

Student Name

Mathematics Stage 4

ALGEBRAIC TECHNIQUES

Book 2 Create algebraic expressions and
evaluate them by substitution

Version: 260207

Report mistakes & suggest improvements:
<https://MrDingMaths.com>

CONTENTS

Overview	2
Forming Expressions from Diagrams	4
Forming Expressions from Descriptions.....	9
Substitution and Evaluation.....	16
Substituting Negative Numbers.....	25
Generating Number Patterns by Substitution	31
Check Your Understanding	35

OVERVIEW

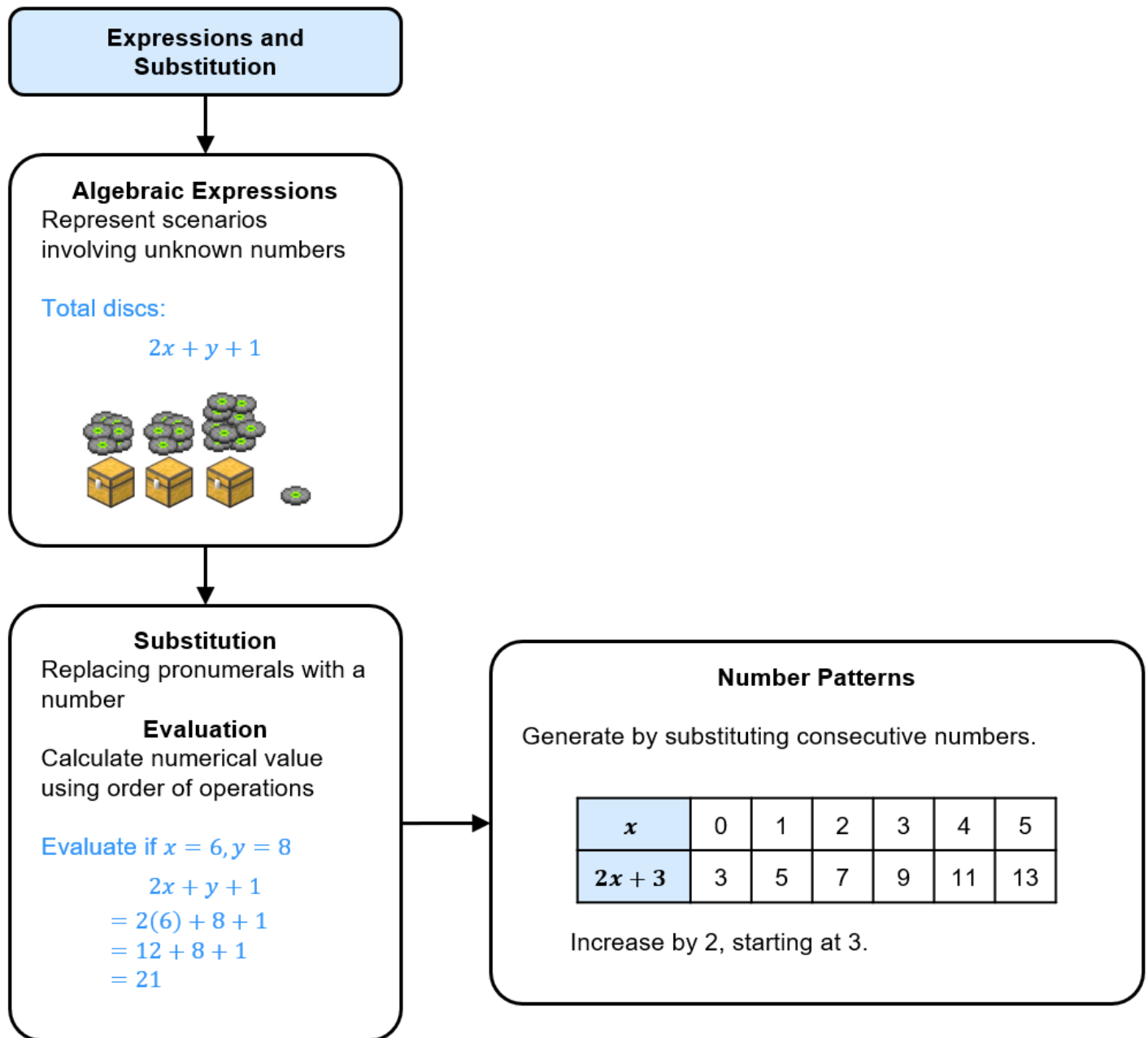
SYLLABUS CONTENT

MA4-ALG-C-01 generalises number properties to operate with algebraic expressions including expansion and factorisation

Create algebraic expressions and evaluate them by substitution

- Generate algebraic expressions by translating descriptions and vice versa
- Substitute numbers into algebraic expressions and evaluate the result
- Generate a number pattern from an algebraic expression

SUMMARY



REVIEW OF PRIOR KNOWLEDGE

1 Write each expression showing all operations.

a $6x$

b $5(2)$

c $3(6 + x)$

d $\frac{x}{6}$

e $\frac{2x}{5}$

f $-x$

2 Follow the order of operations to evaluate these expressions.

a $3 \times 5 + 4$

b $3 \times 5 - 4$

c $12 \div 4 + 7$

d $10 + 4 \times 5$

e $10 - 4 \times 5$

f $4 - 16 \div 2$

g $10 \div (5 + 3)$

h $3 \times 5 + 7 \times 2$

i $9 \times 3 - 8 \div 2$

3 Work out:

a $11 - -2$

b 8×-4

c $3 - 15$

d $18 + -3$

e $-7 + -3$

f $\frac{16}{-4}$

g -5×-2

h $6 \times -3 \times -2$

i $-7 - -4$

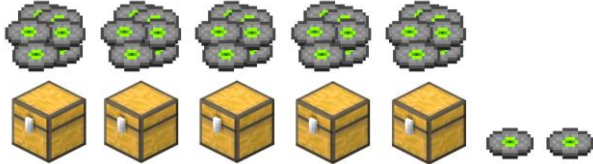
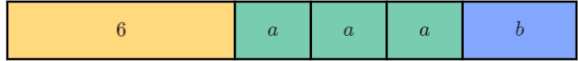
FORMING EXPRESSIONS FROM DIAGRAMS

- **Algebraic expressions** communicate relationships involving numbers.
- The **pronumeral** represents an unknown amount.
- The same **pronumeral** always represents the same number in one expression.

Example Represent a concrete scenario as an algebraic expression.

Write an algebraic expression for the total number of music discs.  $4n + 3$	What does the pronumeral n represent? n represents the of music discs in each chest.
 $2x + y + 1$	Why did we need two pronumerals, x and y in this scenario? One of the chests contains a number of music discs, so we must use a different pronumeral.

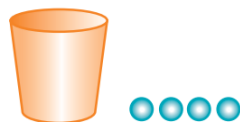
Represent these scenarios as algebraic expressions.

a  $5x + 2$	b  $3x + y + 5$
c  $c + 2t$	d  $16 + 4r$
e  $6 + 3a + b$	f  $40 + 2k + j$

Key ideas

1. A pronumeral represents an of something.
2. The same pronumeral in an expression must represent the amount.

- 1 The diagram shows 1 cup and 4 marbles.



- a If the cup has 6 marbles, total marbles = $6 + 4 = \dots\dots$

$$6 + 4 = 10$$

- b If the cup has 20 marbles, total marbles = $\dots\dots + \dots\dots = \dots\dots$

$$20 + 4 = 24$$

- c If the cup has n marbles, total marbles = $\dots\dots\dots$

$$n + 4$$

- 2 The diagram shows 4 cups.



- a If each cup has 5 marbles, total marbles = $4 \times \dots = \dots\dots$

$$4 \times 5 = 20$$

- b If each cup has 8 marbles, total marbles = $4 \times \dots = \dots\dots$

$$4 \times 8 = 32$$

- c If each cup has n marbles, total marbles = $4 \times \dots = \dots\dots$

$$4 \times n = 4n$$

- 3 The diagram shows 3 cups and 2 marbles.



- a If each cup has 5 marbles, total marbles = $3 \times \dots + 2 = \dots\dots\dots$

$$3 \times 5 + 2 = 17$$

- b If each cup has 10 marbles, total marbles = $3 \times \dots + 2 = \dots\dots\dots$

$$3 \times 10 + 2 = 32$$

- c If each cup has n marbles, total marbles = $3 \times \dots + 2 = \dots\dots\dots$

$$3 \times n + 2 = 3n + 2$$

- 4 These two cups have a different number of marbles in them.

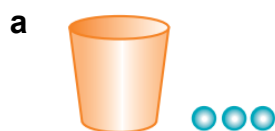
Explain why the total number of marbles is NOT $2n$



$2n$ means the two cups have the same number of marbles.

- 5 There are n marbles in each cup.

Write an algebraic expression for the total number of marbles in each diagram.



Total marbles = + 3

$$n + 3$$



Total marbles = $4 \times \dots\dots\dots = \dots\dots\dots$

$$4 \times n = 4n$$



$$3n + 2$$



$$4n + 3$$



$$2n + 4$$



$$5n + 1$$

- 6 There are x biscuits in each packet. Write an expression for the total number of biscuits.



$$2x + 3$$

- 7 There are x blocks in each bag. Write an expression for the total number of blocks.



$$5x + 4$$

Isaac says:



the total number of blocks is $5x + 4$,
which is the same as $9x$.

Explain why he is incorrect.

There are not 9 bags of blocks.

- 8 x represents the number of marbles in a cup. Draw a diagram to represent:

a $4x + 2$

b $7x + 5$

- 9 Each cup contains x pills. Each bottle contains y pills.
Write the total number of pills in each scenario.



$x + y$



$x + 2y$



$x + 2y + 2$



$3y$



$3 + x + 2y$



$2x + 3y + 1$

g Draw $x + y + 5$

h Draw $5x + 2y + 2$

i Draw $x + 3y$

j Draw $3x + 2y + 4$

- 10 What expressions are represented by the question marks? Match the answers.

$2x - 5$

$5 - 2x$

$2x + 5$

a

?		
x	x	5

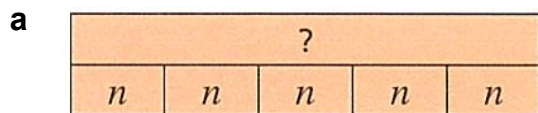
b

x	x
?	5

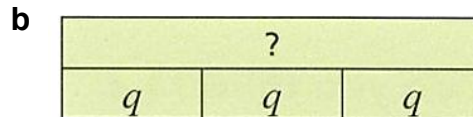
c

5		
?	x	x

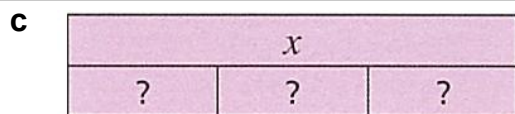
11 What expression is shown by the question mark in these bar models?



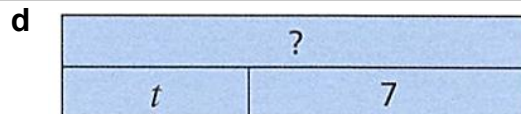
$5n$



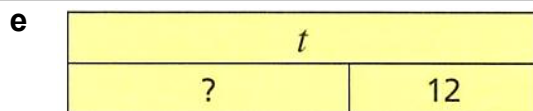
$3q$



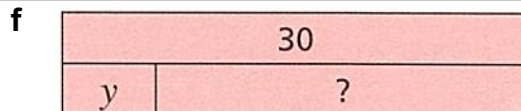
$3x$



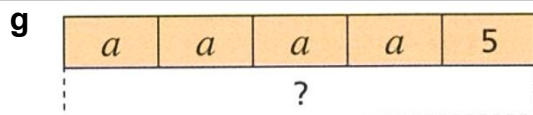
$t + 7$



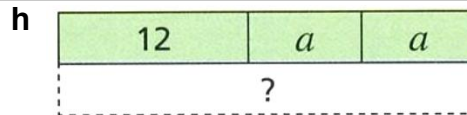
$t - 12$



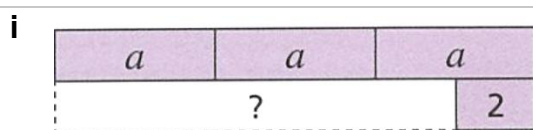
$30 - y$



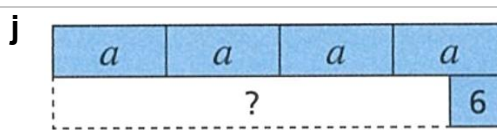
$4a + 5$



$2a + 12$



$3a - 2$



$4a - 6$

12 Draw bar models to represent these expressions.

a $b + 4$

b $b - 4$

c $\frac{b}{4}$

d $4b$

FORMING EXPRESSIONS FROM DESCRIPTIONS

The Language of Operations	
Addition: combining two or more amounts. - Sum - Total - More than - Increased by - Combined 	Subtraction: removing an amount. - Difference between - Less than - Decreased by 
Multiplication: combining equal groups. - Each - Per - Product - Lots of - Double ($\times 2$), Triple ($\times 3$), Quadruple... 	Division: Splitting into equal groups. - Each - Per - Divided by - Shared between - Quotient - Out of - Halve ($\div 2$), Third ($\div 3$), Quarter... 

We often swap number order when we see "than" or "from"

"9 subtracted from 7" $7 - 9$

"12 less than 8" $8 - 12$

Example Represent a description as an algebraic expression.

3 less than a	$a - 3$	How does the words "than" and "from" affect the order of numbers in an expression? <i>"than" and "from" usually mean to the order of the numbers.</i>
5 lots of a	$5a$	
Product of a and b	ab	

Represent these as an algebraic expression.

a 7 less than a	b a divided by 5	c b less than a
$a - 7$	$\frac{a}{5}$	$a - b$
d 14 shared between n	e Product of x and 2	f n subtracted from 11
$\frac{14}{n}$	$2x$	$11 - n$
g Double n , then add 7	h Product of x and 2, then divided by y	i n subtracted from 11, then multiplied by 3
$2n + 7$	$\frac{2x}{y}$	$3(11 - n)$

1 Write an expression for each of the following.

a The sum of 3 and 5 is+ the sum of p and q is + $3 + 5$ $p + q$	b 5 groups of 3 is $\times$ 5 groups of x is $\times$ 5×3 $5 \times x$
c The product of 2 and 6 is$\times$ the product of k and m is$\times$ 2×6 $k \times m$	d 5 more than 4 is+ 5 more than w is+ $5 + 4$ $5 + w$
e 2 less than 8 is- 2 less than c is- $8 - 2$ $c - 2$	f 9 squared is k squared is 9^2 k^2
g Double 7 is Double a is 2×7 $2a$	h Half of 10 is Half of b is $\frac{10}{2}$ $\frac{b}{2}$
i A third of 6 is A third of g is $\frac{6}{3}$ $\frac{g}{3}$	j A quarter of 12 is A quarter of h is $\frac{12}{4}$ $\frac{h}{4}$

2 Write an algebraic expression for each of the following.

a The sum of 2 and x $2 + x$	b The sum of ab and y $ab + y$	c 5 less than x $x - 5$
d The product of x and 3 $3x$	e One quarter of n $\frac{n}{4}$	f Half of s $\frac{s}{2}$
g The product of n and 9 $n + 9$	h y shared between 8 $\frac{y}{6}$	i t subtracted from 10 $10 - t$

3 Write an algebraic expression for each of the following.

a 5 is added to x , then the result is doubled. $2(5 + x)$	b a is tripled, then 4 is added. $3a + 4$	c 3 more than half of n $\frac{n}{2} + 3$
d 6 more than double n $2n + 6$	e The square of the sum of m and n $(m + n)^2$	f The sum of x and y is multiplied by 6. $6(x + y)$
g k is multiplied by 8, then 3 is subtracted. $8k - 3$	h The cube of the sum of m and n $(m + n)^3$	i The sum of x and y is divided by 5 $\frac{x + y}{5}$
j x is multiplied by 7 and the result is halved. $\frac{7x}{2}$	k k is multiplied by 8, then 3 is subtracted. $8k - 3$	l 10 less than the product of 4 and x $4x - 10$
m The square root of the sum of x and y $\sqrt{x + y}$	n The product of x and y is subtracted from 12. $12 - xy$	o 3 is subtracted from k , then the result is multiplied by 8. $8(k - 3)$

4 Write an expression for each of the following.

a 7 more than x $x + 7$	k x more than y $y + x$
b 7 less than x $x - 7$	l x multiplied by y xy
c 9 less than x $x - 9$	m x divided by y $\frac{x}{y}$
d 9 lots of x $9x$	n x divided by 3 $\frac{x}{3}$
e 19 lots of x $19x$	o x divided by 3 and then add 2 $\frac{x}{3} + 2$
f x divided by 19 $\frac{x}{19}$	p x divided by 3 and then subtract 2 $\frac{x}{3} - 2$
g x shared between 19 $\frac{x}{19}$	q x lots of 3 and then subtract 2 $3x - 2$
h x less than 19 $19 - x$	r x lots of 3 and then squared $(3x)^2$
i x less than 3 $3 - x$	s x lots of 3 squared 3^2x
j x less than y $y - x$	t x squared and then multiply by 3 $3x^2$

- 5 Write the calculation in numbers, then write the expression in algebra.

Using Numbers	Using Algebra
Becky begins a game with 12 marbles. She wins 3 and loses 5. How many marbles does she have at the end of the game? $12 + 3 - 5$	Becky begins a game with x marbles. She wins y and loses z . How many marbles does she have at the end of the game? $x + y - z$
In a class of 30 children, 10 have a sister. How many children don't have a sister? $30 - 10$	In a class of a children, b have a sister. How many children don't have a sister? $a - b$
There were 30 questions in a test. Megan got 25 correct. How many did she get wrong? $30 - 25$	There were x questions in a test. Megan got y correct. How many did she get wrong? $x - y$
Lucy went on a journey, going 25 kilometres by train, 4 kilometres by bus, and 2 kilometres on foot. What is the total distance she travelled? $25 + 4 + 2$	Lucy went on a journey, going x kilometres by train, y kilometres by bus, and z kilometres on foot. What is the total distance she travelled? $x + y + z$
Mark buys 7 chocolate bars, each costing 3 dollars. How much does he spend in total? 7×3	Mark buys a chocolate bars, each costing b dollars. How much does he spend in total? ab
Maddie has 5 cards. Emma has 2 times as many as Maddie. How many cards does Emma have? 5×2	Maddie has a cards. Emma has b times as many as Maddie. How many cards does Emma have? ab

6 Write an expression for each of the following:	
a There are 12 people on a bus. x get off. How many people remain?	b A train has 88 passengers. y people get on. How many are on the train?
$12 - x$	$88 + y$
c A can of lemonade costs c dollars. How much do 5 cans of lemonade cost?	d Each ice block is \$3. Each lolly bag is \$2 How much money is earned from selling x ice blocks and y lolly bags?
$5c$	$3x + 2y$
e A rope is x metres long. I cut off four lengths of y metres each. How much rope remains?	f n students go on an excursion. How much should Ms Ali charge each student if she wants to share the bus hire fee of \$200?
$x - 4y$	$\frac{200}{n}$
7 Nicholas buys 10 lolly bags from a supermarket.	
a If there are 7 lollies in each bag, how many lollies does he buy in total?	b If there are n lollies in each bag, how many lollies does he buy in total?
$7 \times 10 = 70$	$10n$
8 Write an algebraic expression for the cost of:	
a x pies at \$6 each.	b 6 pies at \$ x each.
$6x$	$6x$
c 10 pies at \$ n each.	d y drinks at \$2 each.
$10n$	$2y$
e x pies at \$5 and y drinks at \$2	f x pies at \$2 and y drinks at \$6
$5x + 2y$	$2x + 6y$
9 The cost of 10 cans of drink is \$ x . Write an expression for:	
a The cost of one can of drink	b The cost of n cans
$\frac{x}{10}$	$\frac{nx}{10}$

10 If pencils cost $\$x$ each, write an expression for the cost of:

a 10 pencils

$$10x$$

b k pencils

$$kx$$

c 3 packets of pencils,
if each packet contains 5
pencils

$$15x$$

11 Recall that there are 100 centimetres in 1 metre and 1000 metres in 1 kilometre.

Write expressions for each of the following.

a How many metres are
there in x km?

$$1000x$$

b How many centimetres
are there in x metres?

$$100x$$

c How many centimetres
are there in x km?

$$100\,000x$$

12

a Ed is 12 years old. How old will he be in 2 years' time?

$$12 + 2 = 14$$

b Faith is y years old. How old will she be:

i in 2 years' time

$$y + 2$$

ii in x years' time?

$$y + x$$

iii 5 years ago?

$$y - 5$$

12 A rectangular garden bed is 12 m long and 5 m wide.

a Find the area of the garden bed. Show your working out.

$$60\text{ m}^2$$

b The length is increased by x cm and the width are decreased by y cm.
Find the new length and width of the garden.

$$\begin{array}{l} \text{length: } 12 + x \\ \text{width: } 5 - y \end{array}$$

c Write an expression for the area of the new garden bed.

$$(12 + x)(5 - y)$$

13 Mikayla is paid $\$x$ per hour at her job. Write an expression for each of the following:

a How much does Mikayla earn if she works 8 hours?

$$8x$$

b Mikayla gets a pay rise of $\$3$ per hour, what is her new hourly wage?

$$x + 3$$

c Mikayla works 8 hours at the increased hourly rate, write how much she earns as an expression.

$$8(x + 3)$$

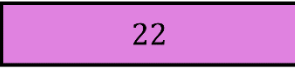
14 Match up each sentence to its algebraic expression.

Multiply b by 2 then subtract from a	•	• $a(b - c)$
Subtract c from a then multiply by b	•	• $4a - c$
Multiply a by b then divide by c	•	• $\frac{c}{a + b}$
Add b to a then divide into c	•	• $\frac{ab}{c}$
Divide c by b then multiply by a	•	• abc
Multiply a by c then divide into b	•	• $b(a - c)$
Add b to a then divide by c	•	• $a - 2b$
Multiply ab by c	•	• $\frac{b}{ac}$
Multiply a by 4 then subtract c	•	• $\frac{a + b}{c}$
Subtract c from b then multiply by a	•	• $\frac{ac}{b}$

SUBSTITUTION AND EVALUATION

- To **substitute** means to **replace** pronumerals with numbers.
- To **evaluate** means to work out the final number by following the **order of operations**.

Example Evaluate expressions after substitution.

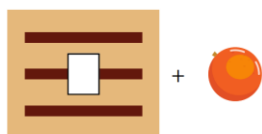
The bar model shows $10 + 4x$ Evaluate the expression when $x = 3$ $10 + 4x$ $= 10 + 4(3)$ $= 10 + 12$ $= 22$	  	What does it mean to “substitute” a number into an expression? <i>Substitute means to a pronumeral with a value.</i> What does it mean to “evaluate” an expression? <i>Evaluate means to calculate the value using the</i>
---	---	--

- a** The total number of oranges is $x + 1$
Evaluate the expression for $x = 5$

$$x + 1$$

$$= \dots + 1$$

$$= \dots$$



6

- b** The number of Lego pieces is $2x + 3$
Evaluate the expression for $x = 6$

$$2x + 3$$

$$= \dots$$

$$= \dots$$



15

- c** Evaluate $7 + 5b$ when $b = 5$

32

- d** Evaluate $\frac{x}{2} + 5$ when $x = 12$

11

- e** Evaluate $\frac{n+7}{3}$ when $n = 8$

5

- f** Evaluate $7 - 4x$ when $x = 2$

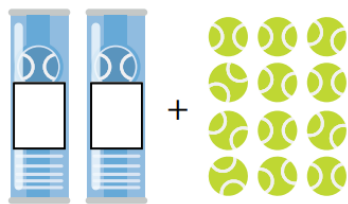
-1

Key ideas

- Substitution means to a letter with a number.
- means to calculate the numerical value of an expression.
- When evaluating, always follow the

1

a The diagram shows $2t + 12$ tennis balls.



Evaluate when $t = 6$.

$$\begin{aligned} &2t + 12 \\ &= 2(\quad) + 12 \\ &= \dots\dots\dots \end{aligned}$$

24

b The diagram shows $3c + 4$ coins.

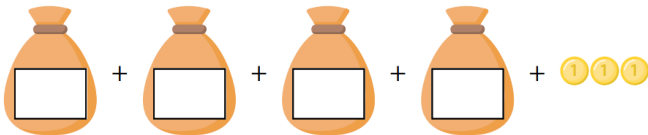


Evaluate when $c = 10$.

$$\begin{aligned} &3c + 4 \\ &= 3(\quad) + 4 \\ &= \dots\dots\dots \end{aligned}$$

34

c The diagram shows $4x + 3$ coins.



How many coins are there if $x = 12$?

51

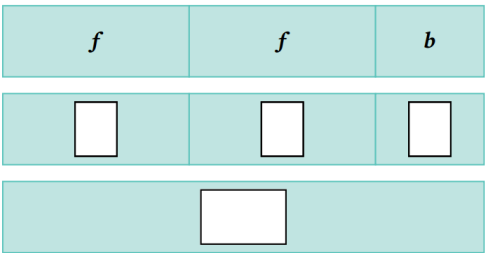
d The diagram shows $2c + 3$ cars.



How many cars are there if $c = 6$?

15

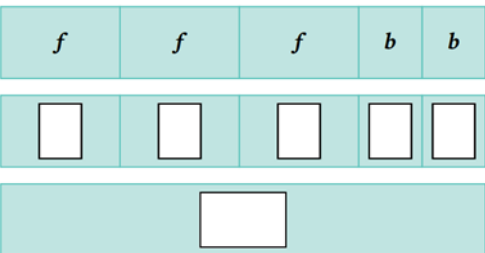
2 The bar model shows $2f + b$. Evaluate when $f = 7, b = 3$



$$\begin{aligned} &2f + b \\ &= 2(\quad) + (\quad) \\ &= \dots\dots\dots \end{aligned}$$

17

3 The bar model shows $3f + 2b$. Evaluate when $f = 7, b = 3$



$$\begin{aligned} &3f + 2b \\ &= 3(\quad) + 2(\quad) \\ &= \dots\dots\dots \end{aligned}$$

27

4 Substitute $c = 5$ to evaluate each expression.

a $c + 10$	b $10 + c$	c $10c$	d $\frac{c}{10}$	e $\frac{10}{c}$
15	15	50	0.5	2

5 Substitute $x = 5$ into these expressions and evaluate the result. Show the substitution step.

a $x + 3$ $= (5) + 3$ $= \dots\dots\dots$	b $14 - x$	c $2x$
8	9	10
d $2x + 4$	e $13 - 2x$	f $3x - 6$
14	3	9
d $10x$	e $10 + x$	f $\frac{x}{10}$
50	15	$\frac{1}{2}$
d $x - 10$	e $10 - x$	f x^2
-5	5	25

6 Evaluate these expressions for $g = 400$.

a $\frac{g}{10}$	b $\frac{10}{g}$	c $\frac{g}{100}$	d $\frac{100}{g}$	e $\frac{g}{20}$	f $\frac{20}{g}$
40	$\frac{1}{40}$	4	$\frac{1}{4}$	20	$\frac{1}{20}$
g $\frac{g}{200}$	h $\frac{200}{g}$	i $\frac{g}{400}$	j $\frac{400}{g}$	k $\frac{g}{1000}$	l $\frac{1000}{g}$
2	$\frac{1}{2}$	1	1	$\frac{2}{5}$	$\frac{5}{2}$

7 Work out the value of each of these expressions when $a = 5$

a $10a + 3$	b $\frac{10}{a} + 3$	c $10a - 3$	d $\frac{a}{10} + 3$
53	5	47	3.5
e $10(a + 3)$	f $\frac{10}{a + 3}$	g $10(a - 3)$	h $3 + \frac{a}{10}$
80	$\frac{5}{4}$	20	3.5
i $\frac{10 - a}{3}$	j $10 - \frac{3}{a}$	k $3 + \frac{10}{a}$	l $3 - \frac{10}{a}$
$\frac{5}{3}$	$\frac{47}{5}$	5	1

8 If $c = 3$, evaluate these expressions.

a	$4 + c$	h	$\frac{5c-7}{2}$
		7	4
b	$4c$	i	$\frac{4c}{4}$
		12	3
c	$4c + c$	j	$\frac{c}{4c}$
		15	$\frac{1}{4}$
d	$5c$	k	c^2
		15	9
e	$5c - 7$	l	$2c^2$
		8	18
f	$(5c - 7) \div 2$	m	$(2c)^2$
		4	36
g	$5c - 7 \div 2$	n	$\frac{4c}{4}$
		11.5	3

9 Evaluate these expressions when $g = 80$.

a $\frac{g}{10} + 5$	b $\frac{g}{10} - 5$	c $10g + 5$	d $5 + \frac{g}{10}$	e $10g - 5$
13	3	805	13	795
f $10 - \frac{g}{20}$	g $10 - \frac{20}{g}$	h $10 + \frac{40}{g}$	i $\frac{10}{g} + 40$	j $g^2 - 1400$
6	$9\frac{3}{4}$	10.5	$40\frac{1}{8}$	5000

10 Find the values of each of these expressions when $c = 100$

a $\frac{c}{5} + 1$	b $\frac{c+1}{5}$	c $\frac{c}{4} + 1$	d $\frac{c+1}{4}$	e $\frac{c}{10} + 1$	f $\frac{c+1}{10}$
21	$\frac{101}{5}$	26	$\frac{101}{4}$	11	$\frac{101}{10}$
g $\frac{c+10}{5}$	h $\frac{c}{5} + 10$	i $\frac{c^2+10}{5}$	j $\frac{c^2}{5} + 10$	k $\frac{c^2-10}{5}$	l $\frac{c^2}{5} - 10$
22	30	2002	2010	1998	1990
m $\sqrt{c}$	n $10\sqrt{c}$	o $\frac{\sqrt{c}}{5}$	p $\sqrt{c} + 5$	q $5 + \sqrt{c}$	r $1 + 5\sqrt{c}$
10	100	2	15	15	51

11 Work out the value of each of these expressions when $x = 3$ and $y = 0$

a $x + y$	b $x + 2y$	c $x - y$	d $x - 2y$
3	3	3	3
e xy	f $\frac{y}{x}$	g $\frac{x}{2+y}$	h $\frac{x}{2-y}$
0	0	1.5	1.5
i $x^2 + y$	j $(x + y)^2$	k $x + y^2$	l $x(y + 2)$
9	9	3	6

12 If $m = 9$ and $n = 16$, find the values of:

a	$\sqrt{m} + \sqrt{n}$	b	$\sqrt{m + n}$	c	$m\sqrt{n}$	d	$n\sqrt{m}$	e	$\sqrt{mn}$
	7		5		36		48		12

13 x and y are two numbers, where $xy = 24$.

- a** What values could x and y equal if they are whole numbers?
Try to list as many as possible.
-
-
- b** What values could x and y be if they can be decimals, fractions, or whole numbers?
-
-

14 Write two different expressions involving x that evaluate to -10 if $x = -5$.

.....

.....

- 15
- a** Find the difference between $4a + 3$ and $4(a + 3)$ when $a = 2$
-
- b** Find the difference between $4a + 3$ and $4(a + 3)$ when $a = 10$
-
- c** Find the difference between $4a + 3$ and $4(a + 3)$ when $a = 100$
-
- d** Find the difference between $4a + 3$ and $4(a + 3)$ when $a = 0.2$
-
- e** What do you notice?
- f** Without any calculations, find the difference between $4a + 3$ and $4(a + 3)$ when $a = 786.174$
-

16 Complete the table.

a	10	0	2	12	5	1
$a + 2$	12	2	4	14	7	3
$2a$	20	0	4	24	10	2
a^2	100	0	4	144	25	1
$2 - a$	-8	2	0	-110	-3	1
$\frac{a}{2}$	5	0	1	6	$\frac{5}{2}$	0.5

17 Complete the table.

x	5	10	7	9	5	8	6
y	3	4	2	5	2	0	-6
$x + y$	8	14	9	14	7	8	0
$x - y$	-6	6	5	4	3	8	12
xy	15	40	14	45	10	0	-36

2 Evaluate each expression for $n = 10$.

$$n = 10$$

$$5n =$$

$$5n + 2 =$$

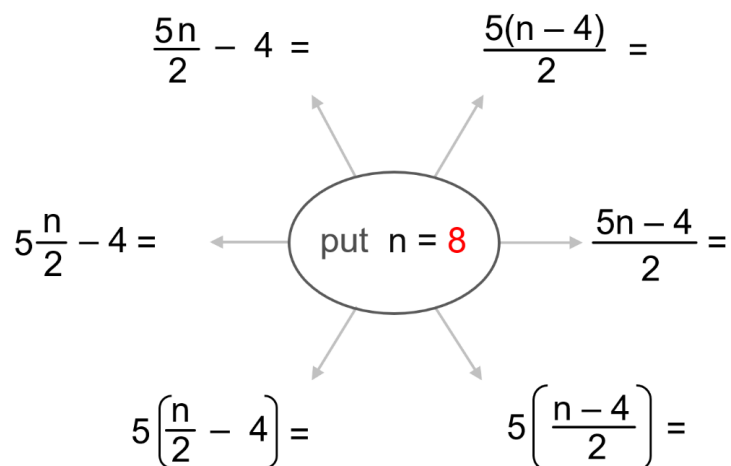
$$\frac{5n + 2}{4} =$$

$$3 \left(\frac{5n + 2}{4} \right) = \frac{3 \left(\frac{5n + 2}{4} \right) + 6}{5} = 7 \left(\frac{3 \left(\frac{5n + 2}{4} \right) + 6}{5} \right) - 1 =$$

$$3 \left(\frac{5n + 2}{4} \right) + 6 =$$

$$\frac{7 \left(\frac{3 \left(\frac{5n + 2}{4} \right) + 6}{5} \right) - 1}{2} =$$

- 18 Substitute $n = 8$ into each expression and evaluate the result.



Which of the expressions are equivalent?

$$\frac{5n}{2} - 4 \text{ and } 5\frac{n}{2} - 4,$$

$$\frac{5(n-4)}{2} \text{ and } 5\left(\frac{n-4}{2}\right)$$

19 **NAPLAN B**

Evaluate the expression $x^2 + 3x - 4$ when $x = 3$

14

- 20 $R = 2d - 3e - 4f$ Find the value of R if $d = 2, e = 3, f = \frac{1}{2}$

-7

- 21 Evaluate $\frac{20}{y} + 3x$ when $x = \frac{1}{5}, y = 2$

$10\frac{3}{5}$

- 22 Evaluate $\frac{20}{x} + 3y$ when $x = \frac{1}{5}, y = 2$

23 Evaluate the expressions for the given value of n

a

$$n = 5$$

$$3n =$$

$$3n + 10 =$$

$$2(3n + 10) =$$

$$2(3n + 10) - 40 =$$

$$6(2(3n + 10) - 40) =$$

$$6(2(3n + 10) - 40) - 55 =$$

5

b

$$n = 2$$

$$5n =$$

$$5n + 4 =$$

$$2(5n + 4) =$$

$$2(5n + 4) - 18 =$$

$$3(2(5n + 4) - 18) =$$

$$3(2(5n + 4) - 18) - 28 =$$

2

c

$$n = 5$$

$$2n - 4 =$$

$$\frac{2n - 4}{3} =$$

$$7 \left(\frac{2n - 4}{3} \right) =$$

14

d

$$n = 9$$

$$3n + 8 =$$

$$\frac{3n + 8}{5} =$$

$$2 \left(\frac{3n + 8}{5} \right) =$$

14

24 Find the value of n by working backwards.

$$n =$$

$$5n =$$

$$5n + 2 =$$

$$\frac{5n + 2}{4} =$$

$$3 \left(\frac{5n + 2}{4} \right) =$$

$$3 \left(\frac{5n + 2}{4} \right) + 6 =$$

$$7 \left(\frac{3 \left(\frac{5n + 2}{4} \right) + 6}{5} \right) =$$

$$\frac{3 \left(\frac{5n + 2}{4} \right) + 6}{5} =$$

$$7 \left(\frac{3 \left(\frac{5n + 2}{4} \right) + 6}{5} \right) - 1 =$$

$$\frac{7 \left(\frac{3 \left(\frac{5n + 2}{4} \right) + 6}{5} \right) - 1}{2} = 10$$

$n = 2$

SUBSTITUTING NEGATIVE NUMBERS

- **Always put brackets around negative numbers when substituting.**
- Without brackets, we might mistake a negative sign for subtraction.
- Brackets help calculators follow the order of operations, especially when raising negative numbers to a power.

Example Evaluate expressions after substituting negative integers.

Evaluate $10 + 4b$ for $b = -2$ $10 + 4b$ $= 10 + 4(-2)$ $= 10 + (-8)$ $= 2$	Evaluate $12 - 3b$ for $b = -5$ $12 - 3b$ $= 12 - 3(-5)$ $= 12 - (-15)$ $= 27$	Evaluate $2b^2$ for $b = -3$ $2b^2$ $= 2(-3)^2$ $= 2(9)$ $= 18$
---	---	--

a Evaluate $x + 1$ for $x = -5$

$$x + 1$$

$$= \dots + 1$$

$$= \dots$$

-4

b Evaluate $2x + 3$ for $x = -3$

$$2x + 3$$

$$= \dots$$

$$= \dots$$

-3

c Evaluate $4b^2$ for $b = -3$

$$4b^2$$

36

d Evaluate $4 - x$ for $x = -3$

7

e Evaluate $\frac{2x+7}{3}$ for $x = -4$

$-\frac{1}{3}$

f Evaluate $12 - 3b$ for $b = -5$

27

Key ideas

1. Always substitute negative numbers in so you are less likely to make a mistake.

1 For $x = -5$, evaluate these expressions.

a $x + 3$ $= (-5) + 3$ $= \dots\dots\dots$	b $14 - x$	c $2x$
d $2x + 4$	e $13 - 2x$	f $3x - 6$
d $\frac{5x}{7}$	e $-3x + 15$	f $10x - 50$

2 For $a = 6$ and $b = -2$, evaluate these expressions.

a $a + b$	b $a - b$	c $b - a$
d $2a + b$	e $5b + 2a$	f $6b - 7a$

3 Flo is working out the value of $c - d$. She is told that $c = 4$ and $d = -9$. She writes:

$$\begin{aligned} c - d \\ = 4 - 9 \\ = -5 \end{aligned}$$

Explain why she is wrong.

She substituted 9, not -9

Provide the correct answer.

4 If $m = -4$ and $n = 20$, find the value of:

a $m + 9$	b $n - 25$	c $7 + m$	d $m + n$
5	-5	3	16
e $n - m$	f $m - n$	g $2m$	h $2n$
24	-24	-8	40
i $-2m$	j mn	k n^2	l m^2
8	-80	400	16

5 If $p = 4$, $q = -3$, $r = 5$, work out the value of:

a $pq + qr$	b $pq - qr$	c $p^2 + q^2$	d $(r - q) \div (p - 6)$
-27	3	25	-4

6 $f = -3$, $g = -10$, $h = 9$. Find the value of:

a $2f + h$	b $3f + h$	c $h - g$	d $f - 2g$
3	0	19	17
e fg	f gh	g fgh	h $fg - 20$
30	-90	270	10

7 For $m = 3$ and $n = 7$, find the value of:

a $\sqrt{6m + n}$	b $\sqrt{10(m + n)}$	c $\sqrt{4(n - m)}$	d $\sqrt{7m + 4n}$
5	10	4	7

8 For $x = 3$, $y = \frac{1}{2}$, $z = -5$ evaluate these expressions.

a	$x + 4$		k	$-2z$	10
		7	l	$-2z - x$	7
b	$x - 4$		m	$-2z - x - y$	6.5
		-1	n	$2(-2z - x - y)$	13
c	$4x$		o	$2(2z + x + y)$	13
		12	p	$2x + x + y$	9.5
d	$4x + y$		q	$\frac{2x+x+y}{2}$	4.25
		12.5	r	$\frac{y+x+2z}{2}$	-3.25
e	$4x + 2y$		s	$\frac{y+x-2z}{2}$	6.25
		13	t	$y + x - \left(\frac{2z}{2}\right)$	8.5
f	$4x - 2y$				
		11			
g	$2y - 4x$				
		-11			
h	$2y + z$				
		-4			
i	$3x + z$				
		4			
j	$3x - z$				
		14			

9 Evaluate these expressions by substituting $a = -2$, $b = 6$, $c = -3$

a	$a^2 - b$		b	$a - b^2$		c	$2c + a$	
		-2			-38			-8
d	$b^2 - c^2$		e	$a^3 + c^2$		f	$3ab + ac$	
		27			1			-30
g	$c - 2ab$		h	$abc - (ac)^2$		i	$c(a + b)$	
		21			0			-12

10 $p = 8$ and $q = -12$. Work out

a $p + q$	b $q - p$	c pq
-4	20	-96
d $p - 2q$	e $\frac{2q}{p}$	f $\frac{pq}{2}$
32	-3	-48
g $p - \frac{q}{2}$	h $\frac{p - q}{2}$	i $\frac{q - p}{4}$
14	10	-5

11 2008 HSC Standard 2 Band 4

What is the value of $\sqrt{\frac{x+2y}{8y}}$ if $x = 5.6$ and $y = 3.1$, correct to 2 decimal places?

- (A) 0.69
- (B) 2.62
- (C) 2.83
- (D) 4.77

A

12 HSC Standard 2 Sample Question Band 4

What is the value of $\frac{a+b}{ab}$ if $a = -2.1$ and $b = -3.6$, correct to 1 decimal place?

-0.8

13 2010 HSC Standard 2 Band 5

If $M = -9$, what is the value of $\frac{3M^2+5M}{6}$?

- (A) -250.5
- (B) -48
- (C) 33
- (D) 235.5

C

25 Evaluate $8p - \frac{14}{q}$ when $p = \frac{-3}{8}$, $q = -7$

-1

14 Complete the table.

x	3	1	0.25	-2	2	$\frac{8}{7}$
$4x + 2$	14	6	3	-6	10	$\frac{46}{7}$
$4 - 3x$	-5	1	3.25	10	-2	$\frac{4}{7}$
$2x - 4$	2	-2	-3.5	-8	0	$-\frac{12}{7}$

15 Complete the table.

a	5	8	2	3	-20	10
b	2	2	1	7	10	-3
$a + b$	7	10	3	10	-10	7
$a + 2b$	9	12	4	17	0	4
$a - b$	3	6	1	-4	-30	13

GENERATING NUMBER PATTERNS BY SUBSTITUTION

- Substituting **consecutive** numbers into an expression can make a **number pattern**.

Example Generate a number pattern by substitution.

x	0	1	2	3	4	5		10		100	Change by 1, starting at 3
$x + 3$	3	4	5	6	7	8		13		103	

x	0	1	2	3	4	5		10		100	Change by 2, starting at -1
$2x - 1$	-1	1	3	5	7	9		19		199	

Complete these tables to generate a number pattern from the algebraic expression.

- a**
- | | | | | | | | | | | | |
|------|---|---|---|---|---|---|--|----|--|-----|--------------------------------------|
| x | 0 | 1 | 2 | 3 | 4 | 5 | | 10 | | 100 | Change by
Starting at |
| $2x$ | | | | | | | | | | | |
- 0 2 4 6 8 10 20 200; change by 2 starting at 0
- b**
- | | | | | | | | | | | | |
|----------|---|---|---|---|---|---|--|----|--|-----|--------------------------------------|
| x | 0 | 1 | 2 | 3 | 4 | 5 | | 10 | | 100 | Change by
Starting at |
| $2x + 3$ | | | | | | | | | | | |
- 3 5 7 9 11 13 23 203 change by 2 starting at 3
- c**
- | | | | | | | | | | | | |
|----------|---|---|---|---|---|---|--|----|--|-----|--------------------------------------|
| x | 0 | 1 | 2 | 3 | 4 | 5 | | 10 | | 100 | Change by
Starting at |
| $5x - 7$ | | | | | | | | | | | |
- 7 -2 3 8 13 18 43 493 change by 5 starting at -7
- d**
- | | | | | | | | | | | | |
|-------|---|---|---|---|---|---|--|----|--|-----|--------------------------------------|
| x | 0 | 1 | 2 | 3 | 4 | 5 | | 10 | | 100 | Change by
Starting at |
| $-4x$ | | | | | | | | | | | |
- 0 -4 -8 -12 -16 -20 -40 -400 change by -4 starting at 0

Key ideas

- By substituting numbers, we can form a number pattern.

1 Complete these tables to generate a number pattern from the algebraic expression.

a

x	0	1	2	3	4	5		10		100
$3x$										

Change by
Starting at

0 3 6 9 12 15 30 300 change by 3 starting at 0

b

x	0	1	2	3	4	5		10		100
$3x + 1$										

Change by
Starting at

1 4 7 10 13 16 31 301 change by 3 starting at 1

c

x	0	1	2	3	4	5		10		100
$3x + 2$										

Change by
Starting at

2 5 8 11 14 17 32 302 change by 3 starting at 2

d

x	0	1	2	3	4	5		10		100
$2 + 3x$										

Change by
Starting at

2 5 8 11 14 17 32 302 change by 3 starting at 2

e

x	0	1	2	3	4	5		10		100
$-3x + 2$										

Change by
Starting at

2 -1 -4 -7 -10 -13 -28 -298 change by -3 starting at 2

f

x	0	1	2	3	4	5		10		100
$2 - 3x$										

Change by
Starting at

2 -1 -4 -7 -10 -13 -28 -298 change by -3 starting at 2

g

x	0	1	2	3	4	5		10		100
$-2x + 2$										

Change by

Starting at

0 -2 -4 -6 -8 -18 -198 change by -2 start at 2

h

x	0	1	2	3	4	5		10		100
$-x - 2$										

Change by

Starting at

-2 -3 -4 -5 -6 -7 -12 -102 change by -1 starting at -2

i

x	0	1	2	3	4	5		10		100
$-x + 1$										

Change by

Starting at

1 0 -1 -2 -3 -4 -9 -99 change by -1 starting at 1

j

x	0	1	2	3	4	5		10		100
$1 - x$										

Change by

Starting at

1 0 -1 -2 -3 -4 -9 -99 change by -1 starting at 1

k

x	0	1	2	3	4	5		10		100
$-x$										

Change by

Starting at

0 -1 -2 -3 -4 -5 -10 -100 change by -1 starting at 0

l

x	0	1	2	3	4	5		10		100
$3x - 5$										

Change by

Starting at

-5 -2 1 4 7 10 25 295 change by 3 starting at -5

2 By looking at the questions in **Q1**, complete the sentences:

In the expression $mx + c$: the coefficient of x is and the constant term is

The coefficient of x represents how much to change as you increase x by

The constant term represents the number to at, i.e., when $x = 0$

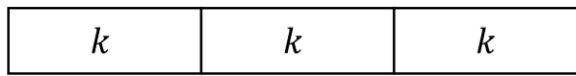
3 Determine the number pattern formed by these equations.

a $3x + 6$ Change by 3 Starting at 6	b $3x + 9$ Change by Starting at Change by 3 starting at 9	c $6x + 18$ Change by Starting at Change by 6 starting at 18
d $3x + 18$ Change by Starting at Change by 3 starting at 18	e $3x - 18$ Change by Starting at Change by 3 starting at -18	f $2x - 18$ Change by Starting at Change by 2 starting at -18
g $18 - 2x$ Change by Starting at Change by -2 starting at 18	h $-2x + 18$ Change by Starting at Change by -2 starting at 18	i $-4x + 18$ Change by Starting at Change by -4 starting at 18
j $-4x - 18$ Change by Starting at Change by -4 starting at -18	k $4x - 18$ Change by Starting at Change by 4 starting at -18	l $-5.4 - \frac{9}{4}x$ Change by Starting at Change by $-\frac{9}{4}$ starting at -5.4

CHECK YOUR UNDERSTANDING

Forming expressions from diagrams

1 What does the bar model represent?



a k^3

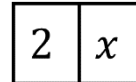
b $k3$

c $k + 3$

d $3k$

2 Tom and Katie are discussing the expression represented by the bar model:

Tom says the bar model represents $2 + x$



Katie says the bar model represents $2x$

Who is correct?

a Only Tom

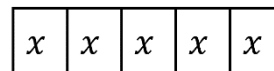
b Only Katie

c Both

d Neither

3 Tom and Katie are discussing the expression represented by the bar model:

Tom says the bar model represents $5x$



Katie says the bar model represents x^5

Who is correct?

a Only Tom

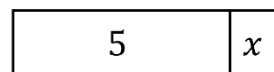
b Only Katie

c Both

d Neither

4 Tom and Katie are discussing the expression represented by the bar model:

Tom says this bar model represents $5x$



Katie says this bar model represents x^5

Who is correct?

a Only Tom

b Only Katie

c Both

d Neither

Forming expressions from descriptions

1	Three more than x			
a	$3 - x$	b	$3x$	c $x + 3$ d $x - 3$
2	5 less than a			
a	$a + 5$	b	$5 + a$	c $5 - a$ d $a - 5$
3	Multiply x by 3 then add 4.			
a	$4x + 3$	b	$3x + 4$	c $3(x + 4)$ d $3x4$
4	a divided by 2			
a	$\frac{2}{a}$	b	$a \div 2$	c $2 \div a$ d $\frac{a}{2}$
5	Half of 1 more than x			
a	$\frac{x + 1}{2}$	b	$\frac{1}{2}x + 1$	c $x + \frac{1}{2}$ d $x + \frac{1}{2}$
6	5 more than 2 lots of a			
a	$7a$	b	$10a$	c $5(a + 2)$ d $2a + 5$
7	2 lots of 3 more than x			
a	$2x + 3$	b	$2(x + 3)$	c $6 + x$ d $6x$
8	The cost of n train tickets at \$5 each.			
a	$5n$	b	$n + 5$	c 5 d n
9	A pen costs \$1.20, how much will n pens cost?			
a	$120n$	b	$1.2 + n$	c $1.2n$ d $120 + n$
10	Uter is n years old. What was his age 10 years ago?			
a	$10n$	b	$10 + n$	c $10 - n$ d None of these.
11	Jane buys 5 packets of sweets. The total cost is \$ x . What is the cost of one packet?			
a	$5 - x$	b	$5x$	c $\frac{x}{5}$ d $x = 5$
12	Jane wants to buy 4 packets of sweets. One packet is \$ x . What is the total cost?			
a	$4 + x$	b	$4x$	c $\frac{x}{4}$ d $x = 4$

Substitution

1	When $h = 10$, $h - 2 =$	a 8	b 5	c -8	d -5
2	When $h = 10$, $\frac{h}{2} =$	a 8	b 5	c -8	d -5
3	When $h = 10$, $h^2 =$	a 100	b 20	c 12	d 1010
4	When $t = 2$ $13 - 2t =$	a $11t$	b 11	c -9	d 9
5	When $x = 4$ $10x - 5 =$	a $5x$	b 35	c 99	d 20
6	When $a = 9$ and $b = 0.5$ $ab =$	a 4.5	b 9.5	c 45	d 9.05

Substitution with negatives

1	When $d = -3$ $15 - 3d =$	a -18	b 6	c 9	d 24
2	When $d = -2$ $10 - 2d =$	a 14	b 6	c -12	d 32
3	When $a = 5, b = -0.2, c = -3$ $ab =$	a -1	b 4.8	c -0.1	d -5.2
4	When $a = 5, b = -0.2, c = -3$ $a - c$	a 2	b -2	c 8	d -8
5	When $a = 5, b = -0.2, c = -3$ $\frac{b}{4} =$	a -0.24	b -0.5	c -0.1	d -0.05
6	When $a = 5, b = -0.2, c = -3$ $c^2 =$	a -6	b 6	c -9	d 9
7	When $a = 5, b = -0.2, c = -3$ $b^2 =$	a 0.04	b -0.04	c 0.4	d -0.4