Student Name

Mathematics Stage 5 Core

TRIGONOMETRY A

Book 2 Apply trigonometry to solve right-angled triangle problems

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

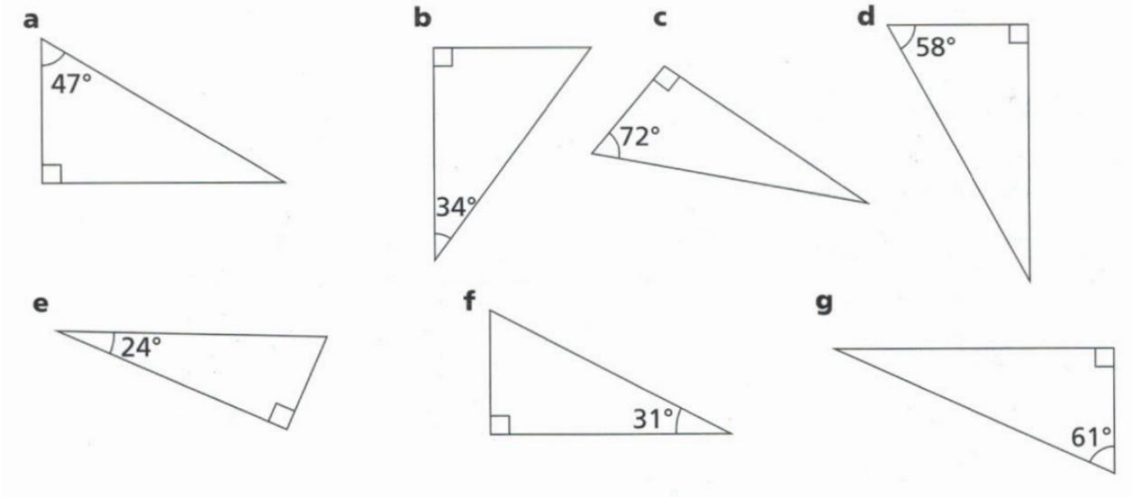
MA5-TRG-C-01 applies trigonometric ratios to solve right-angled triangle problems

Apply trigonometry to solve right-angled triangle problems

- Apply trigonometry to find the lengths of unknown sides in right-angled triangles with a given angle including angles in degrees and minutes
- Apply trigonometry to find the size of unknown angles in right-angled triangles including in degrees and minutes
- Solve a variety of practical problems involving trigonometric ratios in right-angled triangles

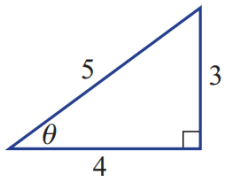
REVIEW OF PRIOR KNOWLEDGE

1 label the hypotenuse (H), the side opposite the given angle (O) and the side adjacent to the given angle (A).



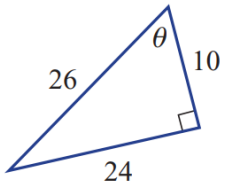
2 Write the three trigonometric ratios, $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$

a



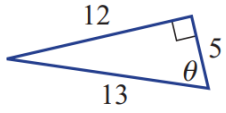
$\sin\theta =$ ☐/☐ $\cos\theta =$ ☐/☐ $\tan\theta =$ ☐/☐

b



$\sin\theta =$ ☐/☐ $\cos\theta =$ ☐/☐ $\tan\theta =$ ☐/☐

c



$\sin\theta =$ ☐/☐ $\cos\theta =$ ☐/☐ $\tan\theta =$ ☐/☐

3 Solve these equations.

a $\frac{x}{6} = 7$

b $\frac{x}{4} = 5$

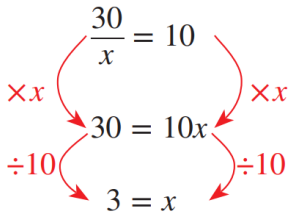
c $\frac{x}{7} = 24$

4 Using a calculator, match the expressions to their value.

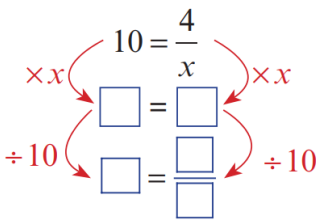
$10 \sin 27°$ •	• 62.7
$\frac{58.5}{\tan 43°}$ •	• 4.6
$\frac{14.13}{\cos 77°}$ •	• 353.6
$11.4 \tan 22°$ •	• 4.5
$58 \cos 39°$ •	• 45.1
$\frac{155}{\sin 26°}$ •	• 62.6
$8.8 \tan 82°$ •	• 62.8

5 To solve an equation with an unknown as the denominator we can first multiply both sides by that unknown. Solve these equations.

a



b



c $\frac{4}{x} = 2$

d $\frac{20}{x} = 4$

e $\frac{15}{x} = 5$

f $2 = \frac{12}{x}$

6 Solve these equations.

a $\sin\theta = 0.4$

$\theta = \sin^{-1}(\quad)$

$=$

b $\cos\theta = \frac{4}{5}$

c $\tan\theta = 32.4$

The sine of an angle is 0.4. What is the angle? .....

The cosine of an angle is 0.4. What is the angle? .....

The tangent of an angle is 0.4. What is the angle? .....

FINDING UNKNOWN SIDES PART 1

TRIGONOMETRIC EQUATIONS

- A **trigonometric ratio** is a number.
- Solve trigonometric equations by treating ratios as numbers.

$$\sin 30° = \frac{x}{7} \qquad \text{is the same as} \qquad 0.5 = \frac{x}{7}$$

Example Solve trigonometric equations: unknown in numerator.

$\cos 60° = \frac{x}{5}$ $\times 5 \quad \times 5$ $5\cos 60° = x$ $2.5 = x$	How did we know to do $\times 5$ to both sides? *The operation done to x is ……., to isolate x, we ……. by 5.* How did we find the value of $5\cos 60°$? *We enter the expression into a ……*

Solve these equations. Round your answer to 2 decimal places.

a $\cos 24° = \frac{x}{10}$

$x = 9.14$

b $\sin 29° = \frac{x}{7}$

$x = 3.39$

c $\cos 82° = \frac{x}{10.1}$

$x = 1.41$

d $\tan 34° = \frac{x}{12}$

$x = 8.09$

e $\tan 38° = \frac{x}{13}$

$x = 10.16$

f $\sin 23° = \frac{x}{0.7}$

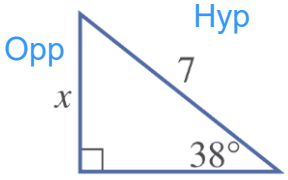
$x = 0.27$

SOLVING FOR AN UNKNOWN SIDE

1. Label the side you want to find and given side of the right-angled triangle.
2. Determine the trigonometric ratio that relates these sides.
3. Write the trigonometric equation.
4. Substitute known values into the equation.
5. Solve the equation.

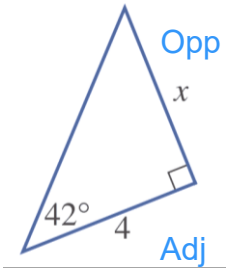
Example Calculate an unknown side using trigonometry; unknown in numerator.

Calculate the value of x in these right-angled triangles.



1. Label
2. sin
3. $\sin\theta = \frac{O}{H}$
4. $\sin 38° = \frac{x}{7}$
5. $7\sin 38° = x$

$x \approx 4.31$

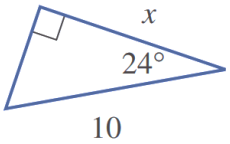


1. Label
2. tan
3. $\tan\theta = \frac{O}{A}$
4. $\tan 42° = \frac{x}{4}$
5. $4\tan 42° = x$

$x \approx 3.60$

Calculate the value of x in these right-angled triangles. Round your answer to 2 d.p.

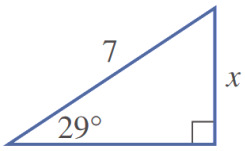
a



1. Label
2.
3.
4.
5.

$x = 9.14$

b



1. Label
2.
3.
4.
5.

$x = 3.39$

Key ideas

1. We solve for unknown sides in a right-angled triangle by:
   a. Identifying a relevant trigonometric ratio that relates the sides and angle.
   b. Write the …… equation.
   c. …… values into the equation.
   d. …… the equation by applying …… operations to both sides.

1 Solve these equations to find the value of x correct to 2 decimal places.

a $\sin 50° = \frac{x}{4}$

$4 \times \sin 50° = x$

$\ldots\ldots\ldots\ldots = x$

3.06

b $\tan 81° = \frac{x}{3}$

$\ldots \times \tan 81° = x$

$\ldots\ldots\ldots\ldots = x$

18.94

c $\cos 33 = \frac{x}{6}$

$\ldots \times \ldots\ldots = \ldots\ldots$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

5.03

d $\cos 75° = \frac{x}{3.5}$

0.91

e $\sin 24° = \frac{x}{4.2}$

1.71

f $\tan 42 = \frac{x}{10}$

9.00

g $\frac{x}{7.1} = \tan 18.4°$

2.36

h $\frac{x}{5.3} = \sin 64.7°$

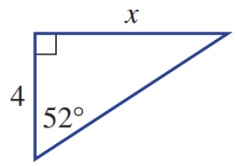
4.79

i $\frac{x}{12.6} = \cos 52.9°$

7.60

2 Determine the two sides given (O, A or H), and hence which trigonometric ratio (sin, cos, or tan) should be used to solve for x.

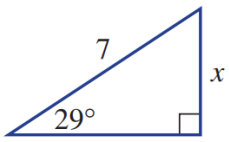
a



○ sin ○ cos ○ tan

tan

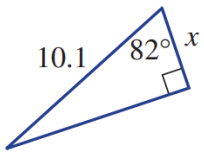
b



○ sin ○ cos ○ tan

sin

c

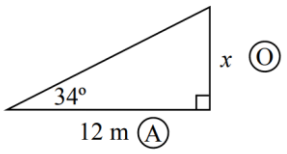


○ sin ○ cos ○ tan

cos

3 Find the side length by setting out your working as shown. Round your answer to 1 d.p.

a



$\tan 34 = \frac{\square\ \text{O}}{\square\ \text{A}}$

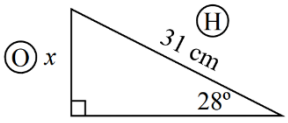
$\times 12 \quad \times 12$

$x = \square \tan \square$

$x = \square$ m 1 d.p.

8.1 m

b



$\sin 28 = \frac{\square\ \text{O}}{\square\ \text{H}}$

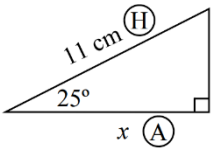
$\times 31 \quad \times 31$

$x = \square \sin \square$

$x = \square$ cm 1 d.p.

4.6 m

c



$\cos 25 = \frac{\square\ \text{A}}{\square\ \text{H}}$

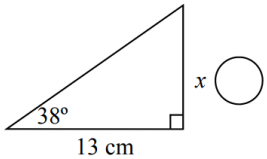
$\times 11 \quad \times 11$

$x = \square \cos \square$

$x = \square$ cm 1 d.p.

10.0 m

d



$\tan \square = \frac{\square\ \text{O}}{\square\ \text{A}}$

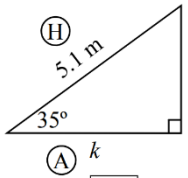
$\times 13 \quad \times 13$

$x = \square \tan \square$

$x = \square$ cm 1 d.p.

10.2 cm

e



$\square = \frac{\square\ \text{A}}{\square\ \text{H}}$

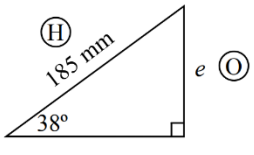
$\times 5.1 \quad \times 5.1$

$k = \square$

$k = \square$ m 1 d.p.

4.4 m

f



$\square = \frac{\square\ \text{O}}{\square\ \text{H}}$

$\times 185 \quad \times 185$

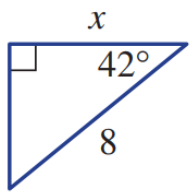
$e = \square$

$e = \square$ mm 1 d.p.

113.9 mm

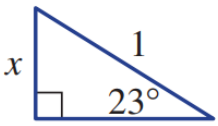
4 Calculate the value of x correct to 2 decimal places using the sine, cosine, or tangent ratios.

a



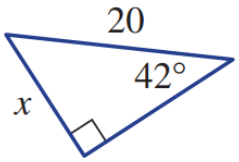
5.95

b



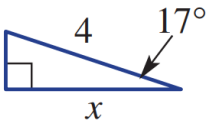
0.39

c



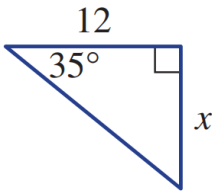
13.38

d



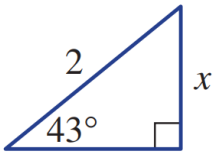
3.83

e



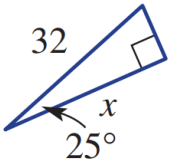
8.40

f



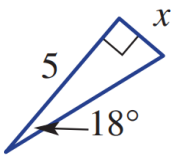
1.36

g



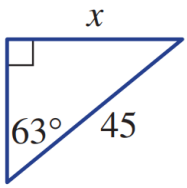
29.00

h



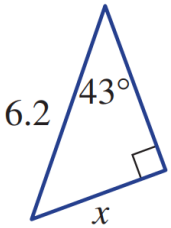
1.62

i



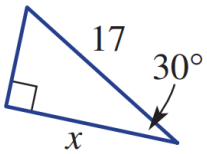
40.10

j



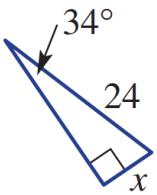
4.23

k



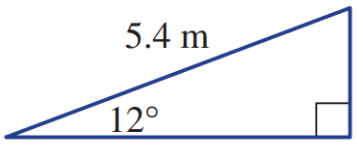
14.72

l



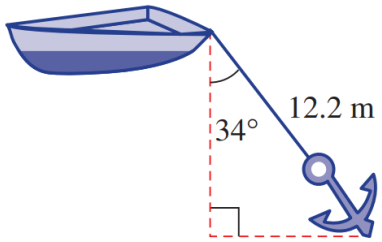
13.42

5 Amy walks 5.4 m up a ramp that is inclined at 12° to the horizontal.
How high (correct to 2 decimal places) is she above her starting point?



1.12 m

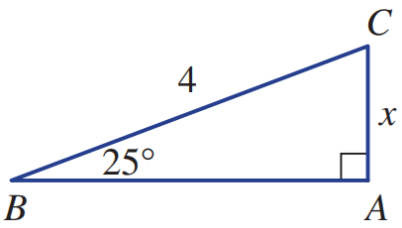
6 One end of a 12.2 m rope is tied to a boat. The other end is tied to an anchor, which is holding the boat steady in the water. If the anchor is making an angle of 34° with the vertical, how deep is the water? Give your answer correct to 2 decimal places.



10.11 m

7 Consider the right-angled triangle on the right.

a Find angle $\angle C$



65°

b Calculate x using the sine ratio.

1.69

c Calculate x using the cosine ratio.

1.69

FINDING UNKNOWN SIDES PART 2

TRIGONOMETRIC EQUATIONS WITH UNKNOWN IN DENOMINATOR

Example Solve trigonometric equations: unknown in denominator.

$$\cos 60° = \frac{5}{x}$$
$$\times x \quad \times x$$
$$x \cos 60° = 5$$
$$\div \cos 60° \quad \div \cos 60°$$
$$x = \frac{5}{\cos 60°}$$
$$= 10$$

When should we do $\times x$ to both sides?

When x *is in the ..... of the fraction.*

Solve these equations. Round your answer to 2 decimal places.

a $\cos 47° = \frac{26}{x}$

$x = 38.12$

b $\sin 53° = \frac{26}{x}$

$x = 32.56$

c $\tan 28° = \frac{25}{x}$

$x = 47.02$

d $\sin 42° = \frac{9}{x}$

$x = 13.45$

e $\tan 44° = \frac{15}{x}$

$x = 15.53$

f $\cos 49° = \frac{6}{x}$

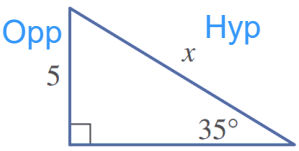
$x = 9.15$

SOLVING FOR AN UNKNOWN SIDE

1. Label sides.
2. Determine trig ratio.
3. Write trig equation.
4. Substitute known values.
5. Solve equation.

Example Calculate an unknown side using trigonometry: unknown in denominator.

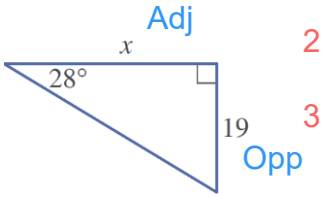
Calculate the value of x in these right-angled triangles.



1. label
2. sin
3. $\sin\theta = \frac{O}{H}$
4. $\sin 35° = \frac{5}{x}$
5. $x \sin 35° = 5$

$$x = \frac{5}{\sin 35°}$$

$$\approx 8.72$$



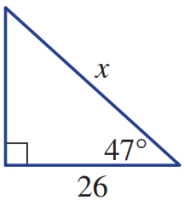
1. label
2. tan
3. $\tan\theta = \frac{O}{A}$
4. $\tan 28° = \frac{19}{x}$
5. $x \tan 28° = 19$

$$x = \frac{19}{\tan 28°}$$

$$\approx 35.73$$

Calculate the value of x in these right-angled triangles. Round your answer to 2 d.p.

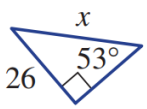
a



1. Label
2.
3.
4.
5.

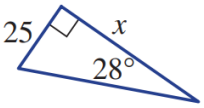
$x = 38.12$

b



$x = 32.56$

c



$x = 47.02$

1 Solve the equation to find the value of x correct to 2 decimal places.

a $\cos 43° = \frac{3}{x}$

$x \cos 43° = 3$

$x = \frac{3}{\cos 43°}$

$= \ldots\ldots\ldots$

4.10

b $\sin 36° = \frac{4}{x}$

$x \sin 36° = 4$

$x = \frac{\quad}{\quad}$

$= \ldots\ldots\ldots$

6.81

c $\cos 33 = \frac{6}{x}$

7.15

2 Emma noticed a shortcut when solving these equations:

Instead of writing:

$2 = \frac{3}{x}$

$2x = 3$

$x = \frac{3}{2}$

We can skip a step:

$2 = \frac{3}{x}$

swap

$x = \frac{3}{2}$

Solve these equations using the shortcut.

a $\tan 64° = \frac{2}{x}$

$x = \frac{2}{\tan 64°}$

$x = \ldots\ldots\ldots$

0.98

b $\cos 67° = \frac{5}{x}$

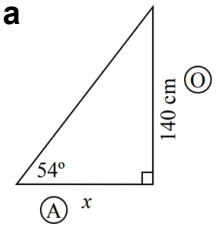
12.80

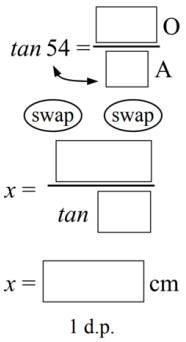
c $\sin 12° = \frac{3}{x}$

14.43

3 Find the side length by setting out your working as shown. Round your answer to 1 d.p.

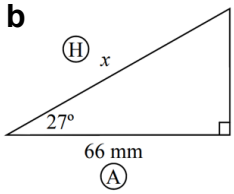
a

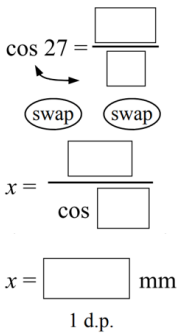




101.7 cm

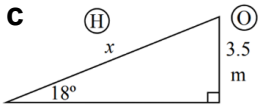
b

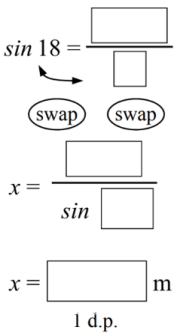




74.1 mm

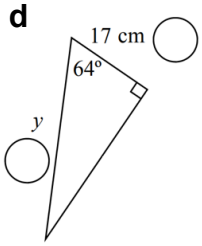
c

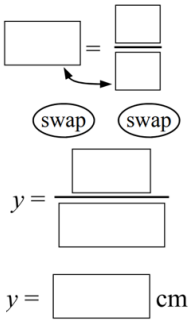




11.3 m

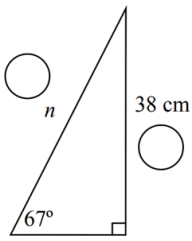
d

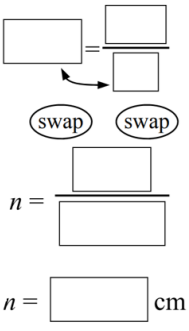




18.9 cm

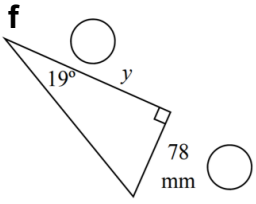
e

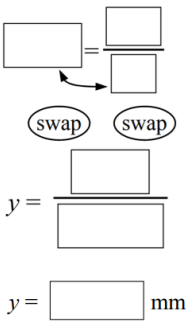




41.3 cm

f

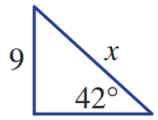




226.5 mm

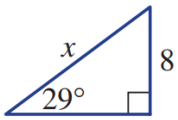
4 Calculate the value of x correct to 2 decimal places.

a



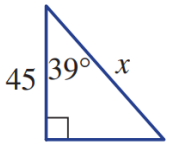
13.45

b



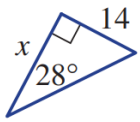
16.50

c



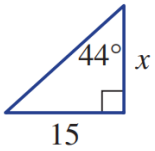
57.90

d



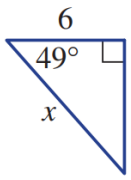
26.33

e



15.53

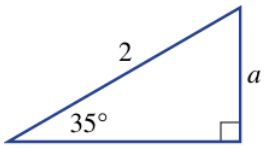
f



9.15

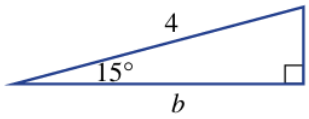
5 Calculate the value of the pronumeral, to 2 decimal places.

a



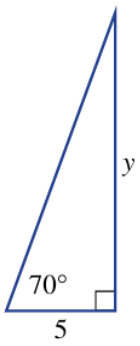
1.15

b



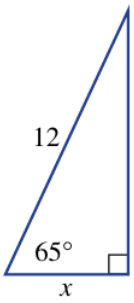
3.86

c



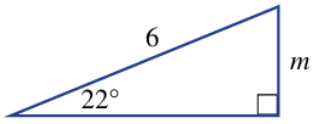
13.74

d



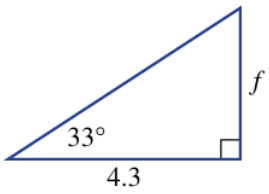
5.07

e



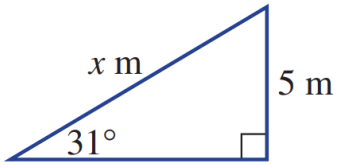
2.25

f



2.79

6 Solve for x.



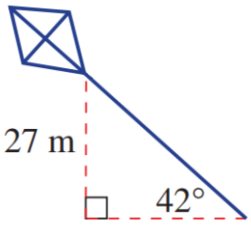
Two students provide different answers in the test. Which student is more accurate? What mistake have they made?

A $x = \dfrac{5}{\sin 31°} = \dfrac{5}{0.52} = 9.62$

B $x = \dfrac{5}{\sin 31°} = 9.71$

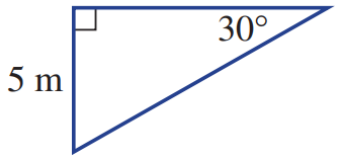
Student B is more accurate. Student A rounded too early.

7 A kite is flying at a height of 27 m above the anchor point. If the string is inclined at 42° to the horizontal, find the length of the string correct to the nearest metre.



40 m

8 Find the perimeter of this triangle, correct to 1 d.p.



23.7 m

9 Find the length of the side labelled x.

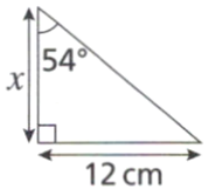
a by using the 54° angle

$\tan 54° = \dfrac{12}{x}$

$x = \dfrac{12}{\tan 54°}$

$= 8.72$ cm

b By using the unmarked angle in the triangle.



Missing angle is $180 - 90 - 54 = 36°$

$\tan 36° = \dfrac{x}{12}$

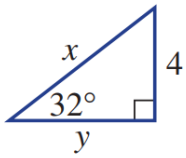
$x = 12 \tan 36°$

$= 8.72$ cm

Which method do you prefer?

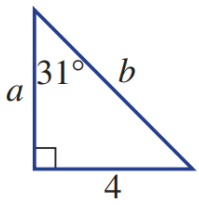
10 Find the value of each pronumeral correct to 1 decimal place.

a



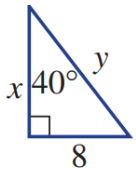
$x = 7.5, y = 6.4$

b



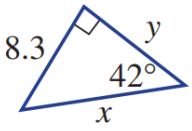
$a = 6.7, b = 7.8$

c



$x = 9.5, y = 12.4$

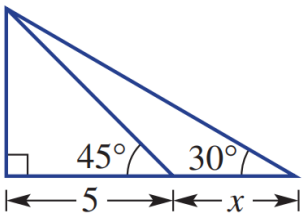
d



$x = 12.4, y = 9.2$

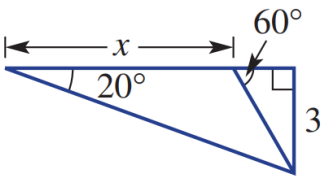
11 Find the value of x, correct to 1 decimal place, in these triangles.

a



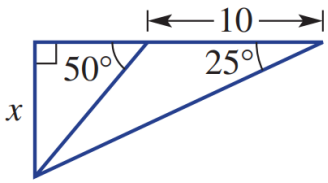
3.7

b



6.5

c



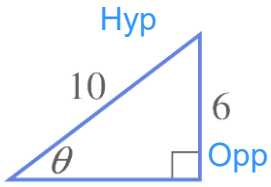
7.7

FINDING UNKNOWN ANGLES

1. Label sides.
2. Determine trig ratio.
3. Write trig equation.
4. Substitute known values.
5. Solve equation.

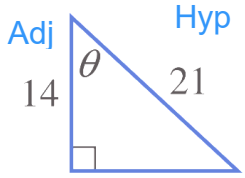
Example Calculate an unknown angle using trigonometry.

Find the unknown angle. Round to 2 d.p.



1. Label
2. sin
3. $\sin\theta = \frac{O}{H}$
4. $\sin\theta = \frac{6}{10}$
5. $\theta = \sin^{-1}\left(\frac{6}{10}\right)$

$\theta \approx 36.87°$

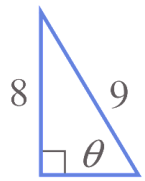


1. Label
2. cos
3. $\cos\theta = \frac{A}{H}$
4. $\cos\theta = \frac{14}{21}$
5. $\theta = \cos^{-1}\left(\frac{14}{21}\right)$

$\theta \approx 48.19°$

Calculate the value of θ in these right-angled triangles. Round your answer to 2 d.p.

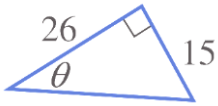
a



1. Label
2.
3.
4.
5.

$\theta = 62.73°$

b



1. Label
2.
3.
4.
5.

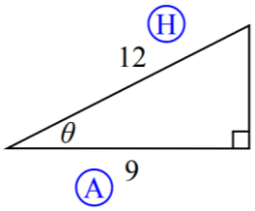
$\theta = 29.98°$

Key ideas

1. To find the unknown …… in a right-angled triangle, we use the …… trigonometric function.

1 Calculate the value of θ to the nearest degree.

a



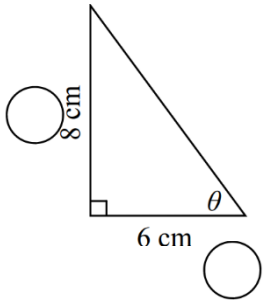
$\cos\theta = \frac{9}{12}$

$\theta = \cos^{-1}\left(\frac{9}{12}\right)$

$\theta =$

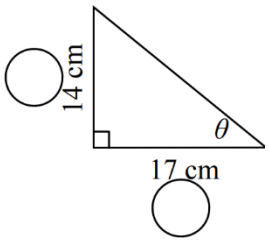
41°

b



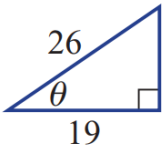
53°

c



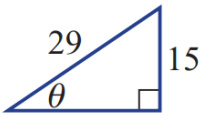
39°

d



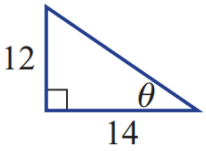
43°

e



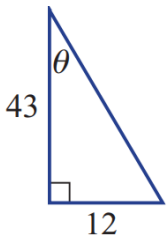
31°

f



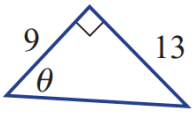
41°

g



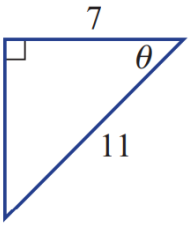
16°

h



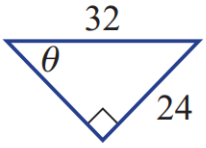
55°

i



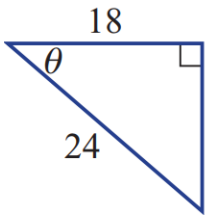
50°

j



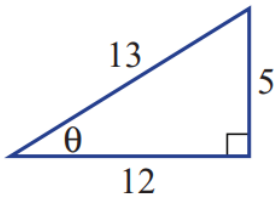
49°

k



41°

2 Refer to the diagram to complete the following, answer to 2 decimal places.



a $\sin\theta = \frac{\square}{\square}$

$\theta =$

22.62°

b $\cos\theta = \frac{\square}{\square}$

$\theta =$

22.62°

c $\tan\theta = \frac{\square}{\square}$

$\theta =$

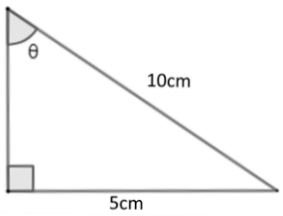
22.62°

What do you notice? Does it make sense?

The answer for all three is 22.62°. This makes sense as you are finding the same angle in each case.

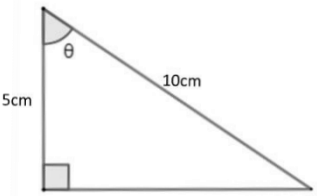
3 Solve for the unknown angle.

a



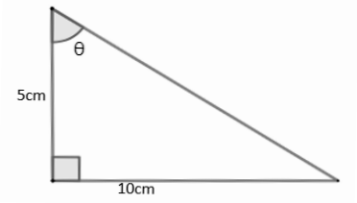
30°

b



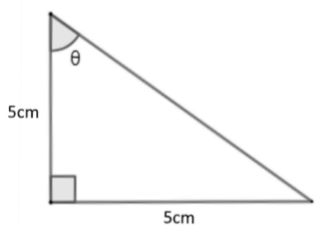
60°

c



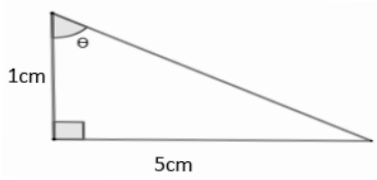
63.4°

d



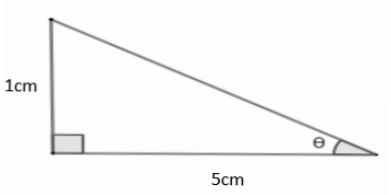
45°

e



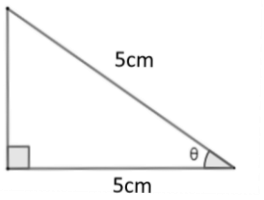
78.7°

f



11.3°

4 Solve for θ



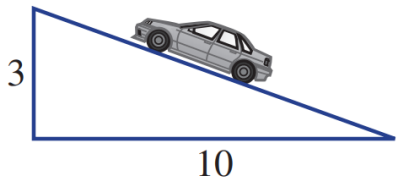
0°

Explain why this triangle is impossible.

It's impossible for a triangle to have an internal angle of zero.

Also, the triangle's hypotenuse and adjacent are equal, which is impossible as the hypotenuse must be the longest side.

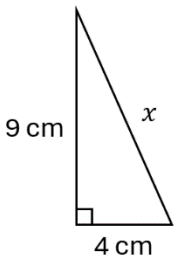
5 A road rises at a grade of 3 in 10.
Find the angle (to the nearest degree) the road makes with the horizontal.



17°

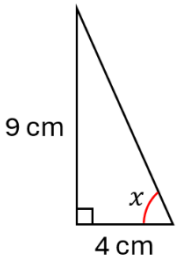
6 Find the value of x. Answer to 2 decimal places.

a



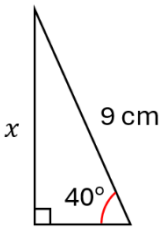
$\sqrt{97} = 9.85$ cm

b



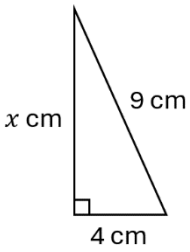
66.04°

c



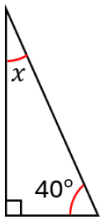
5.79 cm

d



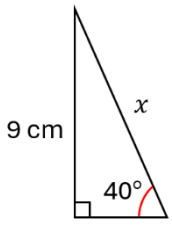
$\sqrt{65} = 8.06$ cm

e



50°

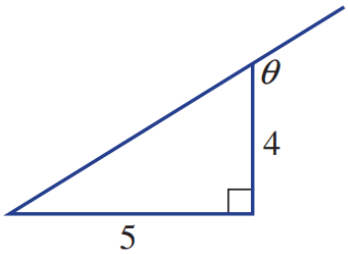
f



14.00 cm

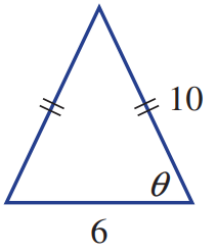
7 Calculate the value of θ in these diagrams. Answer to 1 d.p.

a



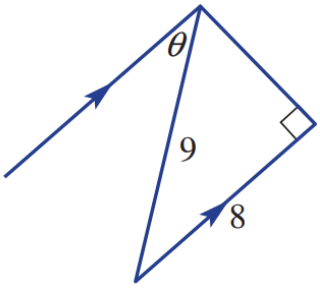
128.7°

b



72.5°

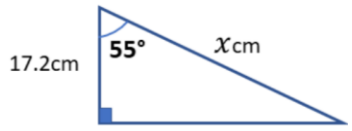
c



27.3°

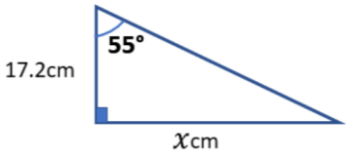
12 Solve for the unknown side. Answer to 1 d.p.

a



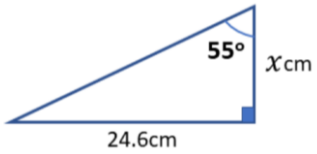
30.0 cm

b



24.6 cm

c

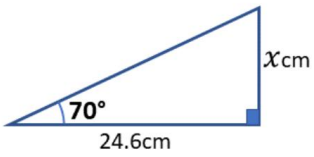


17.2 cm

d

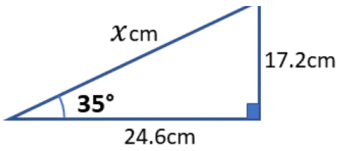
17.2 cm

e



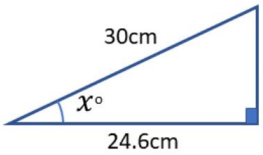
67.6 cm

f



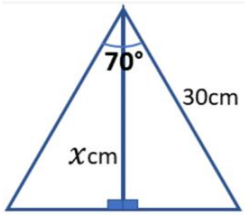
30.0 cm

g



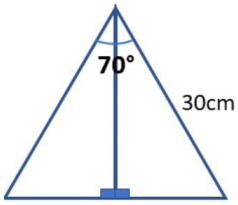
35°

h



24.6 cm

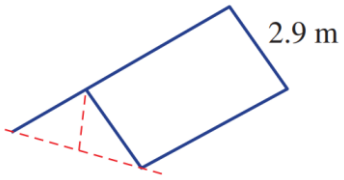
i



Area = ?

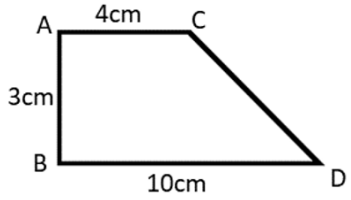
423.1 cm^2

8 A tent with a simple A-frame design is set up as a shelter. The breadth of half of the tent is 2.9 metres, as shown. Find the angle to the ground that the sides of the tent make if the height at the middle of the shelter is 1.5 metres. Round your answer to the nearest 0.1 of a degree.



31.1°

9 Consider the following shape; answer all questions to 1 d.p,



a Calculate the area of ΔBCD

15 cm²

b Calculate the perimeter of the trapezium.

23.7 cm

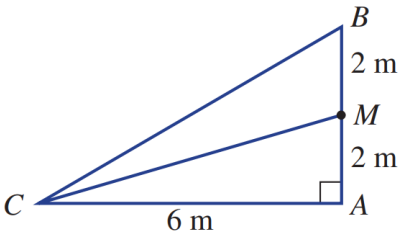
c Find the size of $\angle CDB$ to 1 d.p.

26.6°

d Find the size of $\angle ACD$ to 1 d.p.

153.4°

10 M is the midpoint of AB. Determine whether the angle $\angle ACM$ is exactly half of angle ACB.



∠ACM = 18.4° ∠ACB = 33.7° , no it is not half

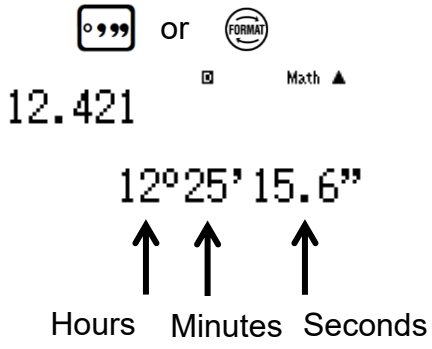
DEGREES MINUTES SECONDS

- Our number system is called the **decimal** system based on tens.
  - Ten ones are ten. Ten tens are a hundred. Ten hundreds is a thousand…
- Angles use the **sexagesimal** system, based on sixties.
  - Sixty seconds are a minute. Sixty minutes are a degree.

Sexagesimal System	
Degree °	$\frac{1}{360}$ of a turn
Minute ′	$\frac{1}{60}$ of a degree
Second ″	$\frac{1}{60}$ of a minute

Sexagesimal Notation

Convert between decimal form and sexagesimal form by using the buttons:



Example Convert between sexagesimal form and decimal form.

Convert these angles in decimal form to sexagesimal form.

$32.5°$ $= 32°\ 30'$ [3] [2] [•] [5] [=] [°,,,]

$89.25°$ $= 89°\ 15'$ [8] [9] [•] [2] [5] [=] [°,,,]

Convert these angles from decimal form to sexagesimal form.

a $44.75° =$ $44°45'$

b $36.6° =$ $36°36'$

c $0.1° =$ $0°6'$

d $77.2° =$ $77°12'$

e $0.5° =$ $0°30'$

f $126.1552° =$ $126°\ 9'\ 18.72''$

ROUNDING IN SEXAGESIMAL FORM

1. Draw a “cut” after the required unit.
2. Circle the **critical unit** right after the cut.
3. Decide to round up to down.
   i. If critical unit is less than 30, round down.
   ii. If critical unit is 30 or more, round up.
4. Round.
   i. rounding down: Remove digits after the cut.
   ii. rounding up: Add 1 to digit before the cut, then remove digits after the cut.

Example Round to the nearest degree and minute.

Round $43° 44′ 8.4″$ to the nearest degree.		Round $88° 30′ 22″$ to the nearest minute.	
1. Draw cut	$43\ °\vert 44\ ′\ 8.4\ ″$	1. Draw cut	$88\ °\ 30\ ′\vert 22\ ″$
2. Critical digit	$43\ °\vert (44)\ ′\ 8.4\ ″$	2. Critical digit	$88\ °\ 30\ ′\vert (22)\ ″$
3. Decide	*Round up*	3. Decide	*Round down*
4. Round	$44°$	4. Round	$88°30′$
$43° 44′ 8.4″ \approx 44°$		$88° 30′ 22″ \approx 88°30′$	

Round these angles to the nearest minute.

a	$51° 59′ 8″$	**b**	$33° 22′ 47″$	**c**	$41° 24′ 47″$
	$51°59′$		$33°23′$		$41°25′$
d	$42.5825°$	**e**	$74.2842°$	**f**	$14.5°$
	$42°35′$		$74°17′$		$14°30′$

Key ideas

1. There are 60 …… in 1 degree.
2. There are 60 …… in 1 minute.
3. When rounding to the nearest degree or minute, we use …… as the halfway mark.

1 Complete these sentences:

a …… degrees make a full turn, or revolution.

b 60 …… make up one degree.

c 60 …… make up one minute.

2 Convert these angles in decimal form to degrees, minutes, and seconds.

a	25.1° = 25°……′ 25° 6' 0"	**b**	25.2° = 25°……′ 25° 12' 0"	**c**	25.25° = 25°……′ 25° 15' 0"
d	25.5° = 25°……′ 25° 30' 0"	**e**	25.625° = 25°……′……″ 25° 37' 30"	**f**	25.75° = …… 25° 45' 0"

3 Round these angles to the nearest degree.

a	11° 38’ 12°	**b**	55°1 6’ 55°	**c**	49° 46’ 50°
d	70° 28’ 70°	**e**	79° 30’ 80°	**f**	18° 27’ 18°
g	42° 21’ 44” 42°	**h**	44° 35’ 21’ 45°	**i**	29° 21’ 45” 29°

4 Round these angles to the nearest minute.

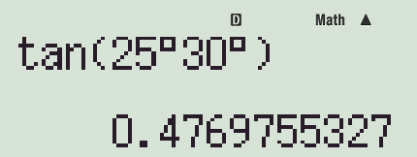
a	43° 52’ 11.71” 43° 52'	**b**	35° 13’ 24.05” 35° 13'	**c**	42° 59’ 28.76” 42° 59'
d	19° 27’ 11.66” 19° 27'	**e**	76° 49’ 23.05’ 76° 49'	**f**	14° 30’ 55.17” 14° 31'

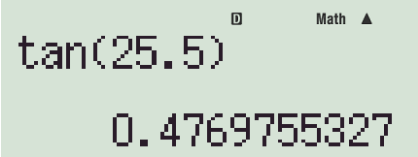
5 Convert these angles in decimal form to sexagesimal form and round to the nearest minute.

a	25.2875° 25° 17'	**b**	45.16666° 45° 10'	**c**	67.4916° 67° 29'
d	72.4215° 72° 25'	**e**	22.3° 22° 18'	**f**	87.252° 87° 15'

TRIGONOMETRY WITH DMS

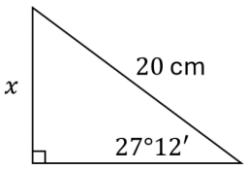
- The °,,, button is used to insert angles in terms of degrees and minutes into the calculator.



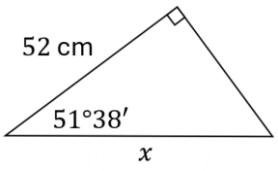


Example Calculate unknown sides when angle is in DMS form.

Calculate the value of x. Round your answer to 2 decimal places.



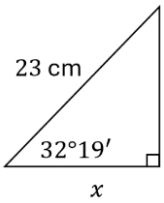
$$\sin(27°12') = \frac{x}{20}$$
$$20\sin(27°12') = x$$
$$9.14 \text{ cm} = x$$



$$\cos(51°38') = \frac{52}{x}$$
$$x\cos(51°38') = 52$$
$$x = \frac{52}{\cos(51°38')}$$
$$= 84.78 \text{ cm}$$

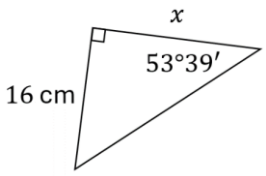
Calculate the value of x. Round to 2 decimal places.

a



19.44 cm

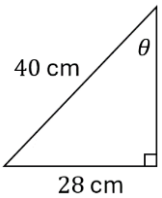
b



$x = 11.77$ cm

Example Calculate an unknown angle and round the result.

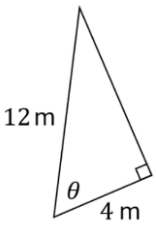
Calculate the value of θ and round to the nearest minute.



$$\sin\theta = \frac{28}{40}$$
$$\theta = \sin^{-1}\left(\frac{28}{40}\right)$$
$$\theta \approx 44.4270$$
$$\approx 44° 25' 37.21''$$
$$\approx 44° 26'$$

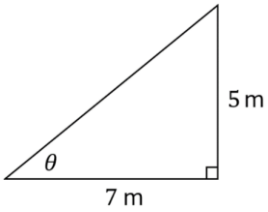
Calculate the value of θ and round to the nearest minute.

a



$\theta = 70°32'$

b



$\theta = 35°32'$

1 Evaluate these trigonometric ratios using the DMS button on the calculator. Answer to 4 d.p.

a cos 89°21′

0.0113

b tan 18°23′

0.3323

c tan 68°23′

2.5236

d 8.3 tan 58°51′

13.7320

e 15 sin 48.18°

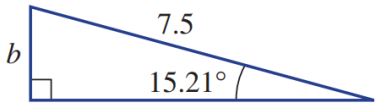
11.1786

f 12.3 cos 27°48′

10.8803

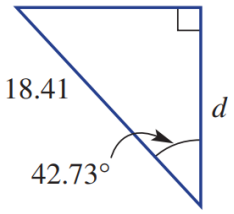
2 Calculate the value of the unknown side in each of the following. Answer to 2 d.p.

a



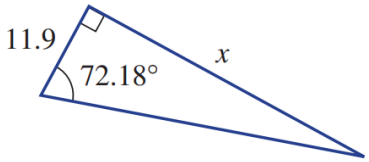
1.97

b



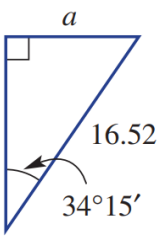
13.52

c



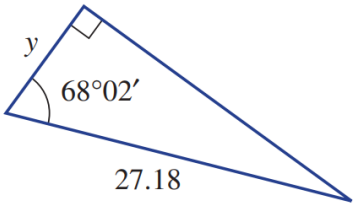
37.02

d



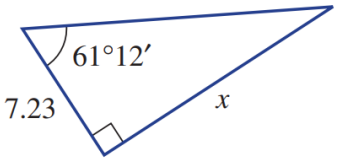
9.30

e



10.17

f



13.15

3 Calculate θ in each of the following, round to the nearest minute.

a $\sin^{-1}(0.7324)$

47° 5'

b $\cos^{-1}(0.9763)$

12° 30'

c $\tan^{-1}(0.3321)$

18° 22'

d $\tan^{-1}(1.235)$

51° 0'

e $\sin^{-1}(0.4126)$

24° 22'

f $\cos^{-1}(0.7462)$

41° 44'

g $\sin\theta = 0.5$

30° 0'

h $\tan\theta = 0.5774$

30° 0'

i $\tan\theta = 1.192$

50° 0'

j $\sin\theta = \frac{4}{7}$

34° 51'

k $\cos\theta = \frac{4}{5}$

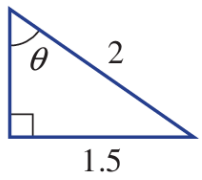
36° 52'

l $\tan\theta = \frac{8}{5}$

58° 0'

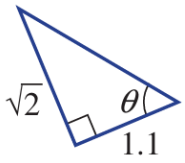
4 Calculate the value of θ in each of the following, round to the nearest minute.

a



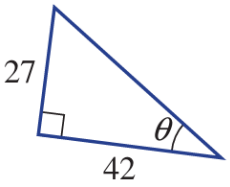
48°35′

b



52°7′

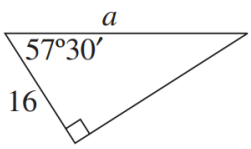
c



32°44′

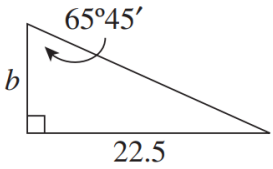
5 Calculate the value of the unknown value, to 2 d.p. for lengths and nearest minute for angles.

a



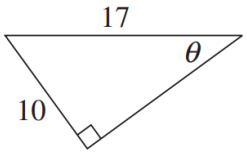
29.78

b



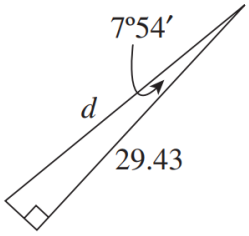
10.14

c



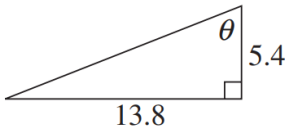
36°2′

d



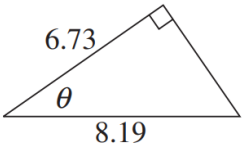
29.71

e



68°38′

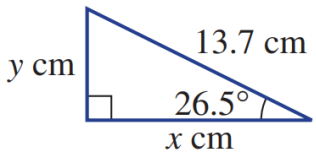
f



34°44′

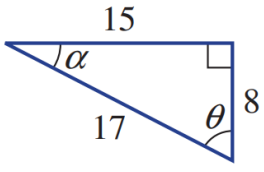
6 Find the values of the pronumerals, to 2 d.p. for lengths and nearest minute for angles.

a



x = 12.26 cm, y = 6.11 cm

b



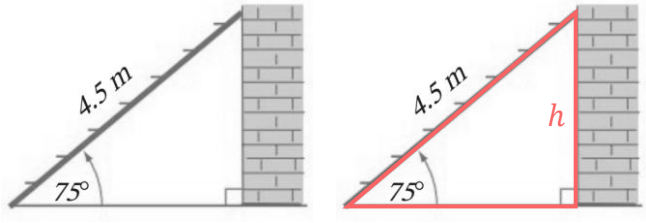
α = 28°4′ , θ = 61°56′

PROBLEM-SOLVING USING TRIGONOMETRY

1. Identify the right-angled triangle.
2. Label the unknown side or angle x or θ, if it isn't already.
3. Solve for the unknown using trigonometry.

Example Solve practical problems involving trigonometry.

Find how high the ladder reaches up the wall.



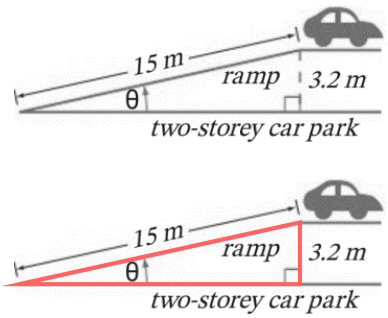
1. Triangle
2. Unknown (h)
3. Solve

$$\sin 75° = \frac{h}{4.5}$$

$$4.5 \sin 75° = h$$

$$x \approx 4.35 \text{ m}$$

Find the angle of the ramp.



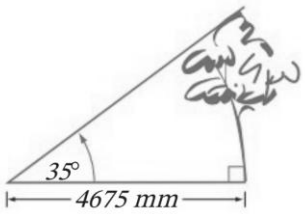
1. Triangle
2. Unknown (θ)
3. Solve

$$\sin \theta = \frac{3.2}{15}$$

$$\theta = \sin^{-1}\left(\frac{3.2}{15}\right)$$

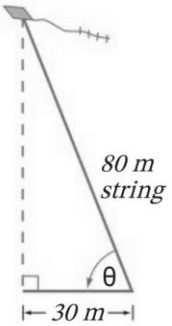
$$\theta \approx 12.32°$$

a Determine the height of this tree.



$x = 3273.47$ mm

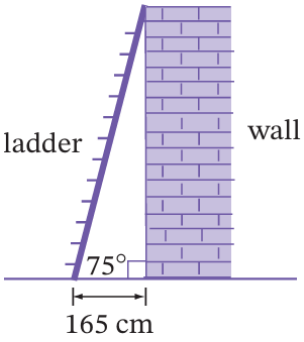
b Find the angle between the kite and the ground.



$\theta = 67.98°$

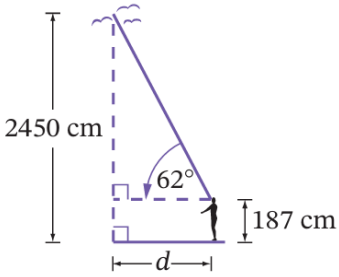
1 For each question, solve for the unknown value.

a Calculate the length of the ladder to the nearest cm.



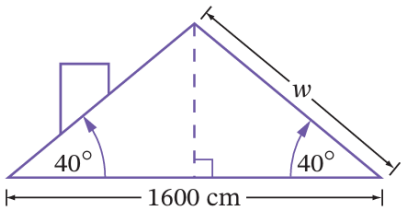
638 cm

b Calculate the horizontal distance of the person from the birds, to 1 d.p.



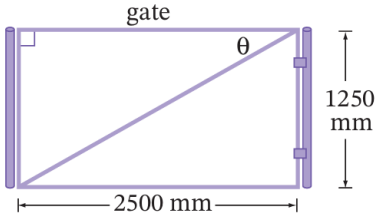
1203.3 cm

c The slant height of the roof, to 1 d.p.



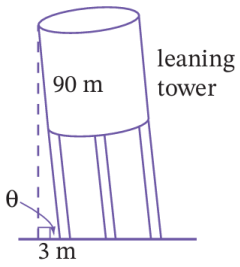
1044.3 cm

d The angle of the gate diagonal, to the nearest minute.



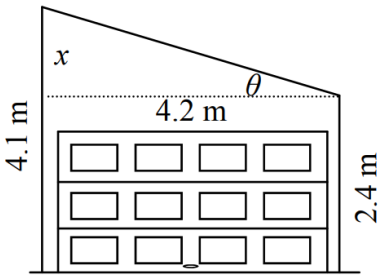
26°34′

e The angle of the leaning water tower, to the nearest minute.



88°5′

2 A garage has a roof shown in the diagram.



a Calculate the height difference between the two walls.

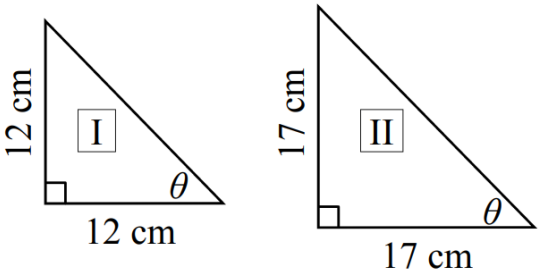
1.7 m

b Find the angle of the roof to the nearest minute.

22°2′

3 Toby constructed two isosceles right triangles.
He says that when the two short sides are the same the angle θ is always $45°$.

a Show he is correct, using trigonometry in the two triangles.



$\tan^1\left(\frac{12}{12}\right) = 45°, \quad \tan^{-1}\left(\frac{17}{17}\right) = 45°$

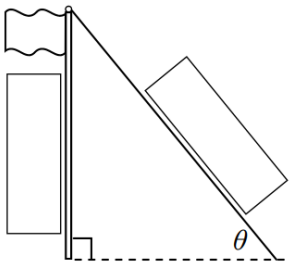
b Show he is correct, using properties of isosceles triangles and angle sum of a triangle.

$\theta + \theta + 90° = 180°$

$2\theta = 90°$

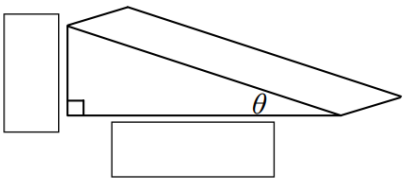
$\theta = 45°$

4 A 4.5 m flagpole has a 5.6 m wire anchored into the ground.
Calculate the angle that the wire makes with the ground to the nearest minute.



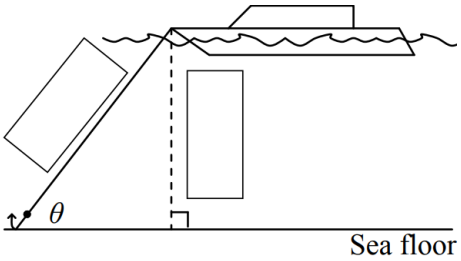
53°28′

5 A triangle skateboard ramp is 3.5 m long and 80 cm high.
Calculate the angle that it makes with the ground to the nearest minute.



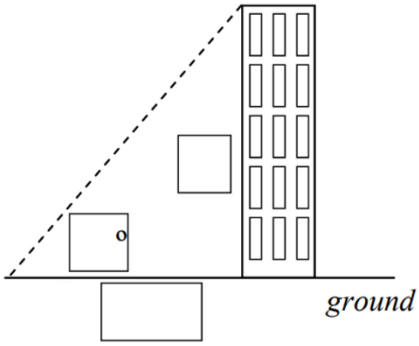
12° 5′

6 A fishing boat is anchored in 65 m deep water. The anchor rope is 90 m long.
Calculate the angle the rope makes with the sea floor, to the nearest minute.



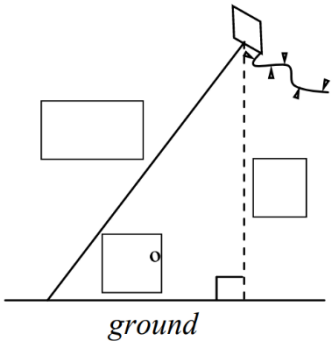
46°14′

7 Jess looks up at a building that is 72 m away. She looks up at an angle of 54°.
Complete the diagram then calculate the height of the building to 1 d.p.



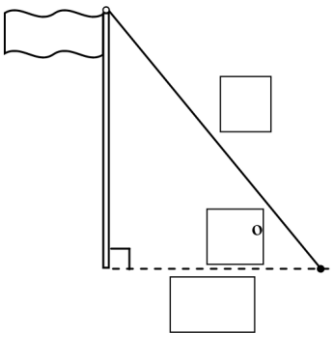
99.1 m

8 A kite string is 55 m long and is fully extended.
The string makes an angle of $64°$ with the ground.
Calculate how high above the ground it is to 1 d.p.



49.4 m

9 A flagpole has a wire that secures it to the ground 6 m from the pole.
The wire makes an angle of $51°$ with the ground.
Calculate the length of the wire.



9.5 m

10 For the following questions, you will need to draw your own diagrams.

a A ladder rests against a wall. The foot of the ladder is 355 cm from the wall and makes an angle of $63°20'$ with the ground.
Calculate the length of the ladder to the nearest cm.

791 cm

b A glider is flying at an altitude (height) of 1.5 km.
To land, it descends at an angle of $18°$ to the ground.
How far must the glider travel before landing? Answer rounded to the nearest 0.1 km.

4.9 km

c The entrance to the school is 60 cm above ground level.
A wheelchair ramp is to be built at an angle of $4°$ to the ground.
How far from the library must the ramp start? Answer to the nearest cm.

858 cm

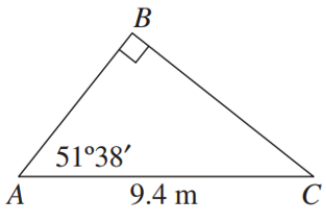
d An aircraft is at an altitude of 770 m. It is 1.5 km horizontally from the airport.
What angle must it descend by to land safely? Answer to the nearest minute.

$27°10'$

e A cannon on level ground shoots at a target, but the cannonball misses by 25 m.
If the target is 1 km away, what angle do they need to adjust the cannon?
Answer to the nearest minute.

$1°26'$

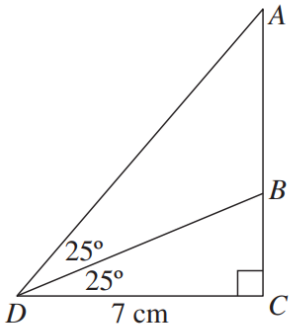
11 A tree snapped into two sections AB and BC in high winds and then fell. The section BA is inclined at $51°38'$ to the horizontal and AC is 9.4 metres long. Find the height of the original tree, in metres correct to one decimal place.



13.2 m

12

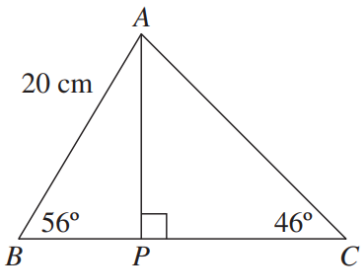
a



Show that $AC = 7\tan 50°$ and $BC = 7\tan 25°$, and hence find the length AB, correct to 1 mm.

51 mm

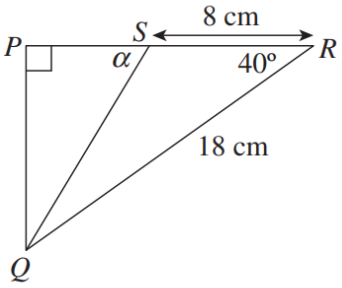
b



Show that $AP = 20\sin 56°$, and hence find the length of PC, giving your answer correct to 1 cm.

16 cm

c

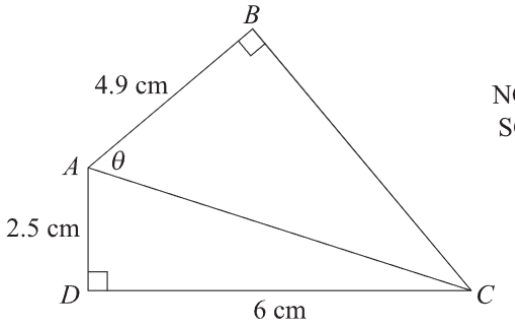


Show that $PR = 18\cos 40°$, find an expression for PQ, and hence find the angle α, correct to the nearest minute.

63°25′

13 **2019 HSC Standard 2 Band 4**

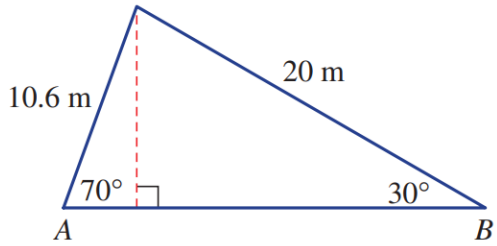
Calculate the size of θ. Answer to the nearest minute.



41°5′

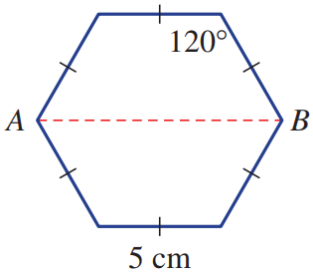
14 Find the length AB in these diagrams. Answer to 2 d.p.

a



20.95 m

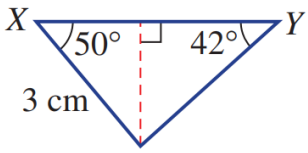
b



10 cm

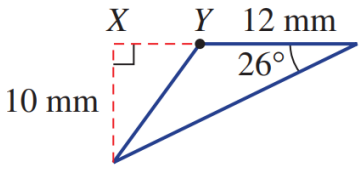
15 Find the length XY in these diagrams. Answer to 1 d.p.

a



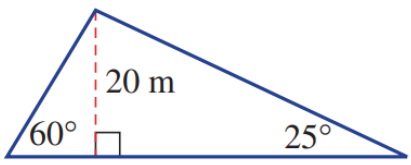
4.5 cm

b



8.5 mm

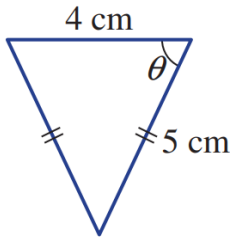
16 Find the perimeter of this triangle.



124.86 m

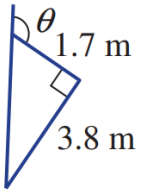
17 Find the angle θ in these diagrams. Answer to the nearest minute.

a



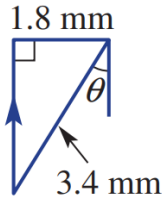
66°25′

b



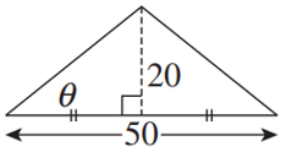
114°6′

c



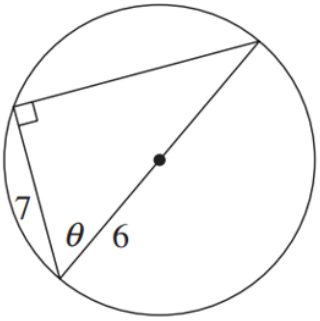
31°58′

d



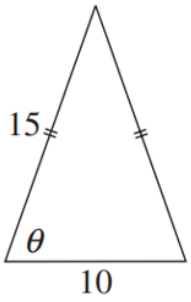
38°40′

e



54°19′

f



70°32′

18 It is given that θ is an acute angle and that $\tan\theta = \frac{\sqrt{5}}{2}$

a Draw a right-angled triangle to show this information.

b Use Pythagoras' theorem to find the length of the unknown side.

$\sqrt{9}$

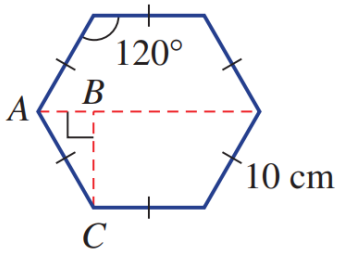
c Hence write the exact values of $\sin\theta$ and $\cos\theta$

$\sin\theta = \frac{\sqrt{5}}{\sqrt{9}}, \cos\theta = \frac{2}{\sqrt{9}}$

d Show that $\sin^2\theta + \cos^2\theta = 1$.

$\left(\frac{\sqrt{5}}{\sqrt{9}}\right)^2 + \left(\frac{2}{\sqrt{9}}\right)^2 = \frac{5}{9} + \frac{4}{9} = 1$

19 Consider a regular hexagon with internal angles of 120° and side lengths of 10 cm.



a Find the length of BC to 2 decimal places.

$\angle BAC = 60°$

$\sin 60° = \frac{BC}{10}$

$BC = 10 \sin 60° = 8.66$ cm

b Find the length of AB.

$\cos 60° = \frac{AB}{10}$

$AB = 10 \cos 60° = 5$ cm

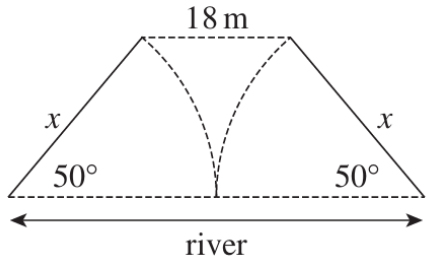
c Find the distance between the parallel sides to 1 decimal place.

$2BC = 20 \sin 60° = 17.3$ cm

d Find the distance between two opposite vertices.

$5 + 10 + 5 = 20$ cm

20 A bridge spans a river, and the two identical sections of the bridge, each of length x metres, can be raised to allow tall boats to pass. When the two sections are fully raised, they are each inclined at 50° to the horizontal, and there is an 18-metre gap between them, as shown in the diagram. Calculate the width of the river in metres, correct to one decimal place.



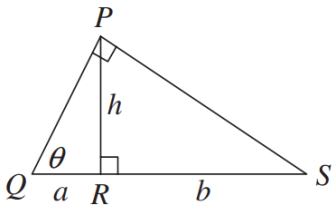
The river is $2x$ m long.

Consider the horizontal projections of the raised bridges: $2x - 2x \cos 50° = 18$

$2x = 50.4$

∴ The river is 50.4 metres long.

21 Consider the diagram.



a Explain why $\angle RPS = \theta$

$\angle QPR = 90° - \theta$, so $\angle RPS = \theta$

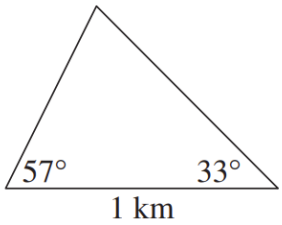
b Find two expressions for $\tan\theta$

$\frac{h}{a}$ and $\frac{b}{h}$

c Hence show $ab = h^2$

$\frac{h}{a} = \frac{b}{h}$, $\therefore h^2 = ab$

22 From the ends of a straight horizontal road 1 km long, a balloon directly above the road is observed to have angles of elevation of 57° and 33° respectively.
Find, correct to the nearest metre, the height of the balloon above the road.
(Hint: create two right-angled triangles by drawing a vertical line)



Draw a vertical line from the apex of the triangle to the base, this represents the height h.

This vertical line splits the base into two sections, which we will label a and b.

$\tan 57° = \frac{h}{a}$, $\tan 33° = \frac{h}{b}$

$a = \frac{h}{\tan 57°}$, $b = \frac{h}{\tan 33°}$

$a + b = 1$ km

$\frac{h}{\tan 57°} + \frac{h}{\tan 33°} = 1$

$h\left(\frac{1}{\tan 57°} + \frac{1}{\tan 33°}\right) = 1$

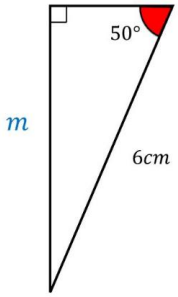
$h = \frac{1}{\left(\frac{1}{\tan 57°} + \frac{1}{\tan 33°}\right)} = 0.45677\ldots$

$\therefore$ 457 metres

CHECK YOUR UNDERSTANDING

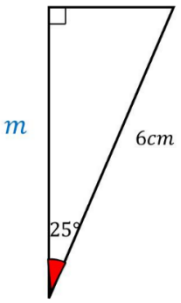
Finding unknown side

1 Calculate the value of m.



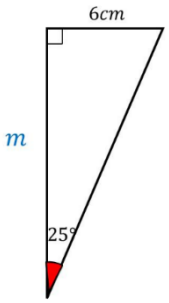
a $\sin(50) \times 6$ **b** $\sin(50 \times 6)$ **c** $\dfrac{6}{\sin(50)}$ **d** $\dfrac{\sin(50)}{6}$

2 Calculate the value of m.



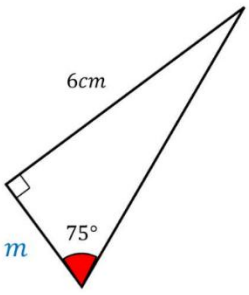
a $\cos(25) \times 6$ **b** $\cos(25 \times 6)$ **c** $\dfrac{6}{\cos(25)}$ **d** $\dfrac{\cos(25)}{6}$

3 Calculate the value of m.



a $\tan(25) \times 6$ **b** $\tan(25 \times 6)$ **c** $\dfrac{6}{\tan(25)}$ **d** $\dfrac{\tan(25)}{6}$

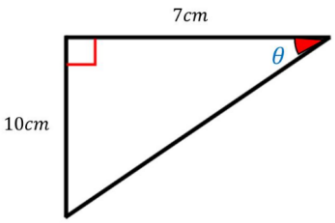
4 Calculate the value of m.



a $\tan(75) \times 6$ **b** $\tan(75 \times 6)$ **c** $\dfrac{6}{\tan(75)}$ **d** $\dfrac{\tan(75)}{6}$

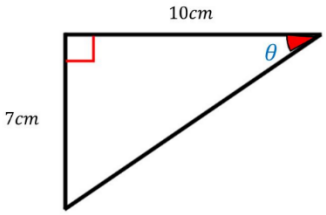
Finding unknown angle

1 Which ratio would you use to find θ?



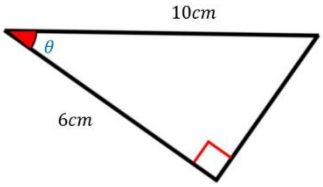
a Hypotenuse **b** Opposite **c** Adjacent **d** None of these

2 Which ratio would you use to find θ?



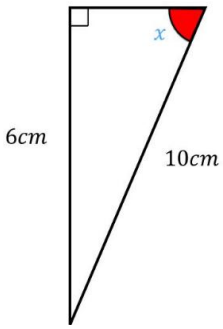
a Hypotenuse **b** Opposite **c** Adjacent **d** None of these

3 Which ratio would you use to find θ?



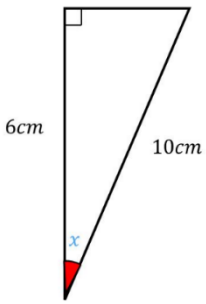
a Hypotenuse **b** Opposite **c** Adjacent **d** None of these

4 Find the value of x.



a $\sin\left(\frac{6}{10}\right)$ **b** $\sin\left(\frac{10}{6}\right)$ **c** $\sin^{-1}\left(\frac{6}{10}\right)$ **d** $\sin^{-1}\left(\frac{10}{6}\right)$

5 Find the value of x.



a $\cos\left(\frac{6}{10}\right)$ **b** $\cos\left(\frac{10}{6}\right)$ **c** $\cos^{-1}\left(\frac{6}{10}\right)$ **d** $\cos^{-1}\left(\frac{10}{6}\right)$