

Student Name

Mathematics Stage 4

PROPERTIES OF GEOMETRICAL FIGURES

Book 2 Classify quadrilaterals and
describe their properties

Version: 260207

Report mistakes & suggest improvements:
<https://MrDingMaths.com>

CONTENTS

Overview	2
Labelling Quadrilaterals	3
Diagonals	4
Convex and Non-Convex Quadrilaterals	5
Properties of Special Quadrilaterals	6
Check Your Understanding	14

OVERVIEW

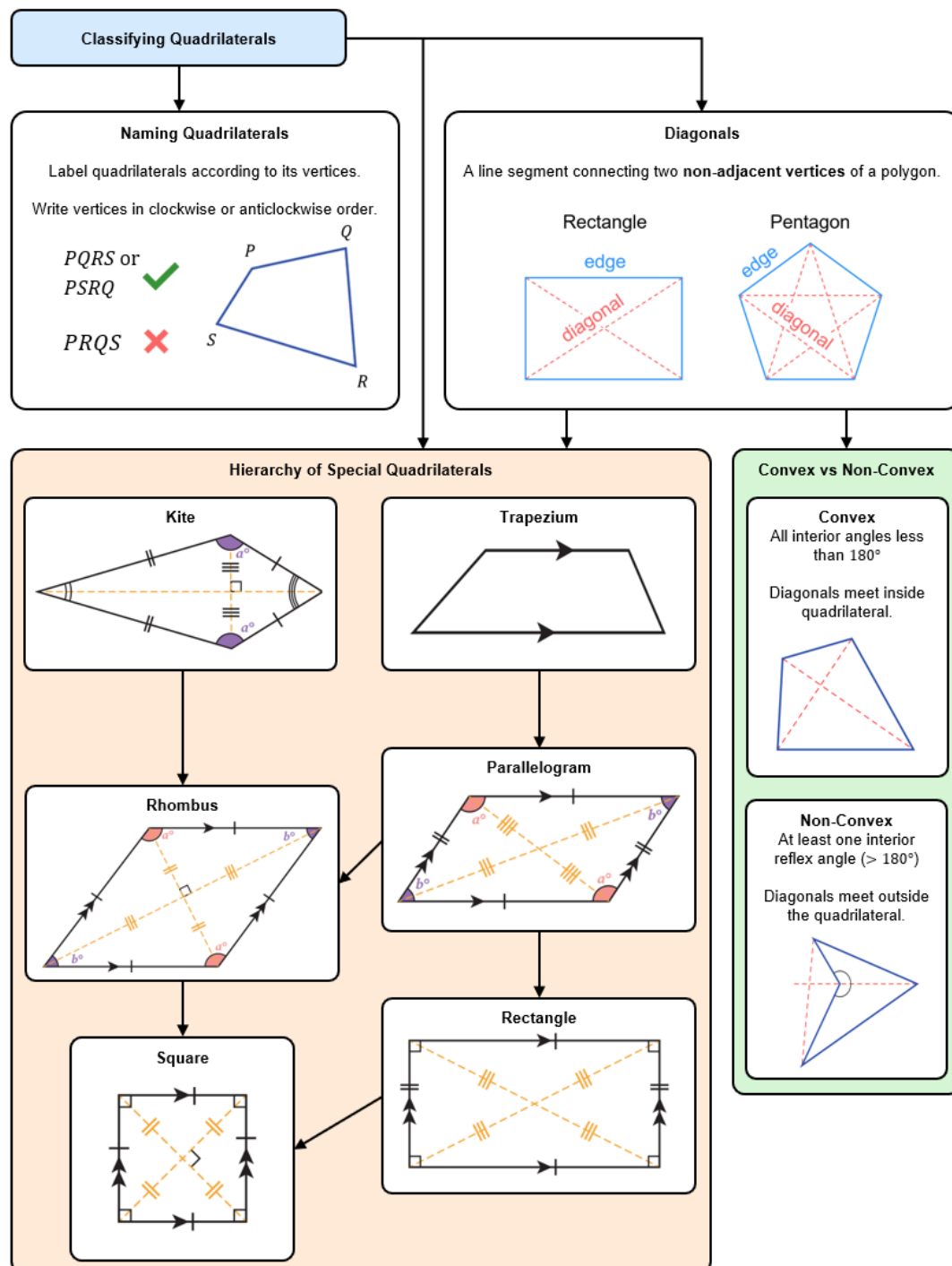
SYLLABUS CONTENT

MA4-GEO-C-01 identifies and applies the properties of triangles and quadrilaterals to solve problems

Classify quadrilaterals and describe their properties

- Identify quadrilaterals using naming conventions
- Distinguish between convex and non-convex quadrilaterals
- Verify and describe the properties of the special quadrilaterals which include parallelograms, rectangles, rhombuses, squares, trapeziums and kites
- Identify and label the properties of the special quadrilaterals using appropriate conventions
- Classify quadrilaterals based on their properties
- Justify why some quadrilaterals may be classified as more than one type of quadrilateral

SUMMARY

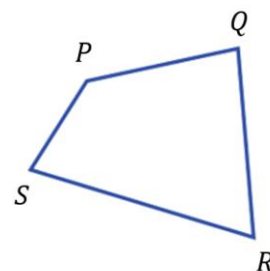


LABELLING QUADRILATERALS

- A **quadrilateral** is a **4-sided** shape.
- We label quadrilaterals using their vertices.
- Write vertices in clockwise or anticlockwise order.

$PQRS$ or $PSRQ$ ✓

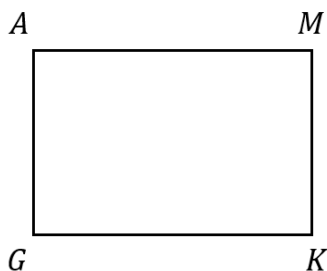
$PRQS$ ✗



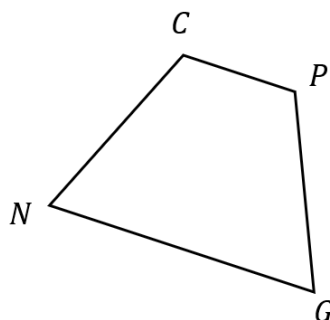
Example Identify quadrilaterals using naming conventions.

Label these quadrilaterals.

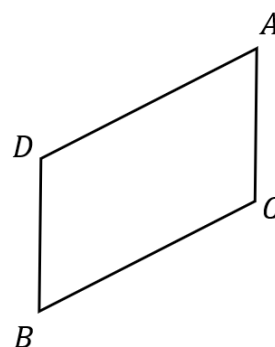
a



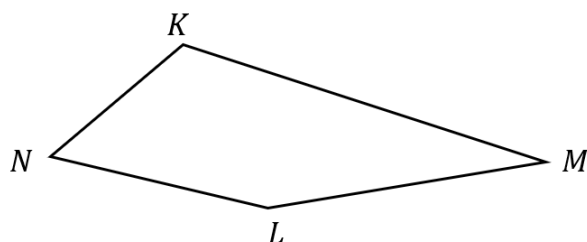
b



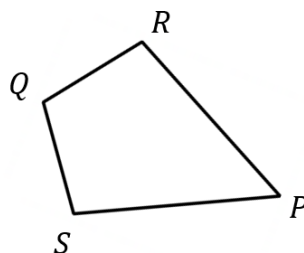
c



d

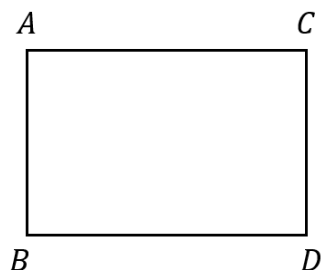


e



f

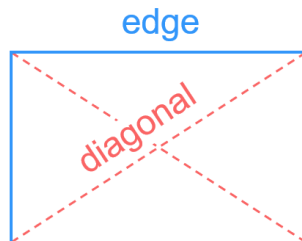
Tommy was asked to draw a rectangle $ABCD$. This is what he sketched. Explain why he is incorrect, and draw rectangle $ABCD$ correctly.



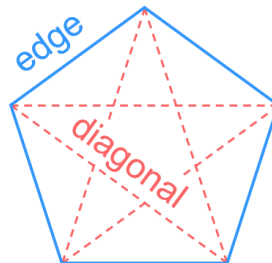
DIAGONALS

- **Adjacent** means next to.
- An **edge** of a polygon joins two **adjacent** vertices.
- A **diagonal** of a polygon joins two **non-adjacent vertices**.
- A diagonal goes from one corner to another, skipping at least one corner.

Rectangle



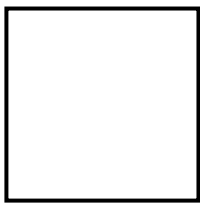
Pentagon



Example Draw the diagonals of a polygon.

Draw the diagonals of these polygons.

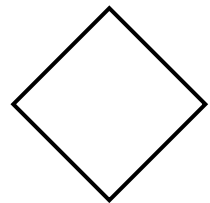
a



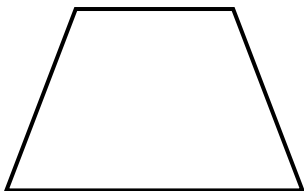
b



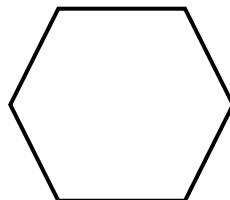
c



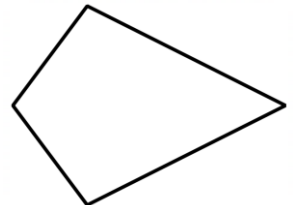
d



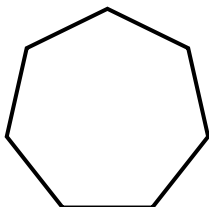
e



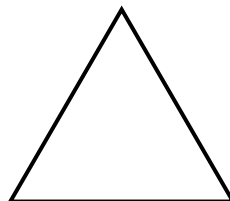
f



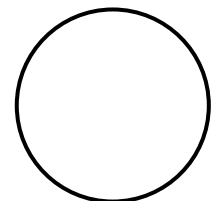
g



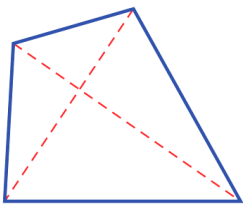
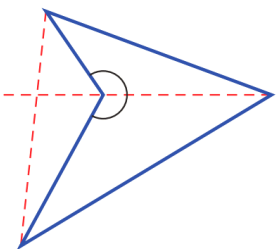
h



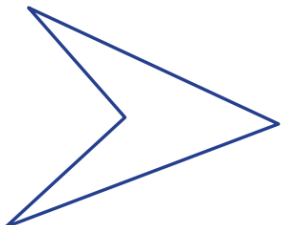
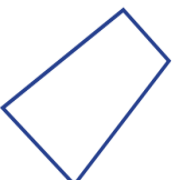
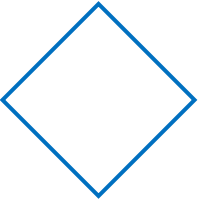
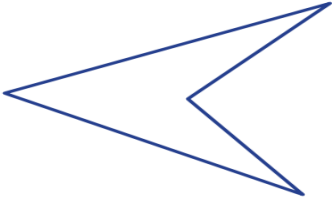
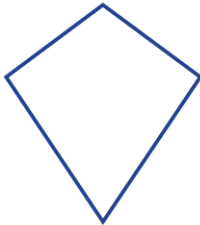
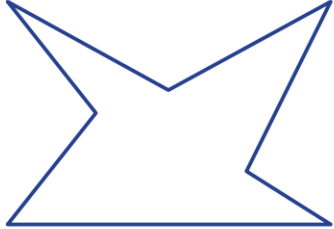
i



CONVEX AND NON-CONVEX QUADRILATERALS

Convex	Non-Convex
	
All interior angles less than 180° Diagonals meet inside the quadrilateral.	At least one interior reflex angle ($>180^\circ$) Diagonals meet outside the quadrilateral.

Example Distinguish between convex and non-convex quadrilaterals.

Determine whether these quadrilaterals are convex or non-convex.		
a  Non-convex	b  Non-convex	c  convex
d  convex	e  convex	f  Non-convex
g  convex	h  Non-convex	i  Non-convex

PROPERTIES OF SPECIAL QUADRILATERALS

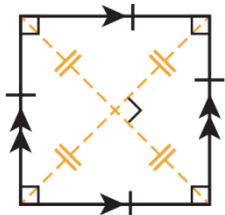
- **Adjacent** means next to.
- **Bisect** means cut into two equal parts.

Investigation Properties of special quadrilaterals.

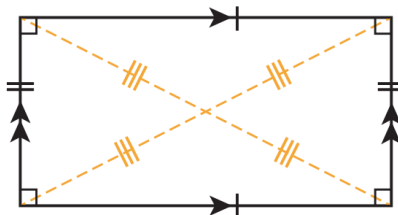
Using the notation on the diagrams, or the GeoGebra links, fill in the table.

 Properties of a Square:	https://www.geogebra.org/m/xq7pbnpe
 Properties of a Rectangle:	https://www.geogebra.org/m/axwkzwwq6
 Properties of a Rhombus:	https://www.geogebra.org/m/wtu2b73s
 Properties of a Parallelogram:	https://www.geogebra.org/m/gmhdqgbn
 Properties of a Parallelogram:	https://www.geogebra.org/m/qem8ddkv
 Properties of a Trapezium:	https://www.geogebra.org/m/h2kxb78k

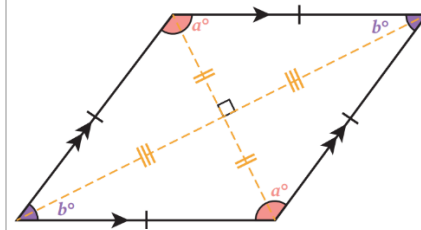
Square



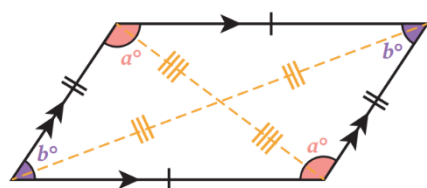
Rectangle



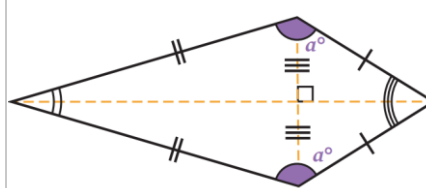
Rhombus



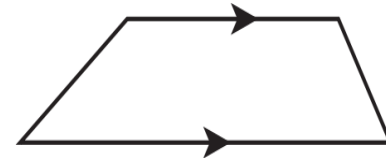
Parallelogram



Kite

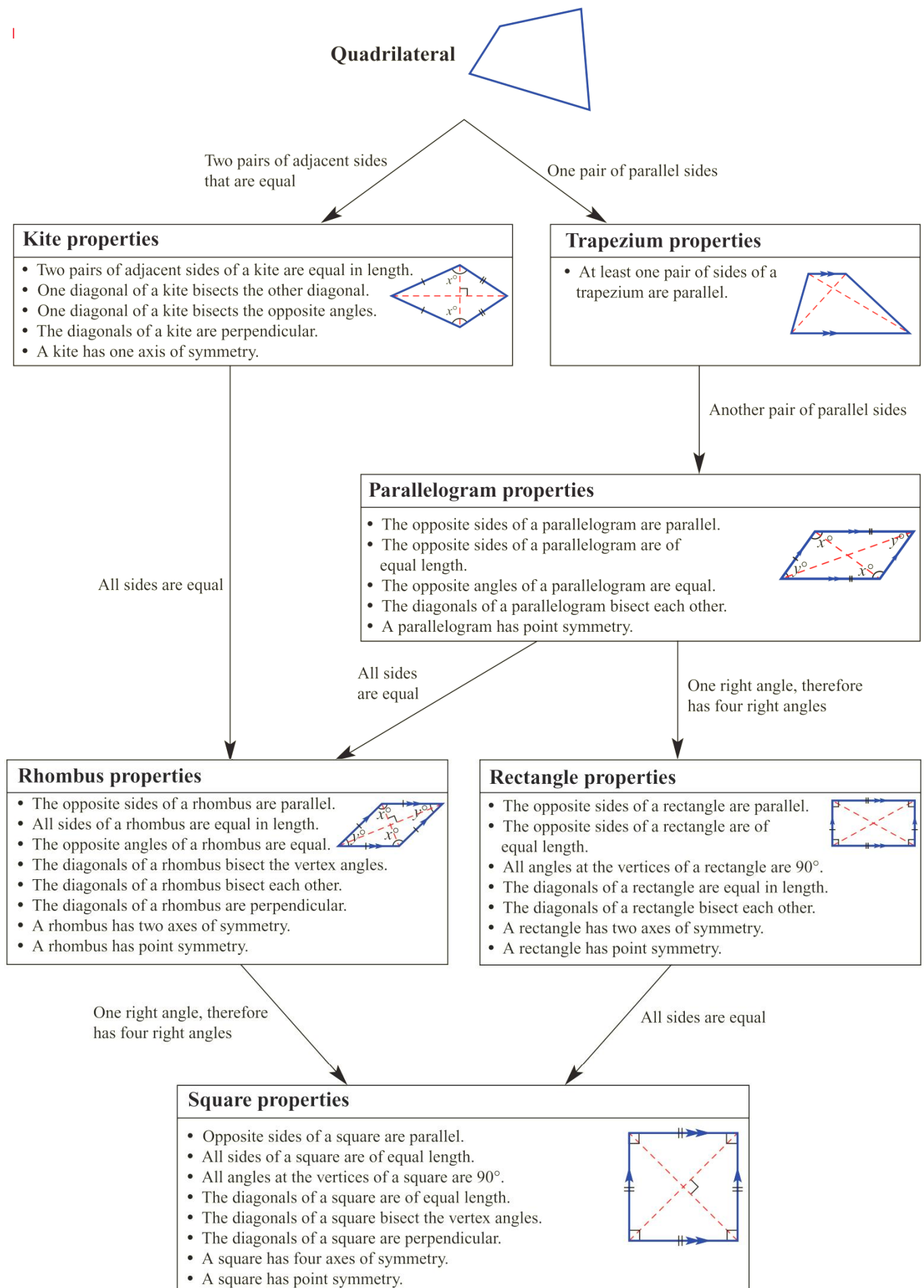


Trapezium

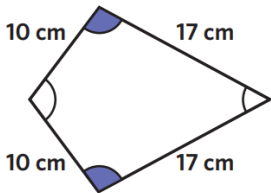
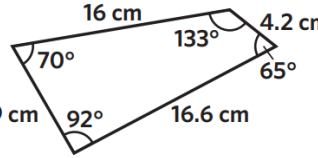
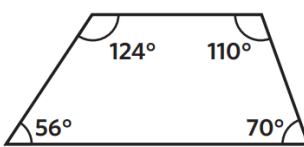


Property (always true?)	Square	Rectangle	Rhombus	Parallelogram	Kite	Trapezium
Opposite sides parallel	Yes	Y	Y	Y	No	1 pair
Adjacent sides equal	Yes	Y	Y	N	2 pairs	N
Opposite sides equal	Yes	Y	Y	Y	No	N
Opposite angles equal	Yes	Y	Y	Y	1 pair	N
Adjacent sides perpendicular	Yes	Y	Y	N	No	N
Diagonals bisect the other diagonal	Yes	Y	Y	Y	1 pair	N
Diagonals intersect at right angles	Yes	N	Y	N	Yes	N
Diagonals equal	Yes	Y	N	N	No	N
Diagonals bisect angles of the quadrilateral	Yes	N	Y	N	1 pair	N

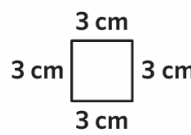
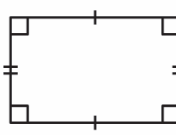
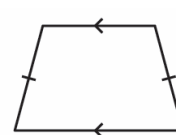
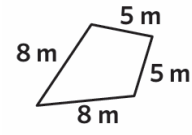
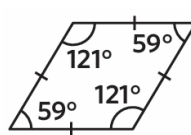
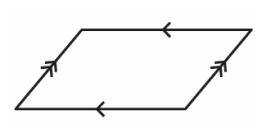
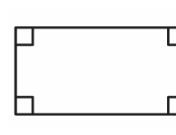
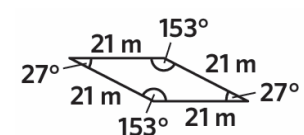
HIERARCHY OF QUADRILATERALS



1 Identify the quadrilateral using its angle properties.
Circle ALL of the shapes the quadrilaterals could be classified as.

a  10 cm 17 cm 10 cm 17 cm Quadrilateral Kite Trapezium Rhombus Parallelogram Square Rectangle Quadrilateral kite	b  Quadrilateral Kite Trapezium Rhombus Parallelogram Square Rectangle Quadrilateral, parallelogram, rectangle
c  16 cm 4.2 cm 9 cm 16.6 cm 70° 133° 92° 65° Quadrilateral Kite Trapezium Rhombus Parallelogram Square Rectangle quadrilateral	d  124° 110° 56° 70° Quadrilateral Kite Trapezium Rhombus Parallelogram Square Rectangle Quadrilateral, trapezium

2 Name these special quadrilaterals using the most appropriate definition.

a  3 cm 3 cm 3 cm 3 cm square	b  rectangle	c  trapezium	d  8 m 5 m 8 m 5 m kite
e  121° 59° 59° 121° parallelogram	f  parallelogram	g  rectangle	h  21 m 153° 27° 21 m 21 m 153° 21 m 27° rhombus

3 Explain why a square is a special type of rectangle.

A square has all the properties of a rectangle

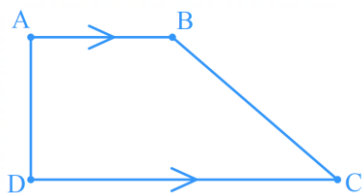
4 Explain why a rectangle is not a type of square.

A rectangle does not have all the properties of a square; it does not have equal sides.

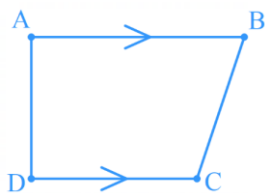
- 5 By considering the properties of special quadrilaterals, decide if the following are **always** true.
- | | | | |
|----------|---|-------|-------|
| a | A square is a type of rhombus. | | true |
| b | A rectangle is a type of parallelogram. | | true |
| c | A parallelogram is a type of square. | | false |
| d | A rhombus is a type of parallelogram. | | true |
| e | A parallelogram is a type of rhombus. | | false |
| f | A rhombus is a type of kite. | | true |

6 Sketch a:

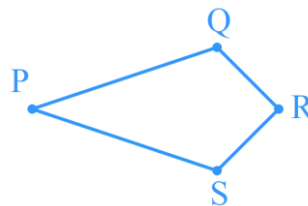
a trapezium ABCD, with AB parallel to CD



b trapezium ABCD, with $\angle ABC$ acute



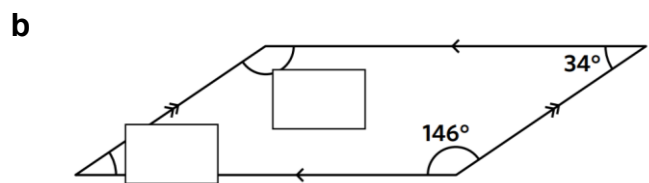
c kite, PQRS, with $\angle PQR$ and $\angle PSR$ both obtuse



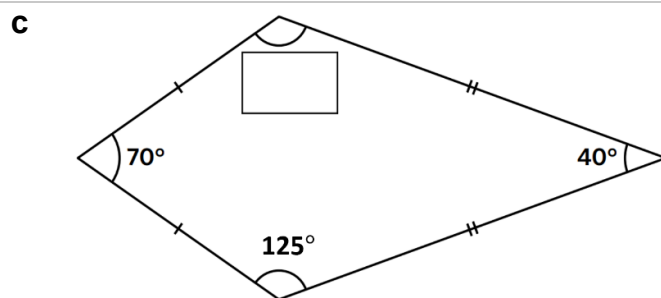
7 Find the missing angles. You will need to use properties of quadrilaterals.



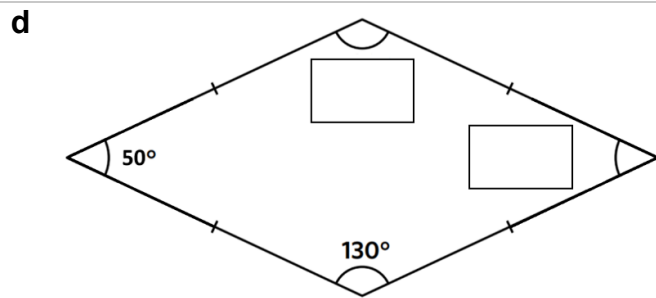
90°, 90°



34°, 146°



125°



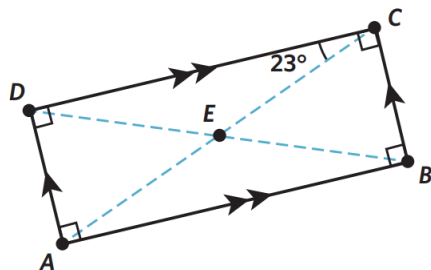
130°, 50°

8 First identify the quadrilateral, then determine the values of the angles.

a Quadrilateral:

.....

$\angle ECB =$

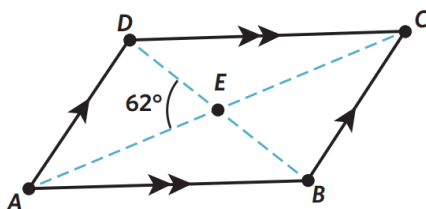


Rectangle
67°

b Quadrilateral:

.....

$\angle CEB =$

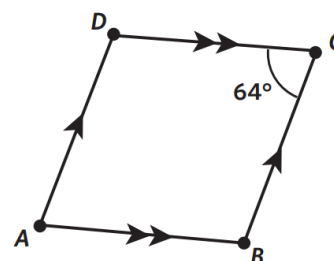


Parallelogram
62°

c Quadrilateral:

.....

$\angle ABC =$

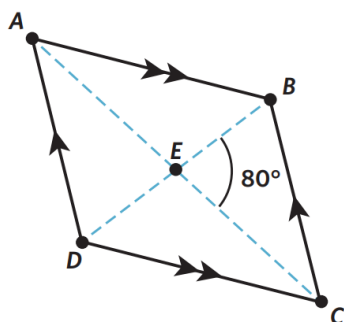


Parallelogram
116°

d Quadrilateral:

.....

e $\angle CED =$

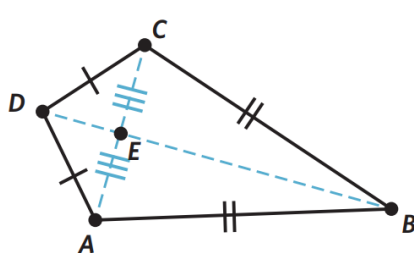


Parallelogram
100°

f Quadrilateral:

.....

g $\angle DEA =$

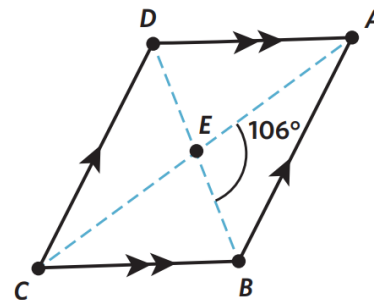


Kite
90°

h Quadrilateral:

.....

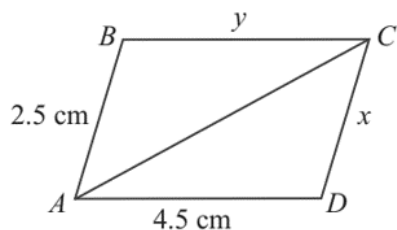
i $\angle ECD + \angle EDC =$



Parallelogram
74°

9 Use the properties of quadrilaterals to find the values of the pronumerals, giving reasons.

a $ABCD$ is a parallelogram.

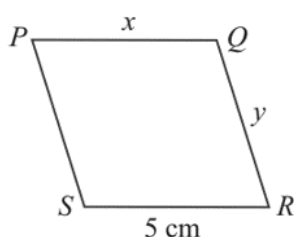


$x = 2.5$ cm (opposite sides of a parallelogram equal)

$y = \dots\dots\dots$ (.....)

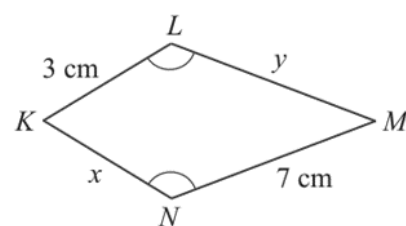
4.5 cm (opposite sides of parallelogram equal)

b $PQRS$ is a rhombus.



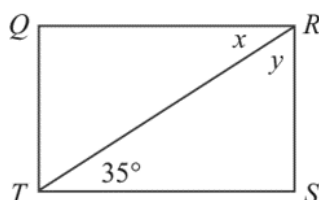
$x = y = 5$ cm
All sides of rhombus equal

c $KLMN$ is a kite.



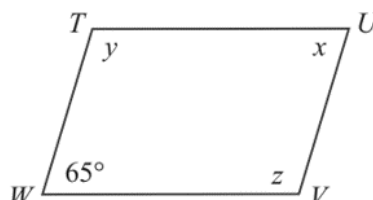
$x = 3$
 $y = 7$
Adjacent sides of kite are equal

d $QRST$ is a rectangle.



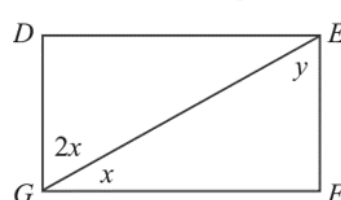
$x = 35^\circ$ Alternate angles on parallel lines
 $y = 55^\circ$ complementary with x

e $TUVW$ is a parallelogram.



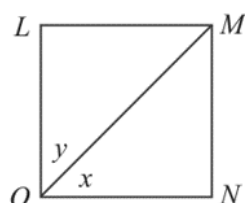
$y = 115^\circ$ Cointerior angles in parallel lines
 $z = 115^\circ$ Cointerior angles in parallel lines
 $x = 65^\circ$ opposite angles in parallelogram equal

f $DEFG$ is a rectangle.



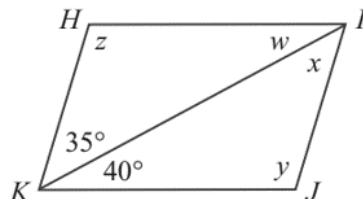
$3x = 90$ angles in a rectangle
 $x = 30^\circ$
 $y = 2x$ alternate angles on parallel lines
 $x = 60^\circ$

g $LMNO$ is a square.



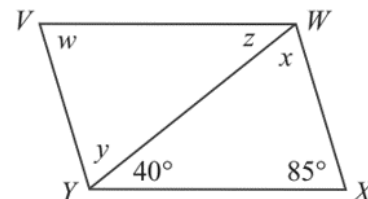
$x = y$ diagonals of square bisect angles at
 $x + y = 90^\circ$ (angles in a square)
 $x = y = 45^\circ$

h $HIJK$ is a parallelogram.



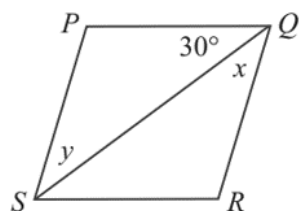
$x = 35^\circ$, $w = 40^\circ$
alternate angles on parallel lines
 $z = 105^\circ$ cointerior angles in parallel lines
 $y = z = 105^\circ$
opposite angles of parallelogram equal

i $VWXY$ is a parallelogram.



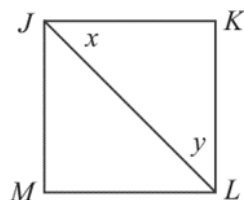
$z = 40^\circ$ alternate angles on parallel lines
 $y = 55^\circ$ cointerior angles on parallel lines
 $x = y = 55^\circ$ alternate angles on parallel lines
 $w = 85^\circ$ opposite angles of parallelogram

j $PQRS$ is a rhombus.



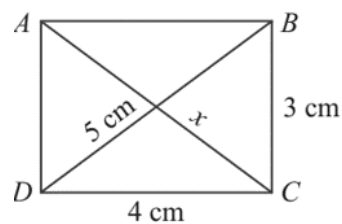
$x = 30^\circ$
 diagonals in rhombus bisect angles at vertex
 $y = x = 30^\circ$ alternate angles on parallel lines

k $JKLM$ is a square.



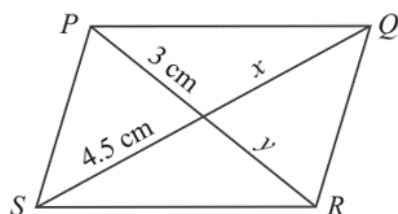
$x = y = 45^\circ$
 diagonals in square bisect angles at vertex

l $ABCD$ is a rectangle.



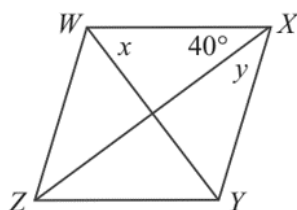
$x = 5$ cm diagonals of rectangle are equal
 and bisect each other.

m $PQRS$ is a parallelogram.



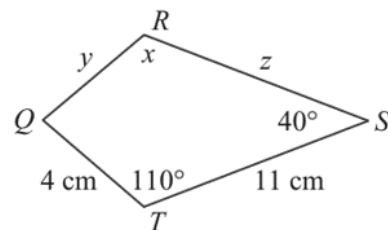
$x = 4.5$ cm, $y = 3$ cm
 Diagonals of parallelogram bisect each other

n $WXYZ$ is a rhombus.



$y = 40^\circ$ diagonals of rhombus bisect angles
 at vertex
 $x = 50^\circ$ angle sum of triangle

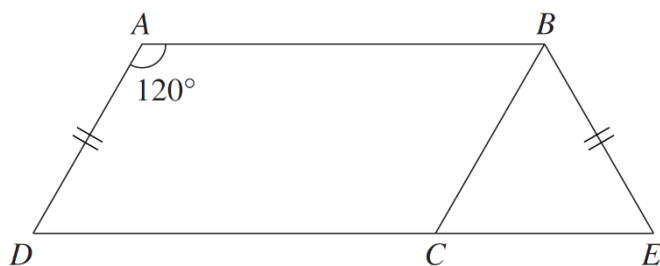
o $QRST$ is a kite.



$x = 110^\circ$ opposite angles of kite between
 unequal sides equal.
 $y = 4$ cm, $x = 11$ cm,
 adjacent sides of kite equal

10 NAPLAN B+

The diagram shows a parallelogram $ABCD$. Prove that $\triangle BCE$ is equilateral.



$\angle BCD = 120^\circ$ opposite angles of parallelogram equal.

$\angle ECB = 60^\circ$ angles on a straight line

$BC = AD$ opposite sides of parallelogram equal.

$\therefore BC = AD = BE$, so $\triangle BCE$ is isosceles

$\therefore \angle BCE = \angle BEC = 60^\circ$ angles opposite equal sides of isosceles triangle are equal

$\angle BCE = 180 - 60 - 60 = 60^\circ$ angle sum of triangle

$\therefore \triangle BCE$ is equilateral, as all angles are 60°

- 11 The length of the diagonals of a rhombus is 24 cm and 18 cm.
Find the length of each side of the rhombus.
(Hint: use Pythagoras' Theorem)

The diagonals form 4 right-angled triangles where the perpendicular sides are 12 cm and 9 cm.

The hypotenuse is the side length of the rhombus.

$$l = \sqrt{12^2 + 9^2} = 15 \text{ cm}$$

- 12 The side of a rhombus is 5 cm.
The length of one diagonal of the rhombus is 8 cm.
Find the length of the other diagonal.

The diagonals form 4 right-angled triangles where the hypotenuse is 5 cm and one of the shorter sides is 4 cm.

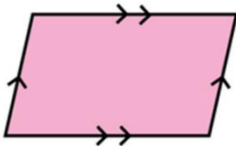
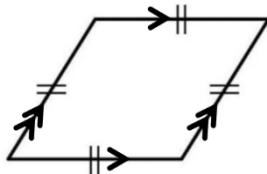
The other diagonal is double the other side length.

$$l = \sqrt{5^2 - 4^2} = 3$$

6 cm

CHECK YOUR UNDERSTANDING

Classifying quadrilaterals

1	A quadrilateral has exactly one pair of parallel sides. How would you classify this quadrilateral?	a	Parallelogram	b	Kite	c	Rhombus	d	Trapezium
2	A quadrilateral has four equal sides. Tom says it could be a rectangle. Katie says it could be a rhombus. Who is correct?	a	Only Tom	b	Only Katie	c	Both	d	Neither
3	Tom says this is a rectangle. Katie says this is a parallelogram. Who is correct?								
		a	Only Tom	b	Only Katie	c	Both	d	Neither
4	Tom says this is a rectangle. Katie says this is a parallelogram. Who is correct?								
		a	Only Tom	b	Only Katie	c	Both	d	Neither
5	Tom says this is a kite. Katie says this is a rhombus. Who is correct?								
		a	Only Tom	b	Only Katie	c	Both	d	Neither
6	Tom says this is a kite. Katie says this is a parallelogram. Who is correct?								
		a	Only Tom	b	Only Katie	c	Both	d	Neither