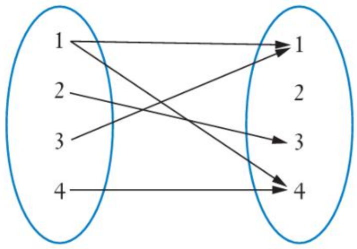

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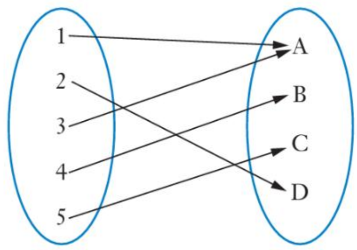


f

x	3	5	8
y	5	± 2	-7

2 Identify whether each set of points represents a function.

a $(2, 4), (-2, 7), (5, 2)$

b $(-1, 7), (-1, 3), (4, 6)$

c $(-6, 3), (5, 1), (4, -5), (-3, -1)$

d $(2, 3), (-5, -3), (4, 4), (8, -3)$

e $(5, 8), (-1, -1), (3, 8), (4, 0)$

f $(7, 2), (-6, 0), (0, 3), (-3, 2)$

g $(3, -1), (3, 5), (3, -6), (3, 0)$

h $(0, 2), (7, 2), (-4, 2), (9, 2), (5, 2)$

FUNCTION NOTATION

- Another way to understand functions is to think of it as a “machine.”
- A “function machine” takes an input number, applies a rule, then gives out an output number.
- x is the **input**, or **independent variable,** y is the **output,** or **dependent variable**.
- An example is the linear function $y = 2x + 3$.

Input $\longrightarrow \boxed{\times 2} \longrightarrow \boxed{+3} \longrightarrow$ Output

x $\qquad\qquad\qquad\qquad$ $y = 2x + 3$

Function notation

input

$$\underbrace{f(x)}_{\text{output}} = \underbrace{2x + 3}_{\text{function rule}}$$

Example Use function notation.

An equation is $y = 2x + 3$. Find the value of y when $x = 3$	A function is $f(x) = 2x + 3$. Evaluate $f(3)$.

What is the same and different about the two questions above?

……………………………………………………………………………………………………

Example Use function notation to find $f(c)$, where c is a constant.

$f(x) = x^2 + 5x$, find $f(3)$ $f(3) = (3)^2 + 5(3)$ $= 24$	What does $f(3)$ mean? *$f(3)$ means: for the rule $f(x) = x^2 + 5x$, what is the ……………… when the ………… is 3?*

a $f(x) = x^2 - 7x$, find $f(4)$

$f(3) = (\quad)^2 - 7(\quad)$

$-f(3) = -12$

b $p(x) = x + 3$, find $p(-5)$

$p(-5) = -2$

c $g(x) = x^2 - 2$, find $g(0)$

$g(0) = -2$

d $f(x) = 16^x$, find $f\left(\frac{1}{2}\right)$

$f\left(\frac{1}{2}\right) = 4$

Example Use function notation to find $f(c)$, where c is an algebraic expression.

$f(x) = x^2 + 5x$, find $f(x+1)$	What does $f(x+1)$ mean?
$f(x+1) = (x+1)^2 + 5(x+1)$	*$f(x+1)$ means: for the rule $f(x) = x^2 + 5x$,*
$= x^2 + 2x + 1 + 5x + 5$	*what is the ……………… when the …………*
$= x^2 + 6x + 6$	*is $x+1$?*

a If $f(x) = -x^2$, find $f(x+1)$

$f(x+1) = -x^2 - 2x - 1$

b If $g(x) = 2x + 3$, find $g(3 - x)$

$g(3-x) = 9 - 2x$

Example Use function notation to solve $f(x) = c$, where c is a number.

$f(x) = 2x - 5$, find x when $f(x) = 13$	What does $f(x) = 13$ mean?
$13 = 2x - 5$	*$f(x) = 13$ means: the ………… of the*
$18 = 2x$	*function rule $f(x) = 2x - 5$ is 13. Find the*
$9 = x$	*input.*

a $f(x) = 4x + 11$, find x when $f(x) = 24$

$z = \frac{13}{4}$

b $g(x) = x^2 + 3$, find x when $g(x) = 28$

$x = \pm 5z$

Key ideas

1. Function notation is a faster way of ……………………… when working with functions.
2. $f(x) = 5x + 7$ means the …………… is x, and the …………… ………… is $5x + 7$

1 Rewrite the following functions using function notation.

a $y = 8x$	**b** $y = 9 - x^2$	**c** $y = \frac{2}{x}$
d $y = x(2x - 3)$	**e** $y + 7 = x$	**f** $2y + 5 = x$

2 $f(x) = 2x + 3$. Find:

a $f(1)$	**b** $f(-2)$	**c** $f(8)$
d $f\left(\frac{1}{2}\right)$	**e** $f(a)$	**f** $f(x + 1)$

3 Find $h(x + 1)$ for each of these functions.

a $h(x) = 2x + 1$	**b** $h(x) = x^2$	**c** $h(x) = x^2 - 2x$

4 $f(x) = (x - 2)(x + 6)$. Find:

a $f(0)$	**b** $f(2)$	**c** $f(-4)$
d $f(a)$	**e** $f(a + 2)$	**f** $f(a - 6)$

5 For the function $L(x) = 3x + 1$, find:

a $L(2) + 1$

b $3L(-1)$

c $L(9) \div L(2)$

6 Given that $f(x) = x^2 - 3x + 5$, find

a $\frac{1}{2}\big(f(2) + f(3)\big)$

b $f(\sqrt{2})$

c $\frac{1}{6}\big(f(-1) + 4f(0) + f(1)\big)$

7 Write the equation as a function with independent variable x.

a $3x + 4y + 5 = 0$

b $4 + xy = 0$

8 A restaurant offers a special deal to groups by charging a cover fee of \$50, then \$20 per person. Write down C, the total cost of the meal in dollars, as a function of x, the number of people in the group.

....

....

9 In each case explain why the function value cannot be found.

a $F(0)$, when $F(x) = \sqrt{x - 4}$

b $g(3)$, when $g(x) = \sqrt{1 - x^2}$

c $h(-2)$, when $h(x) = \frac{1}{2+x}$

d $f(0)$. when $f(x) = \frac{1}{3x}$

10 For $g(x) = 2 - x$, find:

a $g(a)$

b $g(-a)$

c $g(a+1)$

11 For $F(x) = x^2 + 2x$, find:

a $F(x-2)$

b $F(x) - 2$

c $F(x-h)$

12 Given $f(x) = 7x - 9$ and $g(x) = 6 - 2x$, Find:

a $f(2)$

b $g(4)$

c $f(2) + g(4)$

d $f(-2) + 2g(1)$

e $f(g(4))$

f $g(f(4))$

13 Let $f(x) = x^2$. Determine whether the following statements are true or false.

a $f(4x) = 16f(x)$

b $f(xy) = f(x) \times f(y)$

c $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$

d $f(x+y) = f(x) + f(y)$

14 Find $f(k) - f(k-1)$ in simplest form if:

a $f(x) = x^2$

b $f(x) = 3x^2 + 2x$

c $f(x) = \frac{1}{x}$

d $f(x) = 2^x$

15 Let $f(x) = 4^x$. Show that these statements are true.

a $f(2x) = 16^x$

b $4f(x) = f(x+1)$

c $f(x+1) - f(x) = 3f(x)$

16 In each case, explain why the function value cannot be found.

a $F(0)$, where $F(x) = \sqrt{x-4}$

b $H(3)$, where $H(x) = \sqrt{1-x^2}$

c $g(-2)$, where $g(x) = \frac{1}{2+x}$

d $f(0)$, where $f(x) = \frac{1}{x}$

17 $f(x) = 5 - ax$

If $f(8) = 1$, find the value of the coefficient a and hence find the function.

18 $f(x) = \sqrt{b - x^2}$

If $f(2) = 2\sqrt{3}$, find the value of b.

19 Consider $f(x) = x^2 + 5x$

a Complete the square.

b Solve $f(x) = 0$

c Find $f(2x)$

d Evaluate $f(0.5)$

20 $f(x) = 3x - 1$ and $g(x) = x^2 - 8$

a Evaluate $f(-2)$

b Solve $g(x) = -x^2$

c Solve $f(x) = 2(4 - x)$

d Evaluate $g(-3)$

e Solve $2f(x) = 1$

f Solve $24g(x) = 0$

21 $f(x) = px^2 - x$. Find the value of p if $f(-3) = 2f(2)$.

22 $g(x) = x^2 + bx + c$

If $g(2) = -3$ and $g(-1) = 9$, use simultaneous equations to find b and c.

23 Using $P(x) = x^2 - 2x - 4$, find the value of:

a $P(1 + \sqrt{5})$

b $P(\sqrt{3} - 1)$

24 Find $g(a)$, $g(-a)$, and $g(a + 1)$ for each function.

a $g(x) = 2 - x$

b $g(x) = x^2$

c $f(x) = \frac{1}{x-1}$

25 Find $F(t) - 2$ and $F(t - 2)$ for each function.

a $F(x) = 5x + 2$

b $F(x) = x^2 + 2x$

c $F(x) = 2 - x^2$

26 If $f(x) = x^2 + 5x$, find in simplest form:

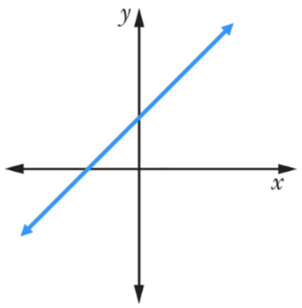
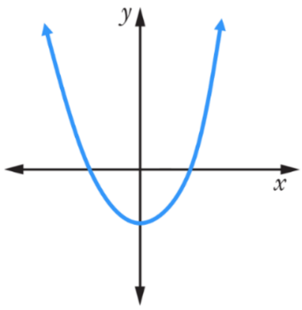
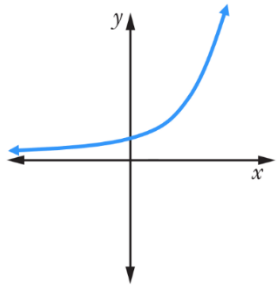
a $\frac{f(1+h)-f(1)}{h}$

b $\frac{f(x+h)-f(x)}{h}$

27 Given the function $f(x) = 2x^2 - 3x - 1$, simplify $\frac{f(x+h)-f(x)}{h}$

VERTICAL LINE TEST

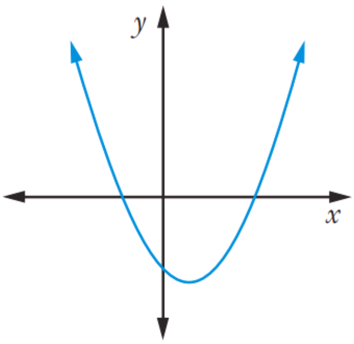
- We can plot relations as coordinates (x, y) to make a graph.
- x are the inputs and y are the outputs.

Linear Function $y = mx + c$	**Quadratic Function** $y = ax^2 + bx + c$	**Exponential Function** $y = ka^x + c$
		

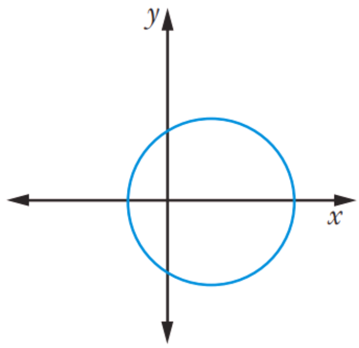
- The vertical line test determines whether a graph represents a function.
- If all possible vertical lines cut the graph at only one point, it passes the test and is a function. This shows that for every value of x, there is only 1 value of y.
- A closed circle ● means the point is on the graph.
- An open circle ○ means it is not.

Example Identify whether a graph is a function or relation.

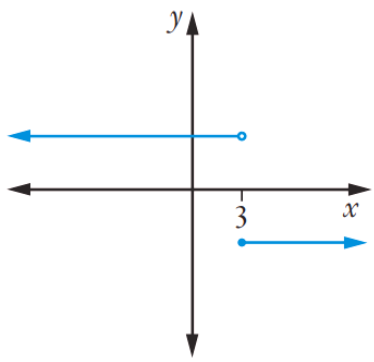
a Show this is a function



b Show this isn't a function

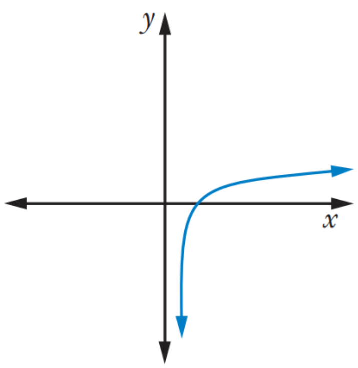


c Show this is a function



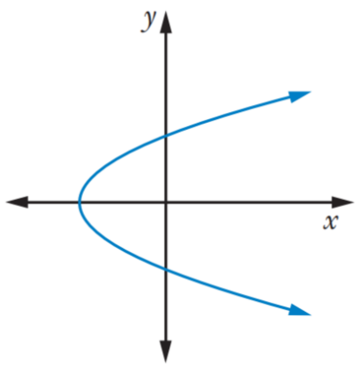
Use the vertical line test to determine whether these graphs represent functions.

a



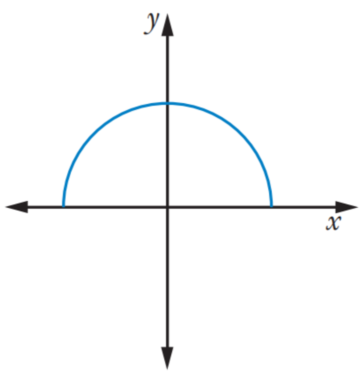
function

b



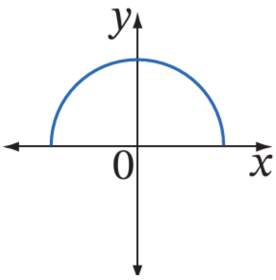
not function

c



function

1 Identify which of these graphs are functions using the vertical line test.

a


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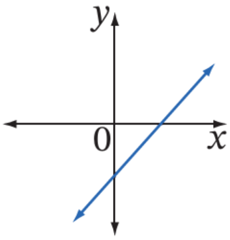

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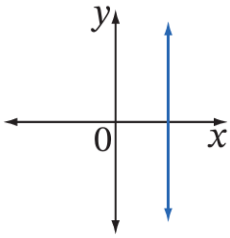

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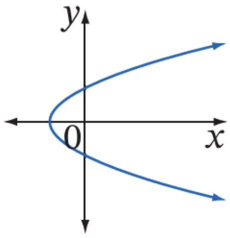

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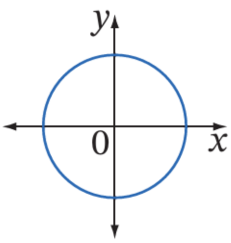

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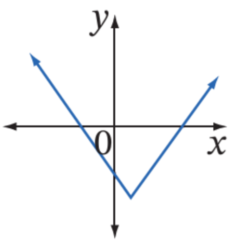

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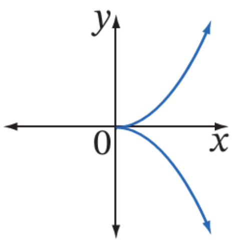

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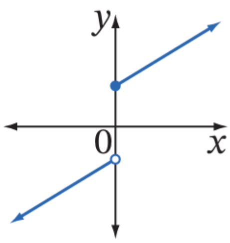

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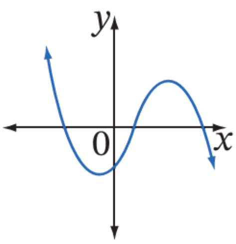

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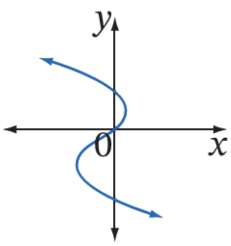

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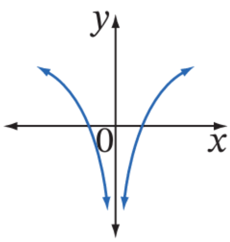

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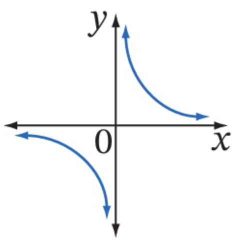

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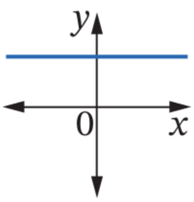

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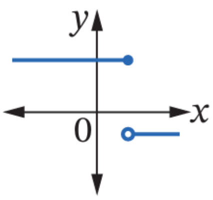

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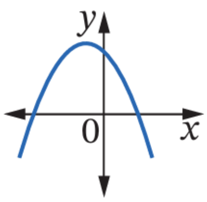

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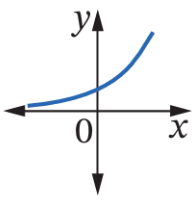
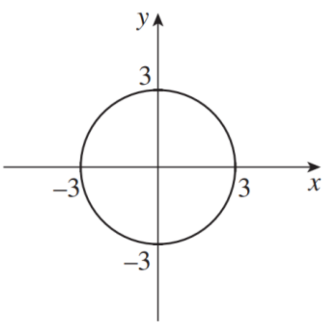
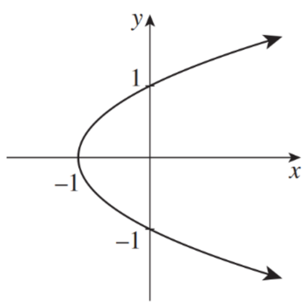


2 These graphs are not functions.
Write down the coordinates of two points on each graph that have the same x-coordinate.

a


b


3 Not all straight lines are functions. Explain why by giving an example.

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