Student Name

Mathematics Stage 5 Paths

TRIGONOMETRY C

Book 2 Apply the sine, cosine and area rules to any triangle and solve related problems

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

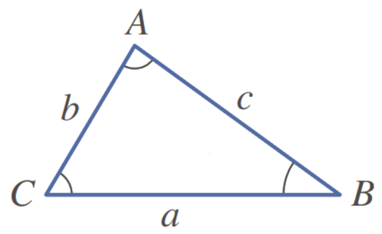
MA5-TRG-P-01 applies Pythagoras' theorem and trigonometry to solve 3-dimensional problems and applies the sine, cosine and area rules to solve 2-dimensional problems, including bearings

Apply the sine, cosine and area rules to any triangle and solve related problems

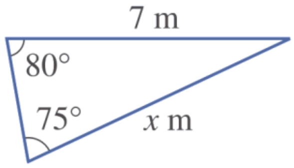
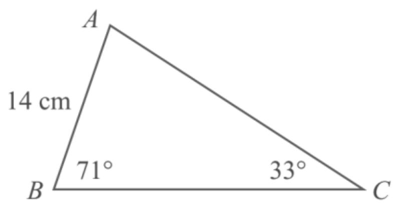
- Use graphing applications to verify the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and that the ratios of a side to the sine of the opposite angle is a constant
- Apply the sine rule in a given triangle ABC to find the value of an unknown side
- Apply the sine rule in a given triangle ABC to find the value of an unknown angle (ambiguous case excluded): $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Use graphing applications to verify the cosine rule $c^2 = a^2 + b^2 - 2ab\cos C$.
- Apply the cosine rule to find the unknown sides for a given triangle ABC.
- Rearrange the formula to deduce that $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ and use this to find an unknown angle
- Use graphing applications to verify the area rule $A = \frac{1}{2}ab\sin C$.
- Apply the formula $A = \frac{1}{2}ab\sin C$, where a and b are the sides that form angle C to find the area of a given triangle ABC
- Solve problems involving finding unknown angles or sides in triangles (excluding right-angled triangles) by selecting and applying the appropriate rule

SINE RULE

- In any triangle:
  - The **longest side** is opposite the **largest angle.**
  - The **shortest side** is opposite the **smallest angle.**
- The **sine rule** expresses this relationship:

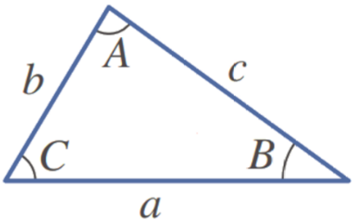
Sine Rule (for finding a side)	
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$	

Example Calculate a side length using sine rule.

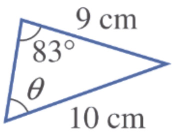
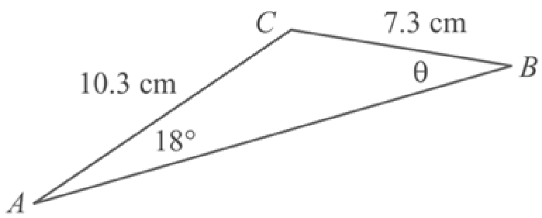
Worked Example	Your Turn
Solve x to 2 d.p. $\frac{x}{\sin 35°} = \frac{12}{\sin 48°}$ $\times \sin 35° \quad \times \sin 35°$ $x = \frac{12 \sin 35°}{\sin 48°}$ ≈ 9.26	Solve x to 2 d.p. $\frac{22}{\sin 45°} = \frac{x}{\sin 60°}$ 26.94
Calculate x to 1 d.p.  $\frac{a}{\sin A} = \frac{b}{\sin B}$ $\frac{x}{\sin 80°} = \frac{7}{\sin 75°}$ $\times \sin 80° \quad \times \sin 80°$ $x = \frac{7 \sin 80°}{\sin 75°}$ ≈ 7.1 m	Calculate AC to 1 d.p.  24.3 cm

Sine Rule (for finding an angle)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Example Calculate an angle using sine rule.

Worked Example	Your Turn
Calculate θ to the nearest minute. $$\frac{\sin\theta}{11} = \frac{\sin 30°}{17}$$ $$\sin\theta = \frac{11 \times \sin 30°}{17}$$ $\times 11 \quad \times 11$ $$\theta = \sin^{-1}\left(\frac{11 \sin 30°}{17}\right)$$ $$= 18°53'$$	Calculate θ to the nearest minute. $$\frac{\sin\theta}{6} = \frac{\sin 77°}{12}$$ 29°9′
Calculate θ to the nearest minute.  $$\frac{\sin A}{a} = \frac{\sin B}{b}$$ $$\frac{\sin\theta}{9} = \frac{\sin 83°}{10}$$ $\times 9 \quad \times 9$ $$\sin\theta = \frac{9 \sin 83°}{10}$$ $$\theta = \sin^{-1}\left(\frac{9 \sin 83°}{10}\right)$$ $$\approx 63°17'$$	Calculate θ to the nearest minute.  25°51′

1 Solve each equation for the unknown, correct to 2 decimal places.

a

$$\frac{a}{\sin 47^\circ} = \frac{2}{\sin 51^\circ}$$

1.88

b

$$\frac{b}{\sin 31^\circ} = \frac{7}{\sin 84^\circ}$$

3.63

c

$$\frac{5}{\sin 63^\circ} = \frac{c}{\sin 27^\circ}$$

2.55

2 Solve each equation for the unknown, correct to the nearest minute.

a

$$\frac{\sin 38^\circ}{4} = \frac{\sin \theta}{5}$$

50° 19'

b

$$\frac{\sin 51^\circ}{11} = \frac{\sin \theta}{9}$$

39° 29'

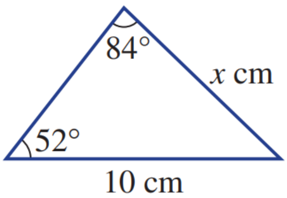
c

$$\frac{12}{\sin \theta} = \frac{18}{\sin 47^\circ}$$

29° 11'

3 Find the value of the unknown in these triangles, correct to 1 decimal place.

a



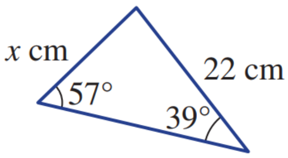
$$\frac{x}{\sin 52^\circ} = \_\_\_\_\_\_$$

$x = \ldots\ldots\ldots\ldots$

$= \ldots\ldots\ldots\ldots$

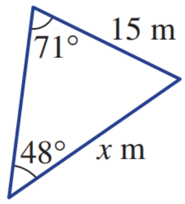
7.9

b



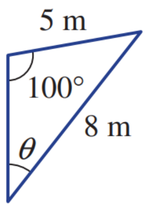
16.5

c



19.1

d



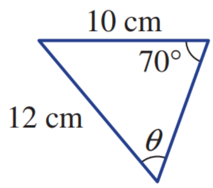
$$\frac{\sin \theta}{5} = \_\_\_\_\_\_$$

$\sin \theta = \ldots\ldots\ldots\ldots$

$\theta = \ldots\ldots\ldots\ldots$

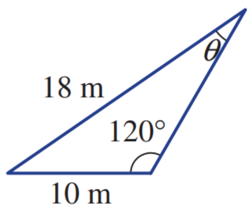
38.0°

e



51.5°

f

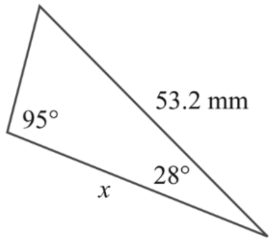


28.8°

4 Calculate the side length x using sine rule to 1 d.p.

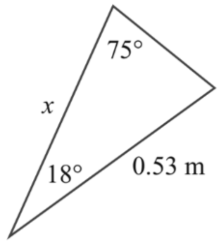
(Hint: Angle sum of a triangle is $180°$)

a



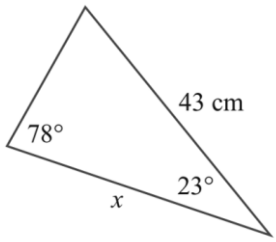
44.8 mm

b



0.5 m

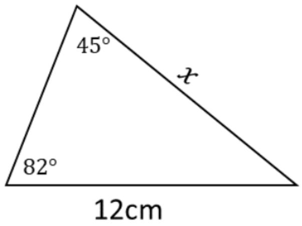
c



43.2 cm

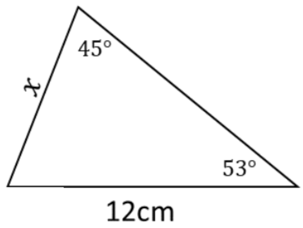
5 Calculate the side length x using sine rule. Round your answer to 1 d.p.

a



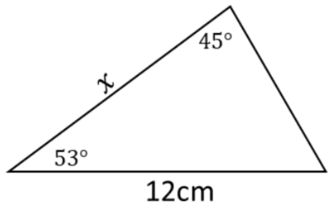
16.8 cm

b



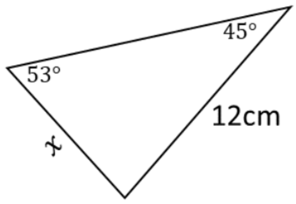
13.6 cm

c



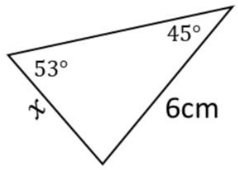
16.8 cm

d



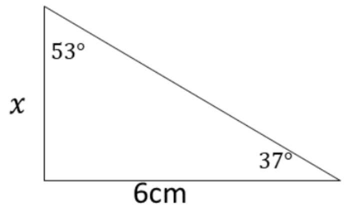
10.6 cm

e



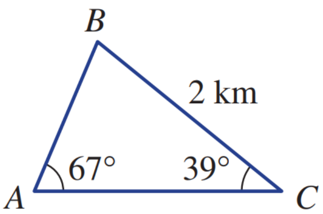
5.3 cm

f



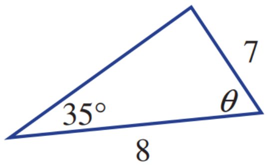
4.5 cm

6 Three markers, A, B and C, map out the course for a cross-country race.
Find the total distance of the race. Round your answer to 2 decimal places.



AB = 1.367 km, AC = 2.089 km, total length: 5.46 km

7 Ryan trying to solve this problem. He wrote $\frac{\sin 35°}{7} = \frac{\sin\theta}{8}$.



a Explain why he is incorrect.

θ is not opposite the 8

b Solve for θ by finding the missing information first.

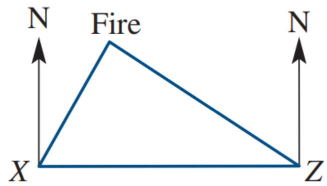
Find top angle using sine rule, $\phi = 40°58'$

$\theta = 180 - 35 - 30°7' = 104°2'$

8 Sienna was located at X and saw a fire in the direction N15°E.
Seven kilometres to the east of X at Z, Dylan saw the fire in the direction N50°W.

a How far is X from the fire? Answer in km to 1 decimal place.

b How far is Z from the fire? Answer in km to 1 decimal place.



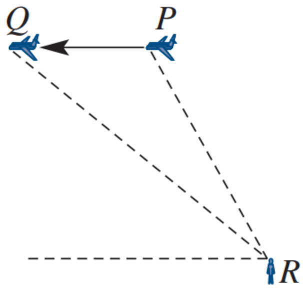
a) 5.0 km

b) 7.5 km

9 A man at R measures the angle of elevation to a plane at P as 45°15′.
The plane travels 15 km from P to Q.
The man then measures the angle of elevation to the plane at Q as 35°42′.

a What is the distance of R to P? Answer in km to one decimal place.

b What is the distance of R to Q? Answer in km to one decimal place.



a) 52.8 km

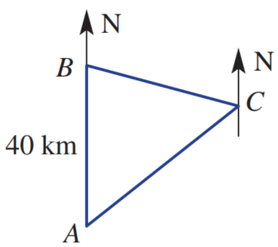
b) 64.2 km

10 A ship heads due north from point A for 40 km to point B, and then heads on a bearing of 100° to point C. The bearing from C to A is 240°.

a Find $\angle ABC$.

b Hence find distance AC, to 1 d.p.

c Hence find distance BC, to 1 d.p.



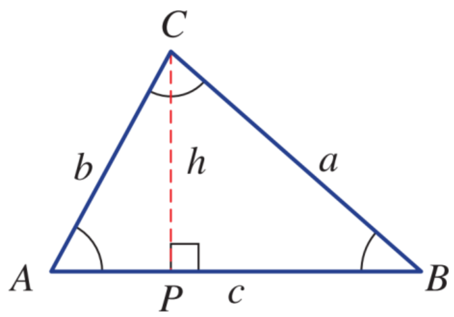
∠ABC = 80°

AC = 61.3 km

BC = 53.9 km

Investigation Proof of sine rule.

Consider the non-right-angled triangle shown.



By looking at right-angled triangle ΔCPB,

$$\sin B = \frac{h}{a}$$
$$\therefore h = a \sin B$$

Similarly, from ΔCPA,

$$\sin A = \frac{h}{b}$$
$$\therefore h = b \sin A$$

By equating our two expressions for h:

$$a \sin B = b \sin A$$
$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B}$$

By constructing a perpendicular line from vertex B to AC and following the steps of the proof above, prove that:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

....

....

....

....

....

....

Hence explain why:

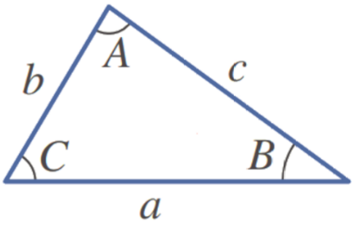
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

....

....

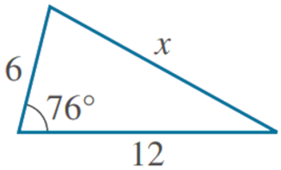
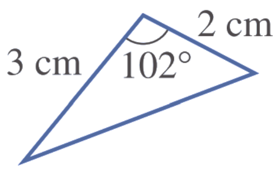
COSINE RULE

- The **cosine rule** relates one angle and three sides of any triangle.

Cosine Rule (for finding a side)	
$c^2 = a^2 + b^2 - 2ab\cos C$	

- Always let the side c or angle C be the side or angle you want to find.

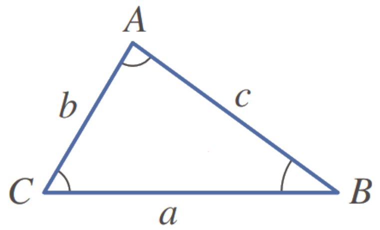
Example Calculate a side length using cosine rule.

Worked Example	Your Turn
Calculate x to 2 d.p. $x^2 = 7^2 + 14^2 - 2(7)(14)\cos 48°$ $x = \sqrt{7^2 + 14^2 - 2(7)(14)\cos 48°}$ $= 10.67$	Calculate x to 2 d.p. $x^2 = 3^2 + 5^2 - 2(3)(5)\cos 64°$ 4.57
Calculate the length of x to 2 d.p.  $c^2 = a^2 + b^2 - 2ab\cos C$ $x^2 = 6^2 + 12^2 - 2(6)(12)\cos 76°$ $x = \sqrt{6^2 + 12^2 - 2(6)(12)\cos 76°}$ $= 12.04$	Calculate the length of the third side to 2 d.p.  3.94 cm

- We can rearrange the cosine rule to find an unknown angle.

Cosine Rule (for finding an angle)

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



Example Calculate an angle using cosine rule.

Worked Example

Calculate θ to the nearest minute.

$$\cos\theta = \frac{12^2 + 5^2 - 8^2}{2(12)(5)}$$

$$\theta = \cos^{-1}\left(\frac{12^2 + 5^2 - 8^2}{2(12)(5)}\right)$$

$$= 28°57'$$

Your Turn

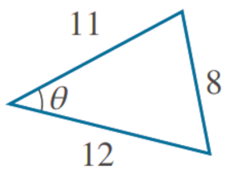
Calculate θ to the nearest minute.

$$\cos\theta = \frac{11^2 + 5^2 - 7^2}{2(11)(5)}$$

$28°8'$

Worked Example

Calculate θ to the nearest minute.



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

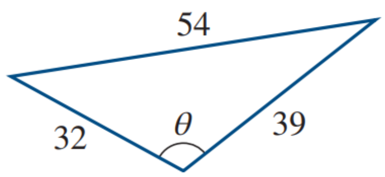
$$\cos\theta = \frac{11^2 + 12^2 - 8^2}{2(11)(12)}$$

$$\theta = \cos^{-1}\left(\frac{11^2 + 12^2 - 8^2}{2(11)(12)}\right)$$

$$\theta = 40°25'$$

Your Turn

Calculate θ to the nearest minute.



$98°33'$

1 Solve for the unknown value in these equations. Answer to 2 d.p. or to the nearest minute.

a $x^2 = 4^2 + 7^2 - 2(4)(7)\cos 120°$

9.64

b $x^2 = 1.5^2 + 1.1^2 - 2(1.5)(1.1)\cos 70°$

1.53

c

$$\cos\theta = \frac{7^2 + 6^2 - 9^2}{2(7)(6)}$$

$87°16'$

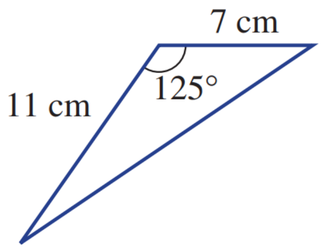
d

$$\cos\theta = \frac{21^2 + 30^2 - 18^2}{2(21)(30)}$$

$36°11'$

2 Find the value of the unknown in these triangles, correct to 2 decimal places.

a



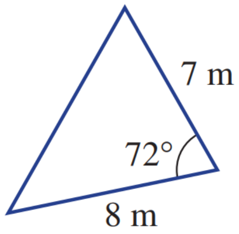
$x^2 = (\quad)^2 + (\quad)^2 - 2(\quad)(\quad)\cos(\quad)$

$x = \sqrt{\qquad\qquad\qquad}$

$x =$ ....

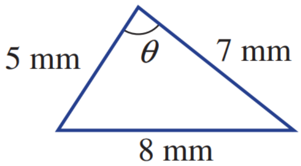
16.07 cm

b



8.85 m

c



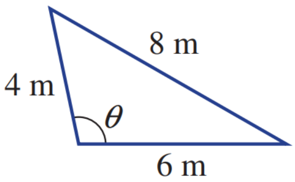
$$\cos\theta = \frac{(\quad)^2 + (\quad)^2 - (\quad)^2}{2(\quad)(\quad)}$$

$$\theta = \cos^{-1}\left(\frac{(\quad)^2 + (\quad)^2 - (\quad)^2}{2(\quad)(\quad)}\right)$$

$\theta =$ ....

81.79°

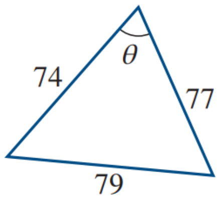
d



104.48°

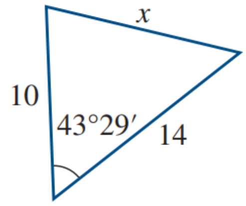
3 Calculate the value x. Answer to 2 d.p. for lengths and the nearest degree for angles.

a



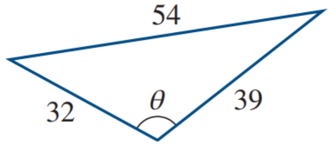
63°

b



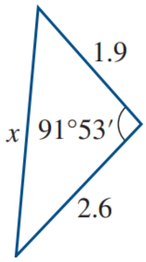
9.64

c



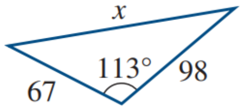
99°

d



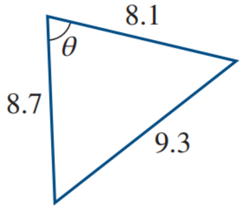
3.27

e



138.65

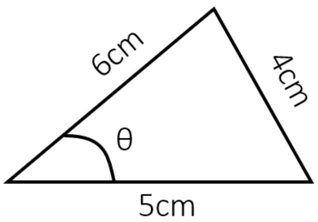
f



67°

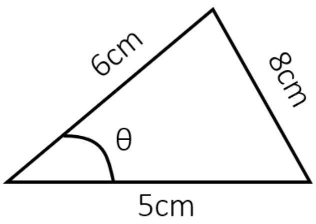
4 Calculate the angle θ using cosine rule. Answer to 2 d.p.

a



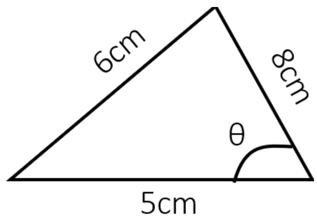
41.40°

b



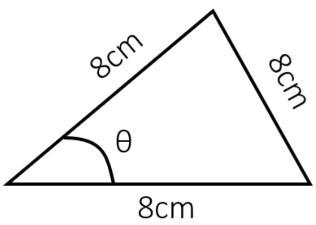
92.87°

c



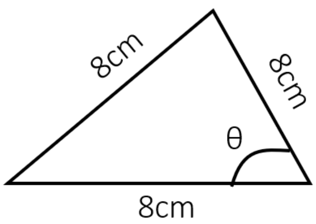
48.51°

d



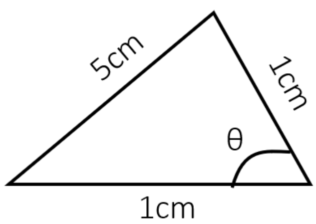
60°

e



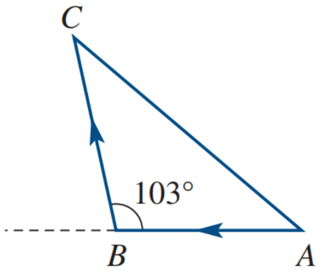
60°

f



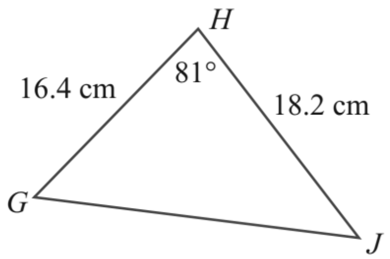
impossible triangle

5 Ruby drives along a 14 km track from point A due west to a point B.
She then turns and travels 19 km to point C.
Calculate the distance from her starting point. Answer correct to one decimal place.



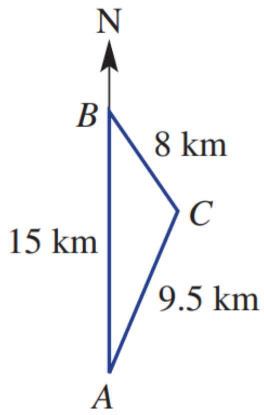
26.0 km

6 Calculate the perimeter of this triangle to 1 d.p.



GJ = 22.5 cm, Perimeter = 57.1 cm

7 Three camp sites, A, B and C, are shown.
If campsite B is due north of camp site A, find the following, correct to 1 decimal place.



a The bearing of campsite C from campsite B

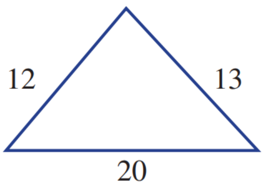
145.9°

b The bearing of campsite A from campsite C

208.2°

8 Consider the triangle shown. Answer to the nearest degree.

a Use cosine rule to find the size of the largest angle.



Largest angle opposite longest side: 106°

b Use sine rule to find the size of the smallest angle.

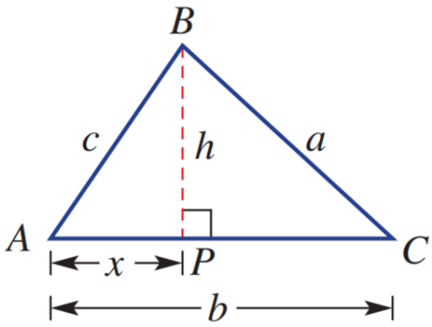
Smallest angle opposite shortest side: 35°

Investigation Proof of cosine rule.

Consider the non-right-angled triangle shown.

Write an expression for length CP

………………………………………………………………



Use Pythagoras' Theorem in ΔABP to write an equation relating c, x, and h

………………………………………………………………

Use Pythagoras' Theorem in ΔCPB to write an equation relating b, a, x, and h

………………………………………………………………

By making h^2 the subject of your two equations above, eliminate h and simplify your result.

………

………

………

………

………

………

Use $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ to write an expression for $\cos A$.

………

Combine your equations from the two parts above to prove cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

………

………

………

………

………

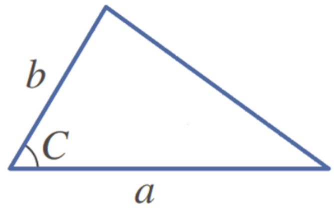
………

AREA OF A TRIANGLE USING TRIGONOMETRY

Side-Angle-Side Area Formula

$$A = \frac{1}{2}ab\sin C$$

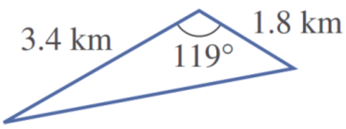
a and b are the sides that form angle C (the included angle).



Example Calculate area of a triangle using sine rule.

Worked Example

Calculate the area to 2 d.p.



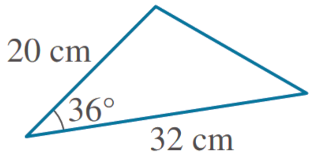
$$A = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(3.4)(1.8)\sin 119°$$

$$= 2.68 \text{ km}^2$$

Your Turn

Calculate the area to 2 d.p.

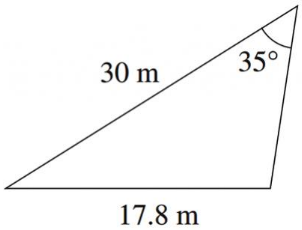


188.09 cm^2

Consider the triangle to the right.

Explain why it is NOT possible to use $A = \frac{1}{2}ab\sin C$

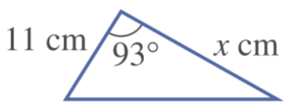
....



Example Calculate side length of a triangle given area.

Worked Example

Calculate x to 2 d.p. given $A = 70$ cm^2



$$A = \frac{1}{2}ab\sin C$$

$$70 = \frac{1}{2}(11)x\sin 93°$$

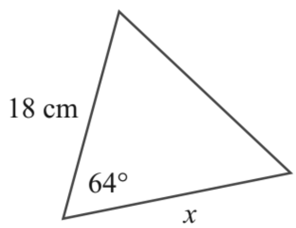
$$\div\frac{1}{2}(11)\sin 93° \qquad \div\frac{1}{2}(11)\sin 93°$$

$$\frac{70}{\frac{1}{2}(11)\sin 93°} = x$$

$$x = 12.74 \text{ cm}$$

Your Turn

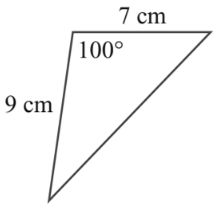
Calculate x to 2 d.p. given $A = 230$ cm^2



28.43 cm

1 Calculate the area of each triangle, correct to one decimal place.

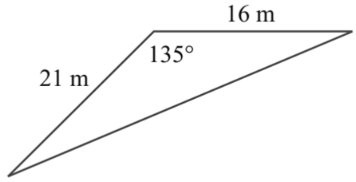
a



$$A = \frac{1}{2}(\quad)(\quad)\sin(\quad)$$
$$= \dots\dots\dots\dots\dots\dots\dots$$

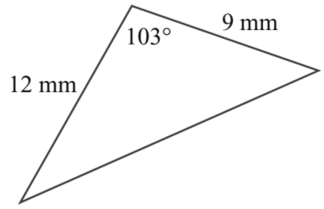
31.0 cm²

b



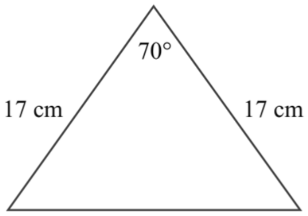
118.8 m²

c



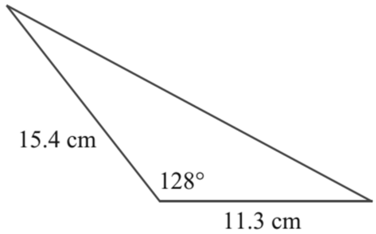
52.6 mm²

d



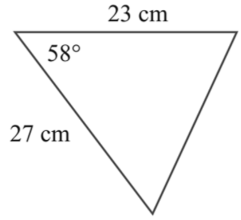
135.8 cm²

e



68.6 cm²

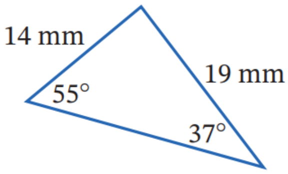
f



263.3 cm²

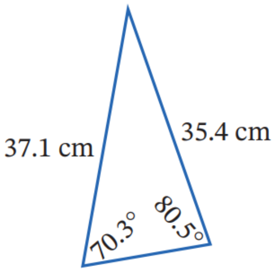
2 Find the area of each triangle. Answer to 1 d.p.

a



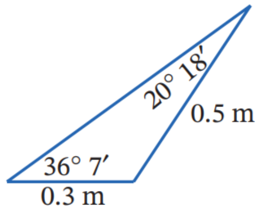
132.9 mm²

b



320.4 cm²

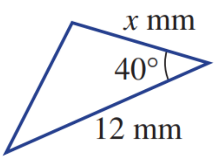
c



0.1 m²

3 Calculate the value of x correct to one decimal place.

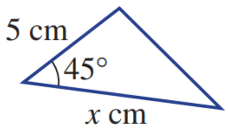
a



Area = 22 mm²

x = 5.7

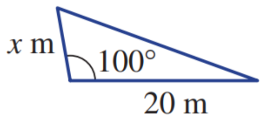
b



Area = 14 cm²

x = 7.9

c

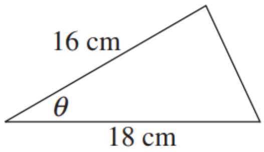


Area = 90 m²

x = 9.1

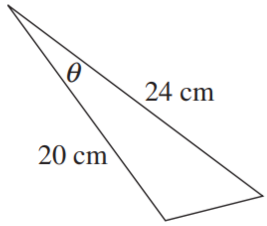
4 Calculate the value of θ given each triangle has area 72 cm^2. Answer to the nearest minute.

a



30°

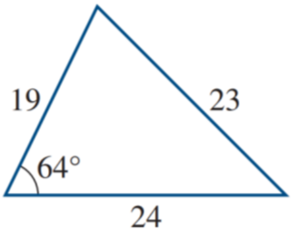
b



17°27′

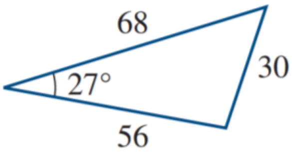
5 Find the area of these triangles.

a



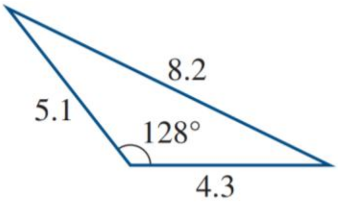
204.9 sq units

b



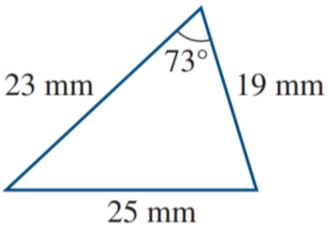
864.4 sq units

c



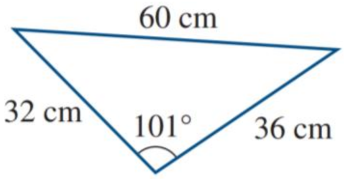
8.6 sq units

d



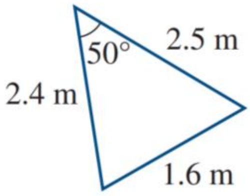
209.0 mm^2

e



565.4 cm^2

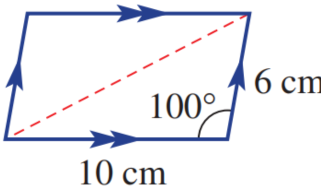
f



2.3 m^2

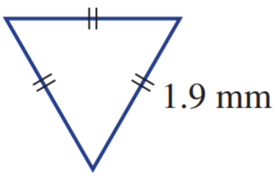
6 Find the area of these shapes.

a



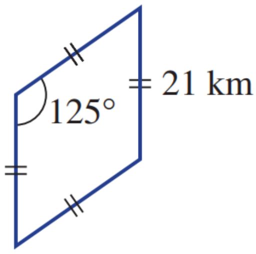
59.09 cm^2

b



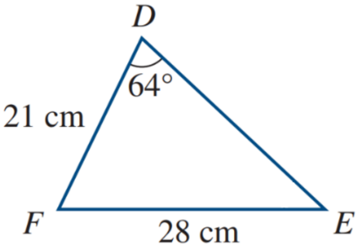
1.56 mm^2

c



361.25 km^2

7 Consider the triangle.



a Explain why we cannot find the area of the triangle.

The angle is not in between two sides.

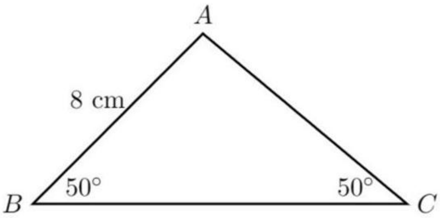
b Using sine rule, find $\angle DEF$ to the nearest degree.

43°

c Hence calculate the area to the nearest square cm.

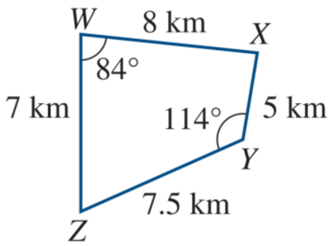
281 cm^2

8 Calculate the area of this triangle to 1 d.p.



31.5 cm^2

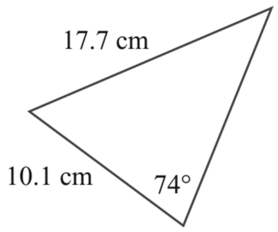
9 A drawing of a farmer's quadrilateral property is shown. By considering ΔWXZ and ΔXYZ, calculate the area of the property in square km correct to one decimal place.



45.0 km^2

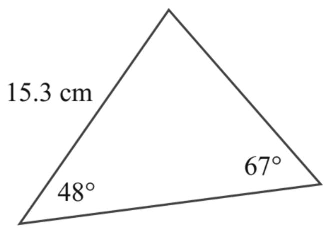
10 Calculate the area of these triangles. Answer in square cm to 1 d.p.
(Hint: Use sine rule and angle sum of a triangle)

a



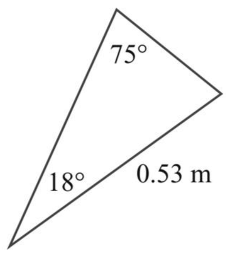
85.4 cm^2

b



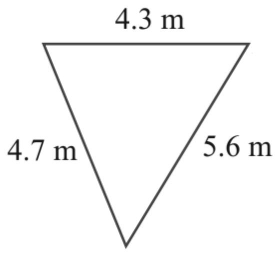
85.6 cm^2

c



448.7 cm^2

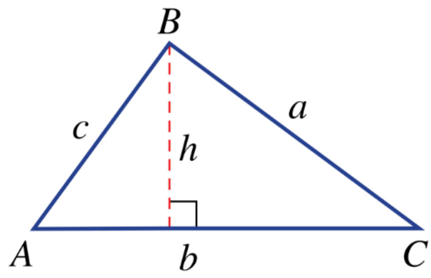
11 Use cosine rule to find an angle, and hence calculate the area of this triangle to 1 d.p.



9.8 m^2

Investigation Proof of SAS Area Formula.

Consider the non-right-angled triangle shown.



The area of a triangle (from Stage 4) is $A = \frac{1}{2}\,bh$

Write the equation for $\sin C$

………………………………………………………

Make h the subject of the equation you wrote above.

………………………………………………………

By combining $A = \frac{1}{2}bh$ and $h = a\sin C$, derive the side-angle-side formula for triangle area.

…………………………………………………………………………………

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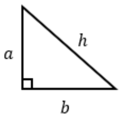
SELECTING AN APPROPRIATE METHOD

Right-Angled Triangle

3 sides

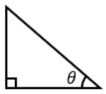


Pythagoras

$h = \sqrt{a^2 + b^2}$

$a = \sqrt{h^2 - b^2}$

2 sides 1 angle



Trig Ratios

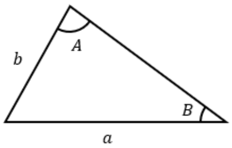
$\sin\theta = \frac{O}{H}$

$\cos\theta = \frac{A}{H}$

$\tan\theta = \frac{O}{A}$

Non-Right-Angled Triangle

2 sides 2 angles

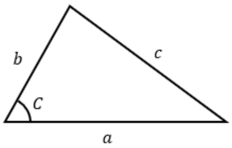


Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

3 sides 1 angle

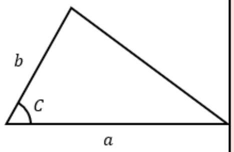


Cosine Rule

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Area



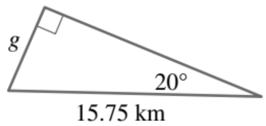
Side-Angle-Side Area Formula

$A = \frac{1}{2}ab\sin C$

Example Select an appropriate method to solve a triangle problem.

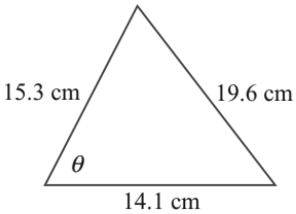
Identify the method you would use to find the unknown in each question.

a



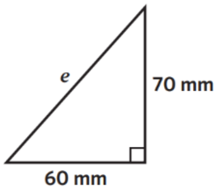
Trig ratio (sin)

b



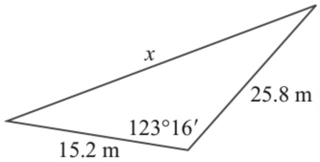
cosine rule

c



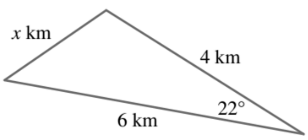
Pythagoras

d



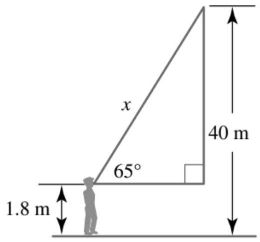
cosine rule

e



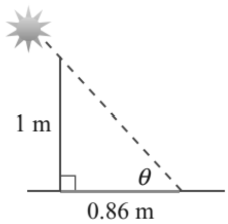
sine rule

f



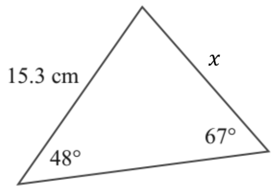
Trig ratio (sin)

g



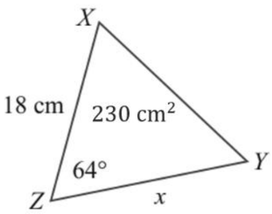
trig ratio (tan)

h



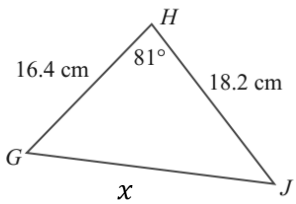
sine rule

i



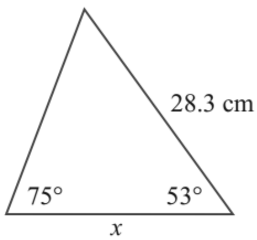
SAS area formula

j



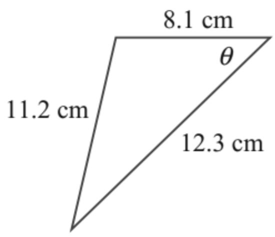
cosine rule

k



sine rule

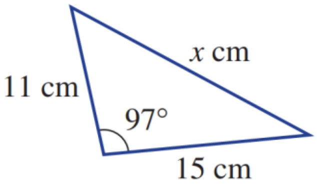
l



cosine rule

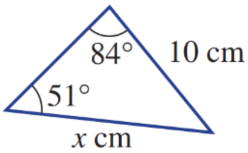
1 Decide whether cosine rule or sine rule should be used to find x. Do not solve for x.

a



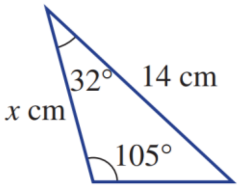
cosine

b



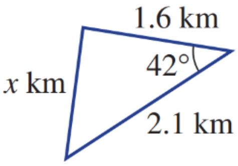
sine

c



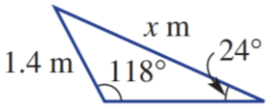
sine

d



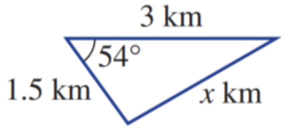
cosine

e



sine

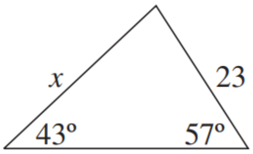
f



cosine

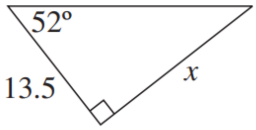
2 Use an appropriate method to find the unknown. Give lengths to 1 d.p. and angles to nearest degree.

a



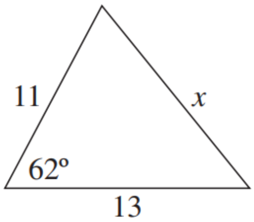
28.3

b



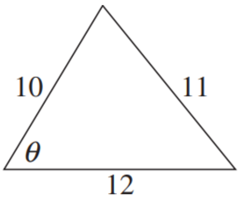
17.3

c



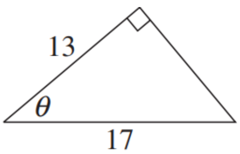
12.5

d



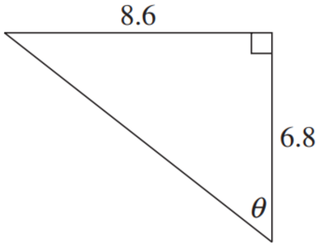
59°

e



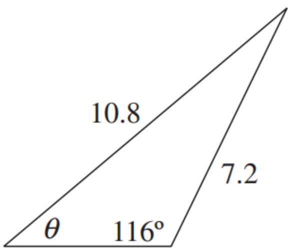
55°

f



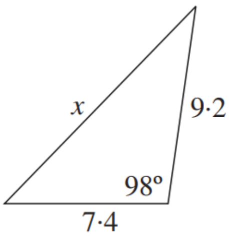
40°

g



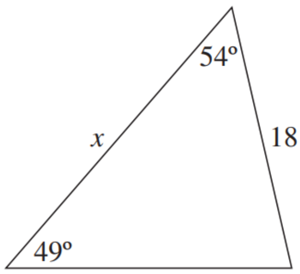
37°

h



12.6

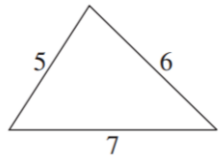
i



23.2

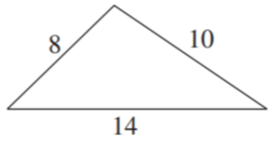
3

a Find the smallest angle, correct to the nearest minute.



$44°25'$

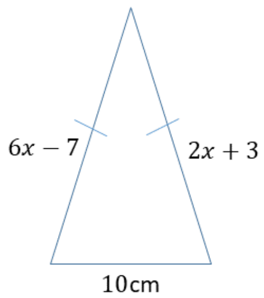
b Find the largest angle, correct to the nearest minute.



$101°32'$

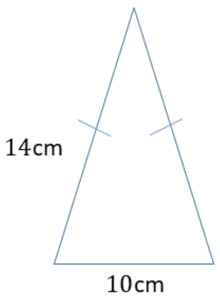
4

a Calculate the perimeter.



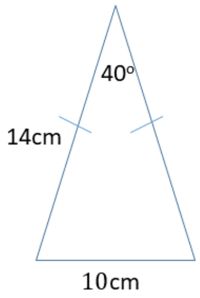
26 cm

b Calculate the height in exact form.



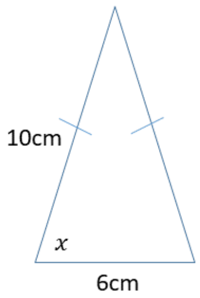
$\sqrt{171}$ cm

c Calculate the area to the nearest whole.



63 cm

d Calculate the angle to 1 d.p.



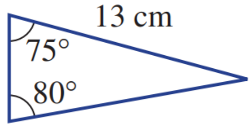
$72.5°$

5 A triangle has sides of 21 m, 17 m, and 10 m.
Find the size of the largest angle, correct to the nearest degree.

$99°$

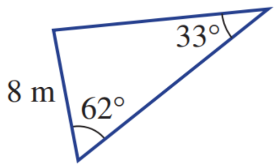
6 Calculate the area of these triangles by using either sine rule or cosine rule to find the required information. Give your answer to 2 d.p.

a



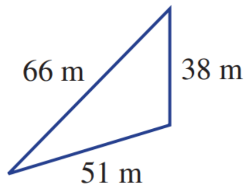
35.03 cm²

b



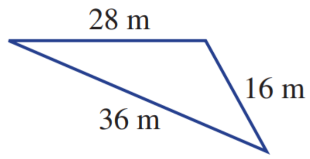
51.68 m²

c



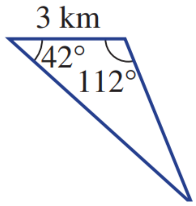
965.88 m²

d



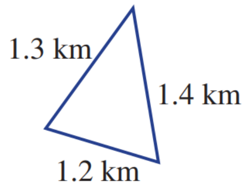
214.66 m²

e



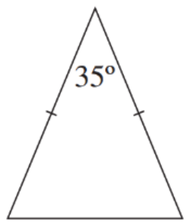
6.37 km²

f



0.72 km²

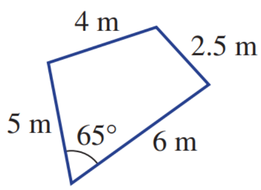
7 The area of this isosceles triangle is 35 cm². Find the length of the equal sides to 2 d.p.



11.05 cm

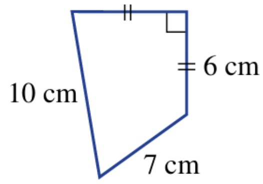
8 Calculate the area of these shapes. Round to 1 d.p.

a



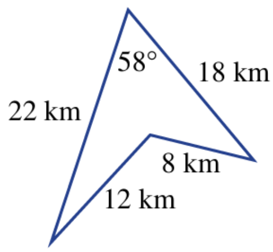
17.3 m²

b



48 cm²

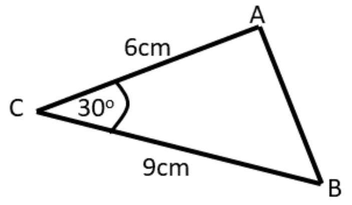
c (hint: use subtraction)



124.8 km²

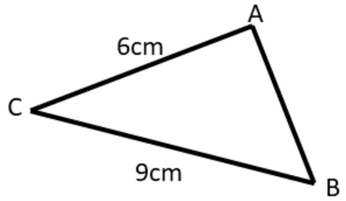
9

a Find the length of AB to 2 d.p.



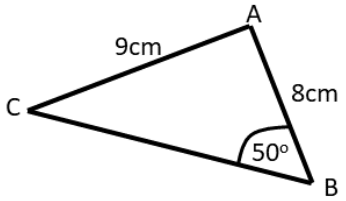
4.84 cm

b The area of the triangle is 9.23 cm^2. Find the size of $\angle ACB$ to the nearest degree.



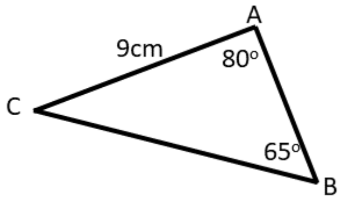
20°

c Find the size of angle $\angle BAC$ to 1 d.p.



87.0°

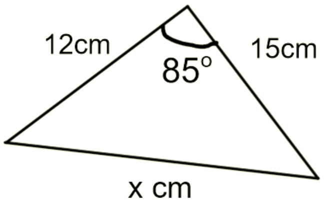
d Find the perimeter of the triangle to 2 d.p.



24.48 cm

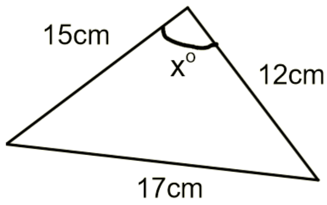
10 Find the unknown value, round lengths to 2 d.p. and angles to the nearest degree.

a



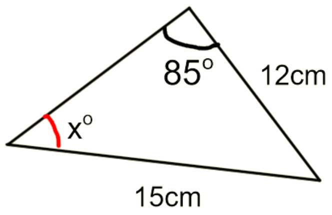
18.37 cm

b



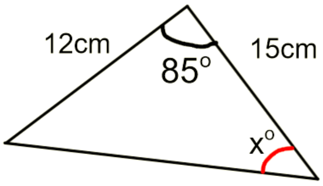
77°

c



53°

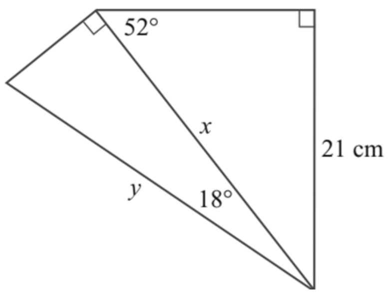
d



41°

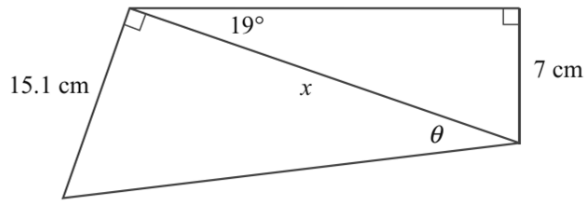
11 Determine the value of the variables to one d.p. for lengths and the nearest minute for angles.

a



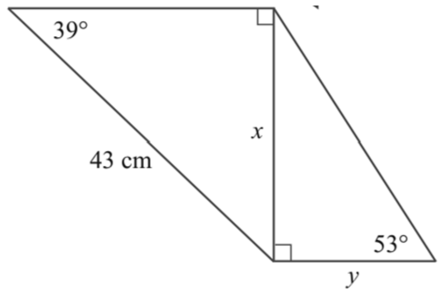
x = 26.6 cm, y = 28.0 cm

b



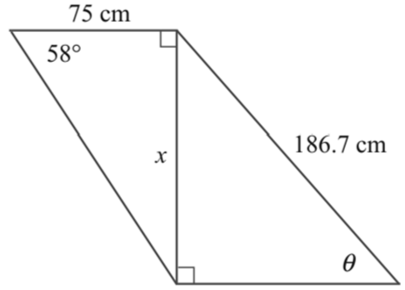
x = 21.5 cm, θ = 35° 5′

c



x = 27.1 cm, y = 20.4 cm

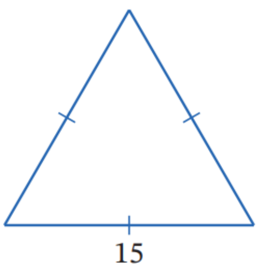
d



x = 120.0 cm, θ = 40° 0′

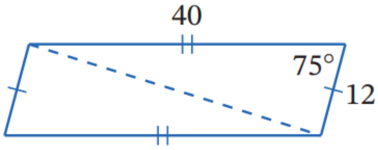
12 Calculate the area of each shape correct to one decimal place. All measurements are in metres.

a Equilateral triangle



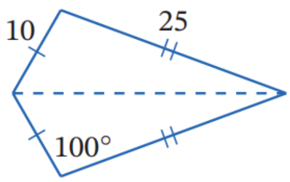
97.4 m^2

b Parallelogram



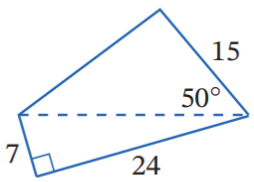
463.6 m^2

c Kite



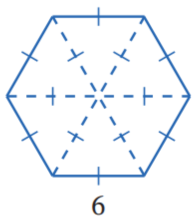
246.2 m^2

d Quadrilateral



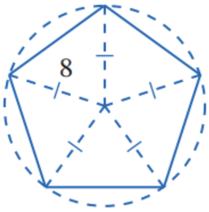
227.6 m^2

e Regular hexagon



93.5 m^2

f Regular pentagon inscribed in a circle



152.2 m^2

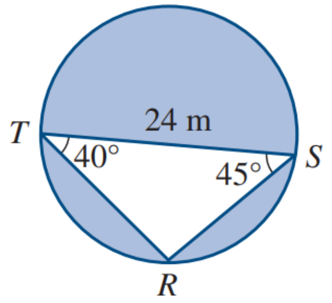
MIXED MULTISTEP TRIGONOMETRY PROBLEMS

FOUNDATION

1 Andrew plans to build a triangular flower bed in the lower part of a circular garden.
The length of TS is 24m, $\angle$TSR is 45° and $\triangle$RTS is 40°.

a What is the size of $\angle$SRT?

b Find the length of TR. Answer correct to the nearest metre.



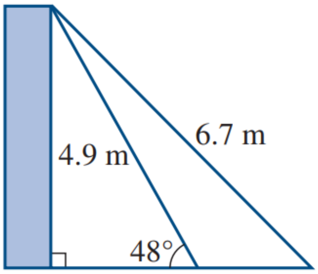
a) 95° b) 17 m

2 Two ladders are the same distance up the wall.
The shorter ladder is 4.9m long and makes an angle of 48° with the ground.
The longer ladder is 6.7m long.

a How far up the wall do the ladders reach?
Answer correct to one decimal place.

b What is the angle, to the nearest degree, that the longer ladder makes with the ground?

c Find the distance between the ladders on the ground. Answer to one decimal place.



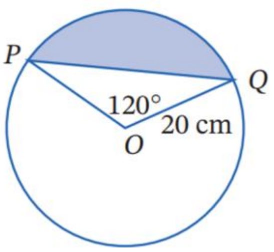
a) 3.6 m, b) 33° c) 2.3 m

3 O is the centre of a circle of radius 20 cm. Calculate, correct to one decimal place, the area of:

a Sector OPQ.

b Triangle OPQ.

c The shaded segment.



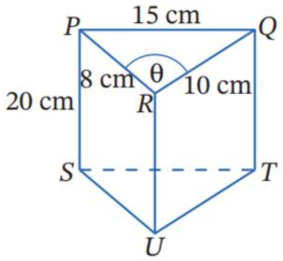
a) 418.9 cm^2 b) 173.2 cm^2 c) 245.7 cm^2

4 A triangular prism has base edge of 8 cm, 10 cm, and 15 cm, and a height of 20 cm.

a Calculate the size of $\angle PRQ$, correct to the nearest degree.

b Find the area of ΔPQR, correct to the nearest cm^2.

c Find the volume of the prism.



a) 112° b) 37 cm^2 c) 740 cm^3

5 The area of an equilateral triangle is 600 cm^2.
Calculate the perimeter of the triangle, correct to 1 d.p.

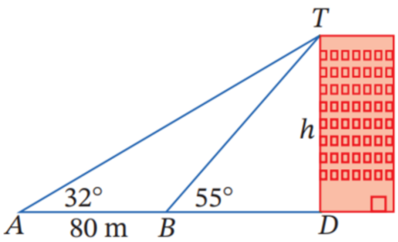
111.7 cm

6 The angles of elevation of a building measured from 2 positions 80 m apart of $32°$ and $55°$.

a Explain why $\angle ATB = 23°$

b Find, correct to 2 decimal places, the length of BT.

c Hence find the height h of the building, correct to the nearest metre.

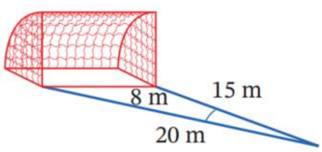


a) ∠ ATB = 55° − 32° (exterior angle theorem), other answers possible

b) 108.50 m

c) 89 m

7 A soccer goal is 8 m wide. A player shoots for goal (along the ground) when 20 m from one post and 15 m from the other post. Within what angle (correct to the nearest 0.1 degree) must the shot be made for the player to have a chance of scoring a goal?

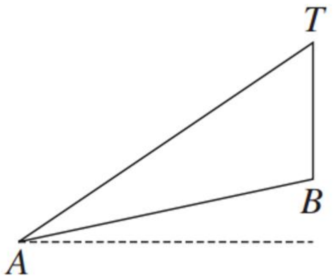


20.8°

8 Melissa is standing at A on a path that leads to the base B of a vertical flagpole.
The path is inclined at $12°$ to the horizontal, and the angle of elevation of the top T of the flagpole from A is $34°$.

a Explain why $\angle TAB = 22°$ and $\angle ABT = 102°$

b Given that $AB = 20$ m, find the height of the flagpole, correct to the nearest metre.

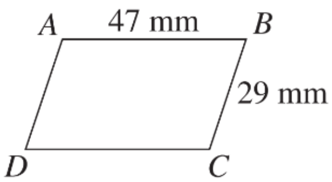


9 metres

9 A parallelogram $ABCD$ has sides $AB = DC = 47$ mm and $AD = BC = 29$ mm.
The longer diagonal BD is 60 mm. Find the following, round your answer to the nearest minute.

a Use the cosine rule to find the size of $\angle BCD$.

b Use co-interior angles to find the size of $\angle ABC$.



a) 101°38'

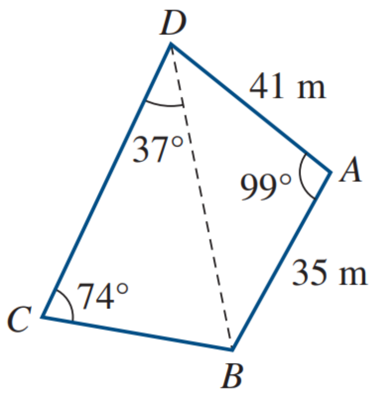
b) 78°22'

10 A survey of a park is shown in the diagram.
A path is proposed from B to D and a fence is required from B to C to D.

a What is the length of the path? Answer correct to the nearest metre.

b Calculate the required length of fencing. Answer correct to the nearest metre.

c What is the area of the park? Answer correct to the nearest square metre.



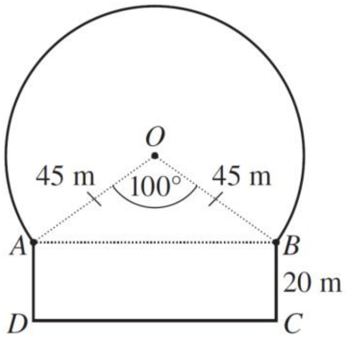
a) 58 m

b) 93 m

c) 1689 m^2

11 2016 HSC Standard 2 Band 4

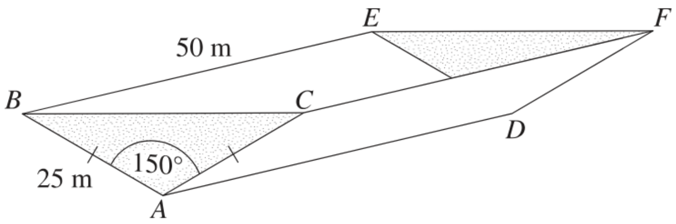
A school playground consists of part of a circle and a rectangle as shown in the diagram.
What is the area of the whole playground, correct to the nearest square metre?



6971 m^2

12 2022 HSC Standard 2 Band 5

A dam is in the shape of a triangular prism which is 50 m long, as shown.
Both ends of the dam, *ABC* and *DEF*, are isosceles triangles with equal sides of 25 metres.
The included angles $\angle BAC$ and $\angle EDF$ are each $150°$
Calculate the number of litres of water the dam will hold when full.



7 812 500 L

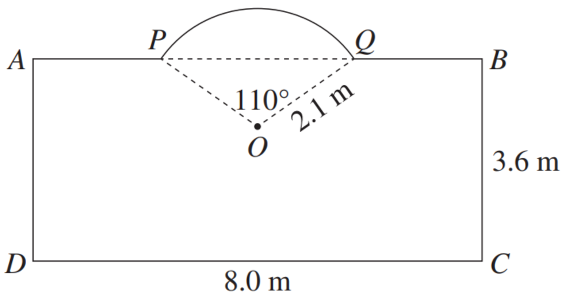
13 2023 HSC Standard 2 Band 5

The diagram shows a shape APQBCD. The shape consists of a rectangle ABCD with an arc PQ on side AB and with side lengths BC = 3.6 m and CD = 8.0 m.

The arc PQ is an arc of a circle with centre O and radius 2.1 m and $\angle$POQ = 110°

What is the perimeter of the shape APQBCD?

Give your answer correct to one decimal place.



23.8 m

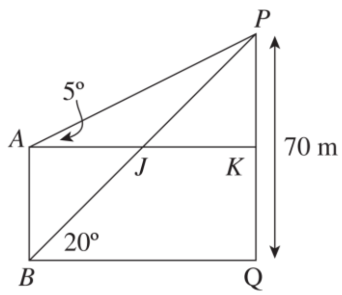
14 Two towers AB and PQ stand on level ground. The angles of elevation of the top of the taller tower from the top and bottom of the shorter tower are 5° and 20° respectively.
The height of the taller tower is 70 metres.

a Explain why $\angle APJ = 15°$.

b Show that $AB = \dfrac{BP \sin 15°}{\sin 95°}$

c Show that $BP = \dfrac{70}{\sin 20°}$

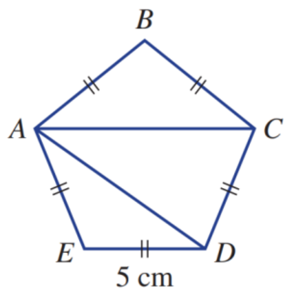
d Hence find the height of the shorter tower, correct to the nearest metre.



53 metres

15 Recall that the sum (S) of the interior angles of a polygon with n sides is given by $S = 180(n - 2)$.

Calculate the area of the pentagon. Answer to 1 d.p.



$\Delta AED = \Delta ABC = 11.89 \text{ cm}^2$

$\Delta ADC = 19.24 \text{ cm}^2$

43.0 cm^2

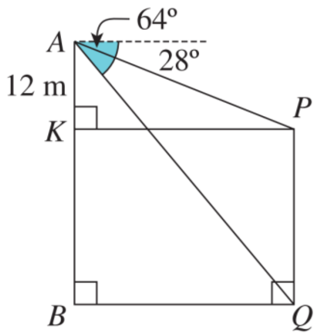
16 Two towers AB and PQ stand on level ground. Tower AB is 12 metres taller than tower PQ. From A, the angles of depression of P and Q are 28° and 64° respectively.

a Use ΔAKP to show that $KP = BQ = 12 \tan 62°$.

b Use ΔABQ to show that $AB = 12 \tan 62° \tan 64°$.

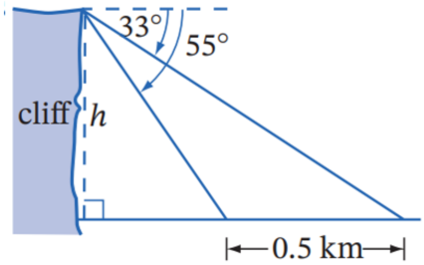
c Hence find the height of the shorter tower, correct to the nearest metre.

d Solve the problem again by using ΔAKP to find AP, and then using the sine rule in ΔAPQ.



Shorter tower is 34 metres

17 From the top of the cliff, the angles of depression of 2 boats at sea that are 0.5 km apart are $55°$ and $33°$. Show that the height of the cliff $h = \frac{0.5 \sin 33° \sin 55°}{\sin 22°}$



Let the distance of the closer boat to the observer be x.

$\sin 55° = \frac{h}{x}, x = \frac{h}{\sin 55°}$

Using sine rule,

$$\frac{0.5}{\sin 22°} = \frac{x}{\sin 33°}$$

$$\frac{0.5}{\sin 22°} = \frac{\left(\frac{h}{\sin 55°}\right)}{\sin 33°}$$

$$\frac{0.5}{\sin 22°} = \frac{h}{\sin 55° \sin 33°}$$

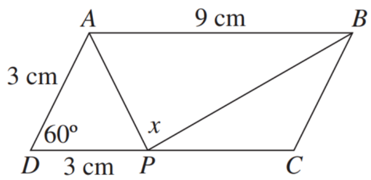
$$\therefore h = \frac{0.5 \sin 33° \sin 55°}{\sin 22°}$$

18 The diagram shows a parallelogram.

a Explain why ΔADP is equilateral, and hence find AP.

b Use the cosine rule in ΔBCP to find BP in exact form.

c Let $\angle APB = x$. Show that $\cos x = -\frac{\sqrt{7}}{14}$



a) isosceles triangle, so $\angle ADP = \angle DPA$ and $\angle ADP + \angle DPA = 120$.

$\therefore \angle ADP = \angle DPA = 60°$

Equilateral as all angles 60

$\therefore AP = 3$ cm

b) $3\sqrt{7}$ cm

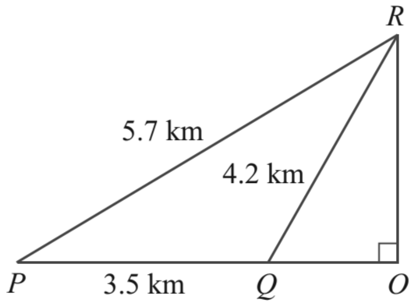
c) $\cos x = \frac{3^2+(3\sqrt{7})^2-9^2}{2(3)(3\sqrt{7})} = \frac{-9}{18\sqrt{7}} = \frac{-9\sqrt{7}}{126} = \frac{-\sqrt{7}}{14}$

1 Refer to the diagram to answer the following questions.

a Calculate $\angle PQR$ and hence $\angle RQO$ to the nearest degree.

b Find the length OR, correct to one decimal place.

c If OR is North, find the true bearing of Q from R



a) $\angle PQR = 95°$,$\angle RQO = 85°$

b) 4.2 km

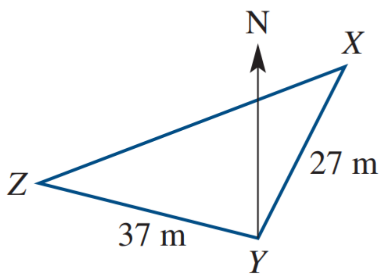
c) 185°

2 XYZ represents a triangular area of land.
The bearing of X from Y is 041° and the bearing of Z from Y is 305°.
The distance XY is 27m and the distance YZ is 37 m.

a What is the size of angle XYZ? Answer correct to the nearest degree.

b Calculate the area to the nearest square metre.

c What is the length of the boundary XZ, to the nearest metre?



a) 96°

b) 497 m^2

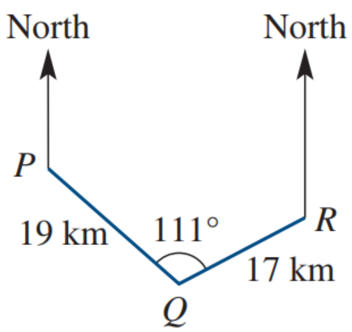
c) 48 m

3 The diagram shows information about the locations of towns P, Q and R.
Amber takes 2 hours and 30 minutes to walk directly from Town P to Town Q.

a What is Amber's walking speed, correct to the nearest km/h?

b What is the distance from P to R? Answer correct to the nearest kilometre.

c How long would it take Amber to walk from P to R? Answer to the nearest minute.



a) 8 km/h

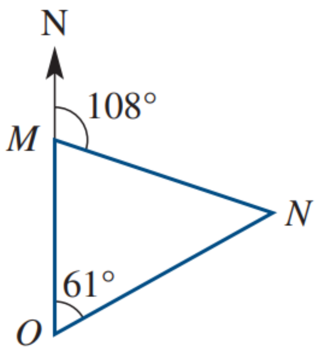
b) 30 km

c) 3 h 45 min

4 M is 50km north of O. The bearing of N from M is 108° and from O it is 061°.
Answer the following questions to the nearest kilometre.

a What is the distance between M and N?

b What is the distance between N and O?



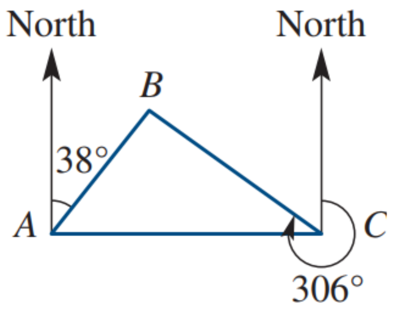
a) 60 km

b) 65 km

5 A ship is located at B and two radar stations 75 km apart are located at A and C.
The bearing of the ship from one radar station is 038° and from the other radar station it is 306°.
C is due east of A.

a How far is the ship from the radar station at A, to the nearest kilometre?

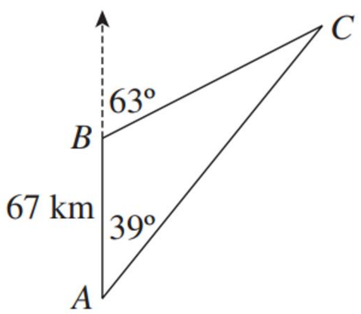
b How far is the ship from the radar station at C, to the nearest kilometre?



a) 44 km

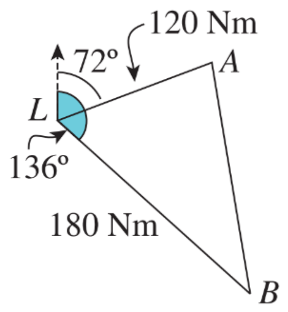
b) 59 km

6 A salesman drove from town A to town B, then to town C, then finally directly home to town A.
Town B is 67 km north of town A, and the bearings of town C from towns A and B and $039°\mathrm{T}$ and $063°\mathrm{T}$ respectively. Find how far the salesman drove, correct to the nearest kilometre.



317 km

7 In the diagram to the right, ship A is 120 nautical mules from lighthouse L on a bearing of $072°\text{T}$, while ship B is 180 nautical miles from L on a bearing of $136°\text{T}$.
Calculate the distance between the two ships, correct to the nearest nautical mile.

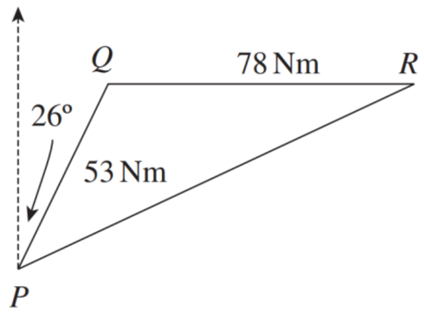


167 nautical miles

8 A ship sails 53 nautical miles from P to Q on a bearing of $026°\text{T}$.
It then sails 78 nautical miles due east from Q to R.

a Explain why $\angle PQR = 116°$

b How far apart are P and R, correct to the nearest nautical mile?



a) draw a north-south line at Q, $\angle PQR = 90 + 26 = 116$ due to alternate angles

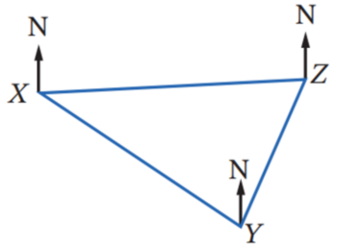
b) 112 nautical miles

9 A yacht sails from X to Y on a bearing of 130° for 4.2 km.
It then turns and travels to Z on a bearing of $025°$ for 2.9 km.

a Label the diagram.

b Explain why $\angle XYZ = 75°$

c Calculate the distance XZ, correct to 1 d.p.



b) ∠ XYN = 180° – 130° = 50° (co-interior angles on parallel lines)

c) 4.4 km

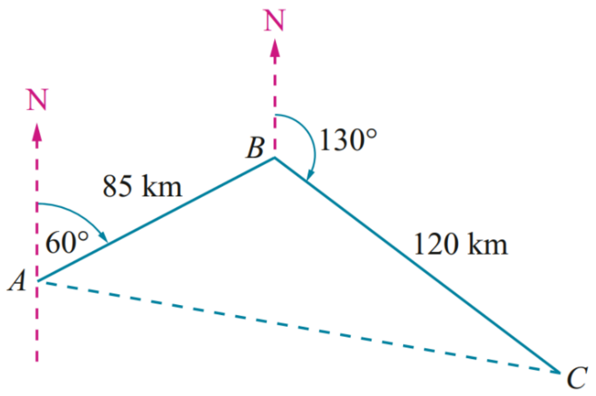
10 A ship sails 85 km from A to B on a bearing of 060° T.
It then turns and sails 120 km to C on a bearing of 130° T.

a Find the size of ∠ABC.

b How far is the ship from its starting point?

c Calculate ∠BAC, to the nearest minute.

d What is the bearing of the ship from its starting point to the nearest minute?



a) 110°

b) 169 km

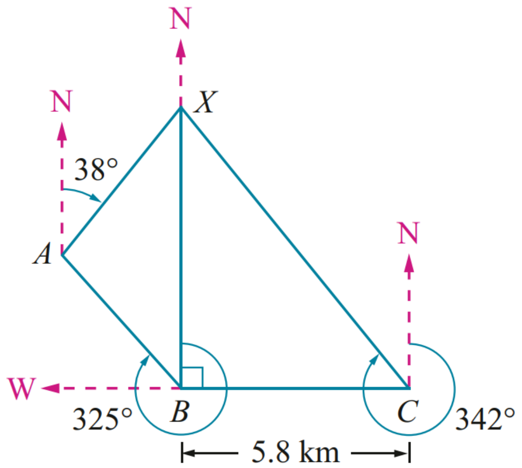
c) 41° 41'

d) 101° 51' T

11 Consider the diagram.

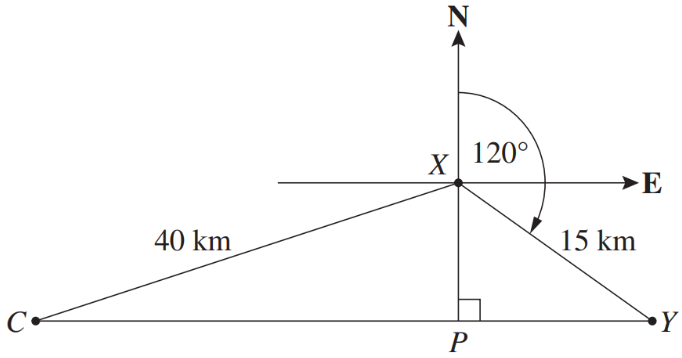
a Show that the distance BX is 17.9 km.

b Show that the distance from B to X via A is 22.2 km.



12 **2023 HSC Standard 2 Band 5**

The diagram shows the location of three places X, Y and C.
Y is on a bearing of 120° and 15 km from X.
C is 40 km from X and lies due west of Y.
P lies on the line joining C and Y and is due south of X.



a Find the distance from X to P.

7.5 km

b What is the bearing of C from X, to the nearest degree?

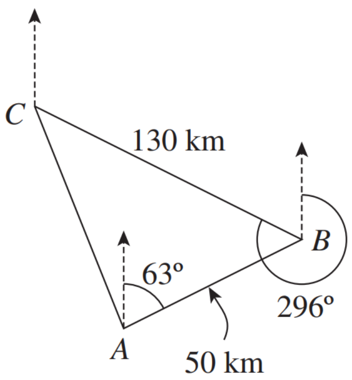
259°

13 A ship sails 50 km from port A to port B on a bearing of $063°\mathrm{T}$,
then sails 130 km from port B to port C on a bearing of $296°\mathrm{T}$.

a Show that $\angle ABC = 53°$

b Find, correct to the nearest km the distance of port A from port C.

c Find $\angle ACB$, and hence find the bearing of port A from port C, correct to the nearest degree.



a) extend north line at B, $\angle ABC = 296 - 63 - 180 = 53°$

b) 108 km

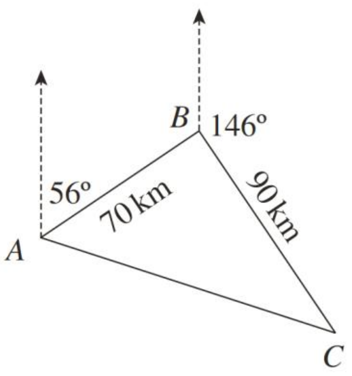
c) $\angle ACD \approx 22°$, bearing $\approx 138°$

14 A motorist drove 70km from town A to town B on a bearing of 056°T, and then drove 90km from town B to town C on a bearing of 146°T.

a Explain why $\angle ABC = 90°$

b How far apart are the towns A and C, correct to the nearest kilometre?

c Find $\angle$BAC, and hence find the bearing of town C from town A, to the nearest degree.



a) angle co-interior to 56° is 124°.

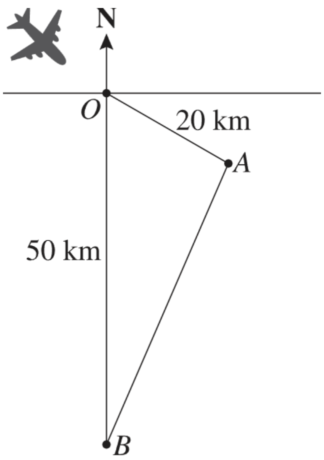
$\angle ABC = 360 - 124 - 146 = 90°$

b) 114 km

c) 108°

15 **2022 HSC Standard 2 Band 5**

The diagram shows an aeroplane that was flying towards an airport at A on a bearing of 135°T. When it was at point O, 20 km away from the airport at A, the flight course was changed. The aeroplane landed at an airport at B directly south of O. The distance from O to B is 50 km.



a Show that the distance between airport A and airport B is 38.5 km, correct to 1 d.p.

38.5 km

b Find $\angle OBA$ to the nearest degree.

22°

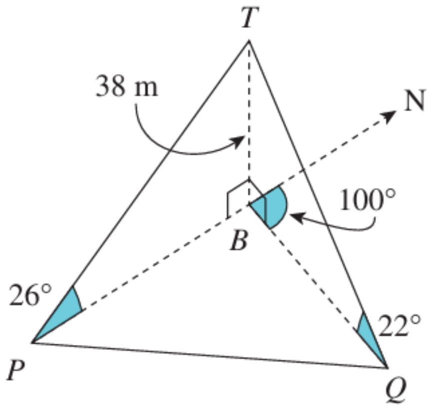
c What is the bearing of the airport at B from the airport at A?

202°

16 From two points P and Q on level ground, the angles of elevation of the top T of a 38 m tower are 26° and 22° respectively.
Point P is due south of the tower, and the bearing of Q from the tower is 100°T.

a Show that $PB = 38 \tan 64°$, and find a similar expression for QB.

b Hence determine, correct to the nearest metre, the distance between P and Q.



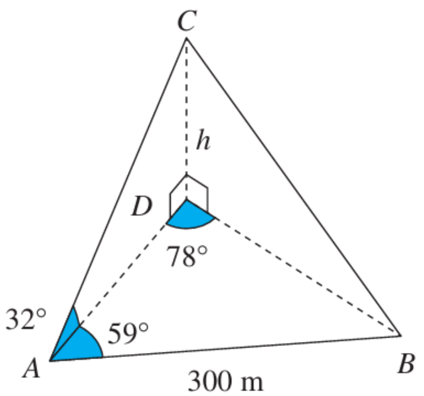
a) 38 tan 68°

b) 111 m

17 Two observers at A and B on horizontal ground are 300 m apart.
From A, the angle of elevation of the top C of a tall building DC is 32°.
It is also known that $\angle DAB = 59°$ and $\angle ADB = 78°$.

a Show that $AD = \dfrac{300 \sin 43°}{\sin 78°}$

b Hence find the height of the building, correct to the nearest metre.



131 m