

Student Name

Mathematics Stage 4

PROPERTIES OF GEOMETRICAL FIGURES

Book 3 Apply the properties of triangles
and quadrilaterals

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CONTENTS

Overview	2
Angle Sum of a Triangle	5
Exterior Angle Theorem.....	13
Angle Sum of a Quadrilateral	21
Mixed Practice.....	29
Check Your Understanding	36

OVERVIEW

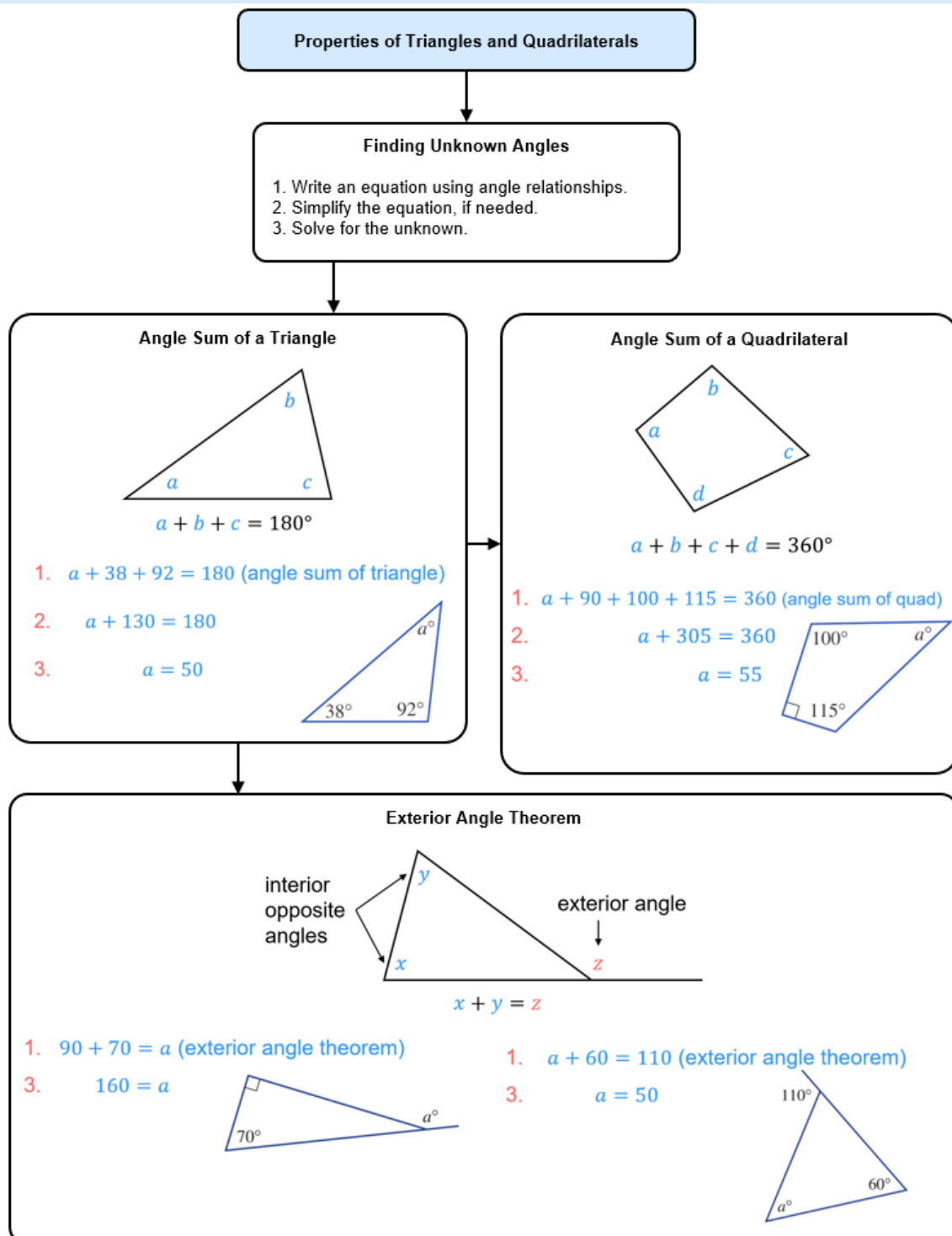
SYLLABUS CONTENT

MA4-GEO-C-01 identifies and applies the properties of triangles and quadrilaterals to solve problems

Apply the properties of triangles and quadrilaterals

- Prove that the interior angle sum of a triangle is 180° with or without digital tools
- Prove that any exterior angle of a triangle equals the sum of the 2 interior opposite angles
- Apply the angle sum of a triangle to prove that the angle sum of a quadrilateral is 360°
- Apply the properties of triangles and quadrilaterals to determine unknown sides and angles to solve problems, giving reasons

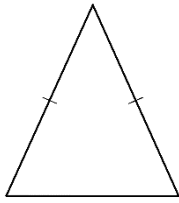
SUMMARY



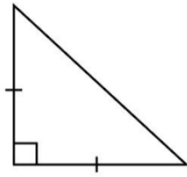
REVIEW OF PRIOR KNOWLEDGE

1 Show the two equal angles in these isosceles triangles.

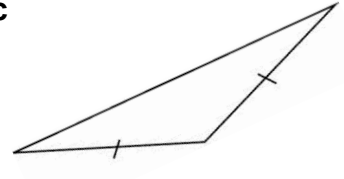
a



b



c



2 Solve these equations.

a $x + 140 = 180$

b $x + 95 = 180$

c $x + 71 + 90 = 180$

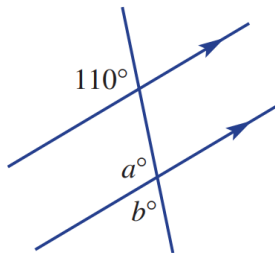
d $2x + 70 = 180$

e $2x + 30 + 65 = 180$

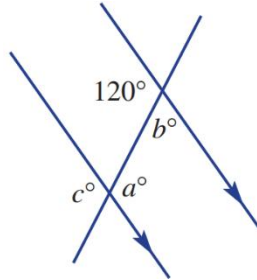
f $x + x + 90 = 180$

3 Find the value of the pronumeral in these diagrams.

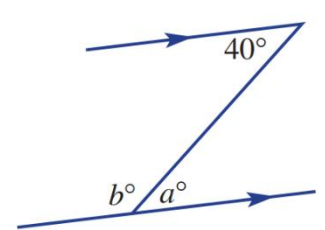
a



b



c



4 Solve these equations.

a $123 = 75 + x$

b $102 = 47 + x$

c $x = 18 + 123$

5

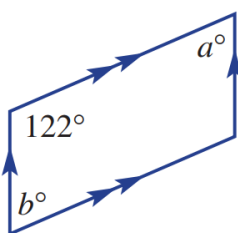
a $305 + x = 360$

b $70 + 90 + a = 360$

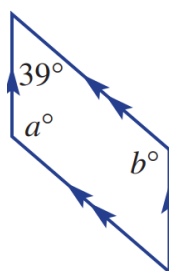
c $x + 50 + 110 = 360$

6 Find the value of the pronumerals, using co-interior angles.

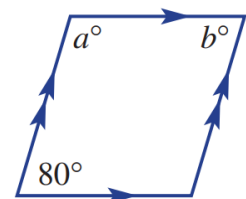
a



b

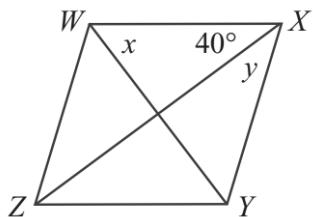


c

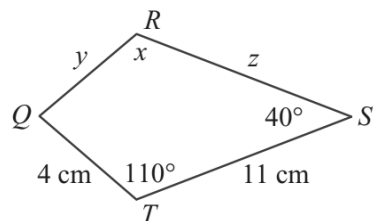


7 Use the properties of quadrilaterals to find the values of the pronumerals.

a WXYZ is a rhombus.

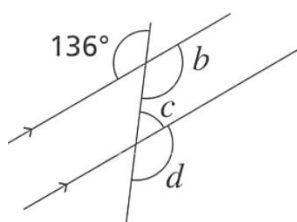


b QRST is a kite.

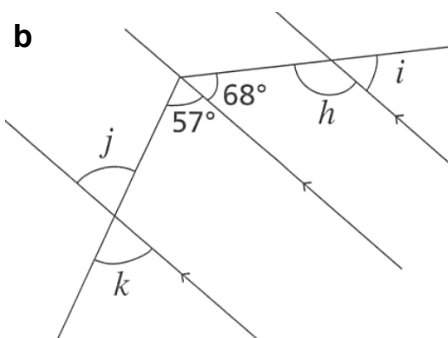


8 Find the value of each pronumeral.

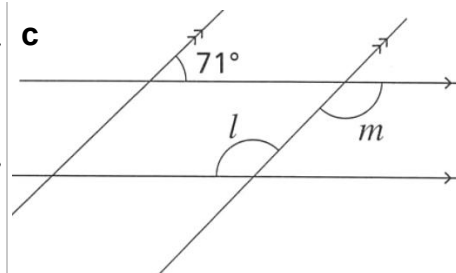
a



b



c

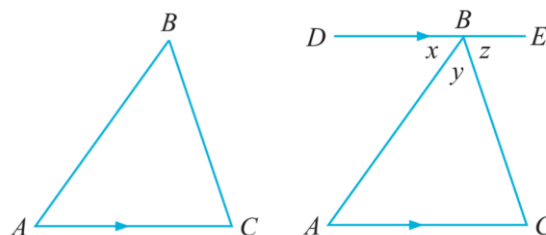


ANGLE SUM OF A TRIANGLE

Investigation Prove the angle sum of a triangle is 180° .

Consider the triangle $\triangle ABC$:

Construct line DBE parallel to AC .



By considering $\angle DBE$: $x + y + z = \dots\dots\dots^\circ$ ($\dots\dots\dots$)

$\angle DBA = \angle BAC$ ($\dots\dots\dots$)

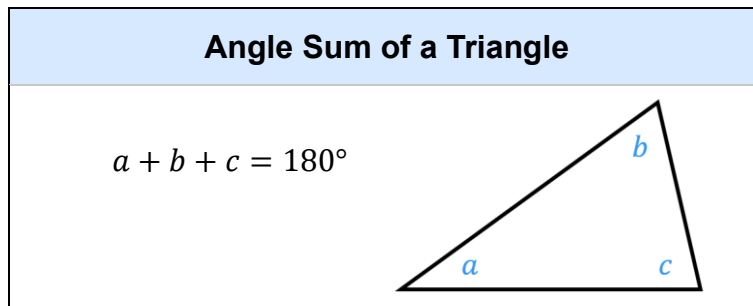
$\angle EBC = \angle \dots\dots\dots$ (alternate angles on parallel lines)

By considering the sum of the angles in $\triangle ABC$:

$x + y + z = \dots\dots\dots^\circ$

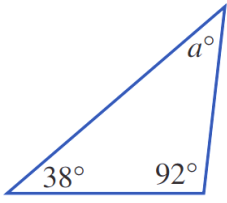
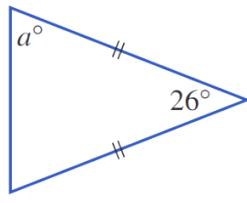
Key ideas

1. All the interior angles of a triangle add up to $\dots\dots\dots$

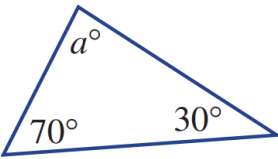
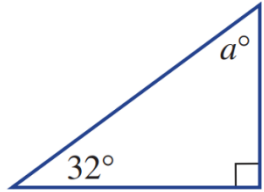
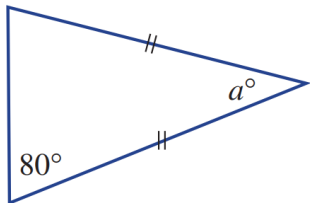
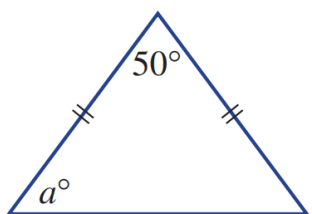


- To solve angle relationship problems:
 1. Write an equation using angle relationships.
 2. Simplify the equation, if needed.
 3. Solve for the unknown using inverse operations.

Example Solve problems involving angle sum of a triangle.

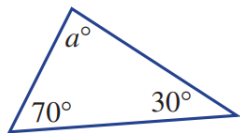
 1. $a + 38 + 92 = 180$ (angle sum of triangle) 2. $a + 130 = 180$ 3. $a = 50^\circ$	 1. $a + a + 26 = 180$ (angle sum of triangle) 2. $2a + 26 = 180$ 3. $2a = 154$ $a = 77^\circ$
--	---

Find the value of the pronumerals in these diagrams.

a  $a + 70 + 30 = 180$ $\dots\dots\dots$ $\dots\dots\dots$ $a = 80^\circ$	b  $\dots\dots + \dots\dots + \dots\dots = 180$ $\dots\dots\dots$ $\dots\dots\dots$ $a = 58^\circ$
c  $a = 20^\circ$	d  $a = 65^\circ$

1 Use the angle sum of a triangle to find the value of the pronumeral. Show full working out.

a

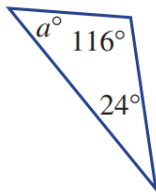


$$a + 30 + 70 = 180$$

.....
.....

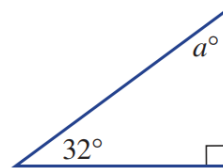
$$a = 80^\circ$$

b



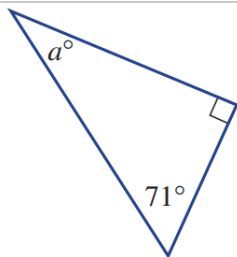
$$a = 40^\circ$$

c



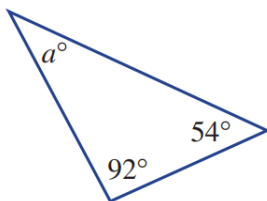
$$a = 58^\circ$$

d



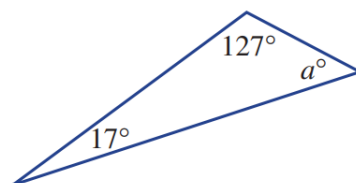
$$a = 19^\circ$$

e



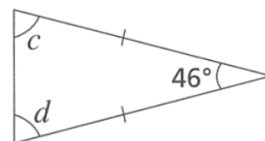
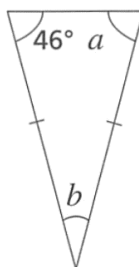
$$a = 34^\circ$$

f



$$a = 36^\circ$$

2 Both of the triangles shown are isosceles.



a What part of the diagram tells you the triangles are isosceles?

The dashes

b Determine the value of the missing angles a, b, c, d .

$$a = 46^\circ$$

$$b = 88^\circ$$

$$c = d = 67^\circ$$

c Explain why angles a and b are different to c and d , even though both triangles have similar information.

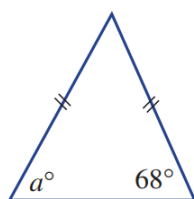
In the first triangle, 46° was one of the equal angles.

In the second triangle, 46° was the non-equal angle.

DEVELOPMENT

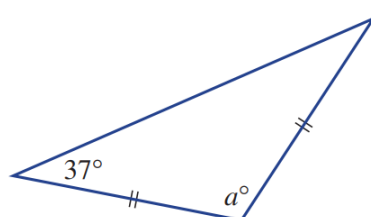
- 3 Use the angle sum of a triangle to find the value of the pronumeral in these isosceles triangles. You do not need to show working out.

a



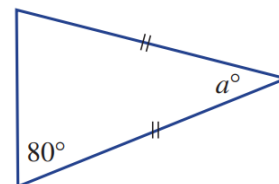
68°

b



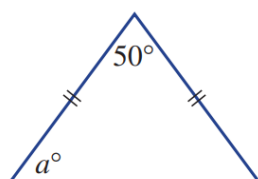
106°

c



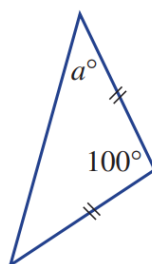
20°

d



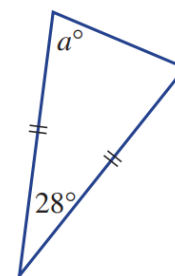
65°

e



40°

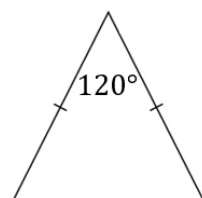
f



76°

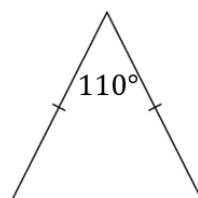
- 4 Determine the value of the unknown angles in these triangles. You do not need to show working out.

a



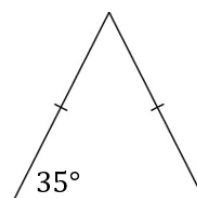
30, 30

b



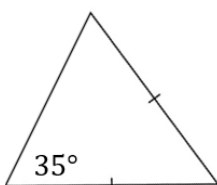
35, 35

c



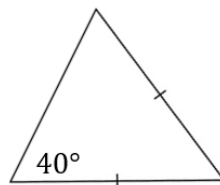
110, 35

d



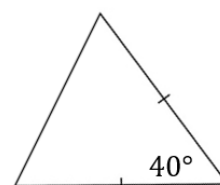
35, 110

e



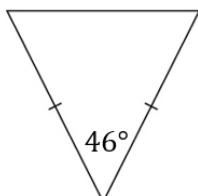
40, 100

f



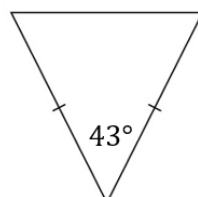
70, 70

g



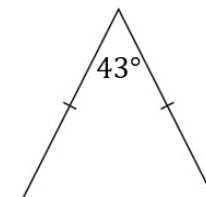
67, 67

h



68.5, 68.5

i



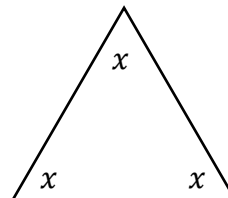
68.5, 68.5

- 5 Two of the angles in a triangle are 74° and 104° . Work out the size of the third angle.

$$a = 74 + 74 = 180^\circ$$

$$a = 2^\circ$$

- 6 Use algebra to explain why every angle in an equilateral triangle is 60° .



$$3x = 180^\circ$$

$$x = 60^\circ$$

- 7 One of the angles in an isosceles triangle is 80° . Work out the sizes of the other two angles.

If 80° is the non-equal angle, then the other angles are 50°

If 80° is one of the equal angles, then the other angle is 20°

Explain why there are two possible sets of answers.

The question doesn't specify whether 80° is one of the equal angles or not

- 8 One of the angles in an isosceles triangle is 160° . Work out the sizes of the other two angles.

Other two angles are both 10°

Explain why there is only one possible set of answers.

160° can't be one of the equal angles because it would exceed the angle sum of a triangle

- 9 An isosceles triangle has an interior angle of 60° . Find all the other angles.

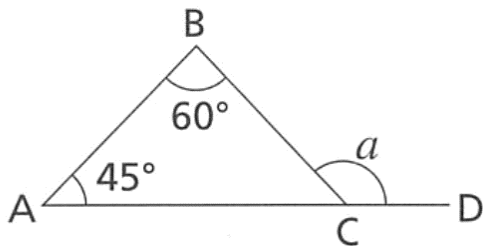
The other two angles are 60°

What do you notice?

If an isosceles triangle has 60° , it must be equilateral

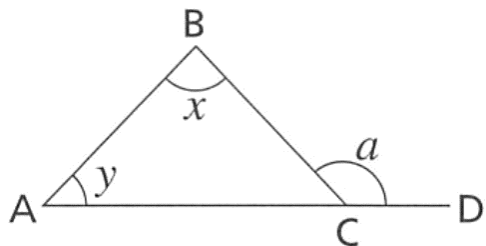
10 Provide full reasoning for both parts below.

a Show that $a = 105^\circ$



$\angle ACB = 75^\circ$ (angle sum of a triangle)
 $a = 180 - 75 = 105^\circ$
(angles on a straight line supplementary)

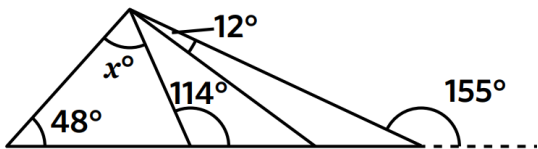
b Prove that $a = x + y$



$\angle BCA = 180 - x - y$ (angle sum of triangle)
 $a = 180 - (180 - x - y)$ (angles on a straight line)
 $a = x + y$

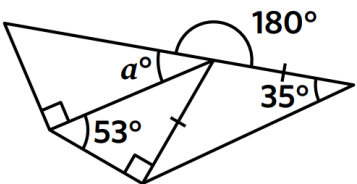
11 Determine the value represented by the pronumeral. You do not need to show working out.

a



$x = 66^\circ$

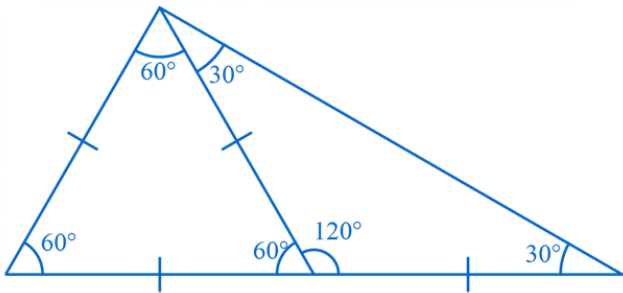
b



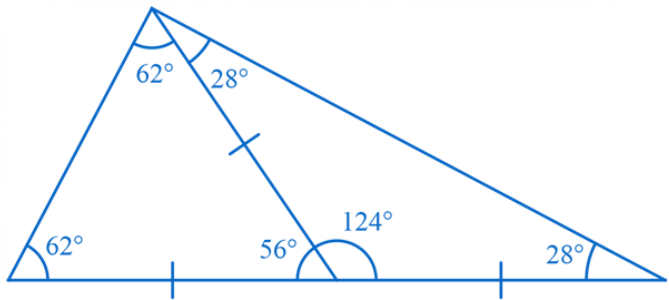
$a = 33^\circ$

12 Determine the size of the missing angles in each diagram. You do not need to show working out.

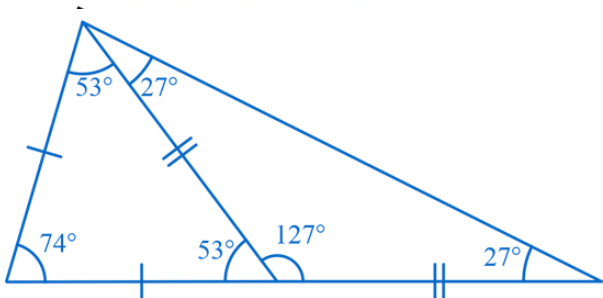
a



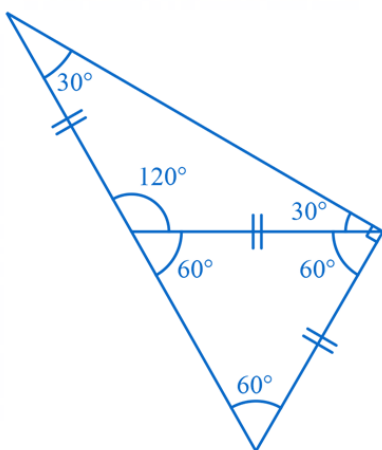
b



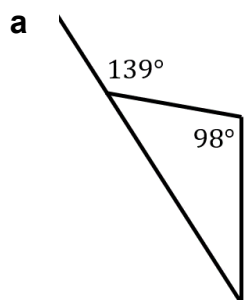
c



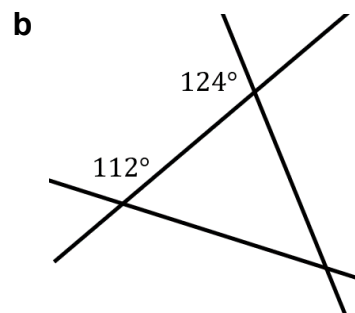
d



13 Decide whether the triangles shown are isosceles.



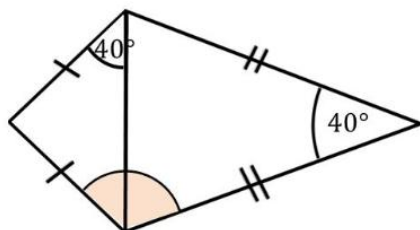
yes



yes

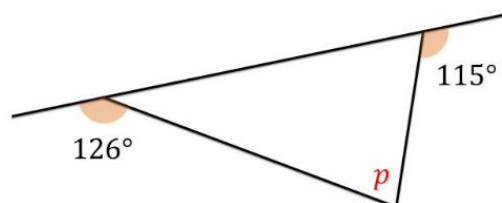
14

a Find the size of the shaded angle.



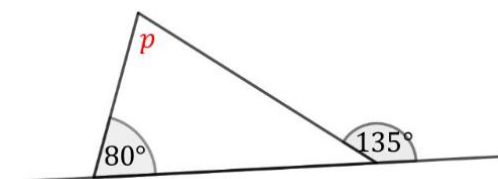
110°

b Find the size of angle p .



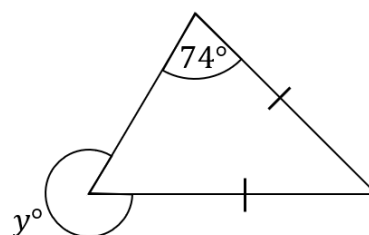
61°

c Find the size of angle p .



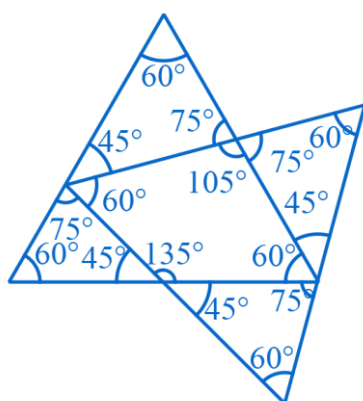
55°

d Find the size of angle y .

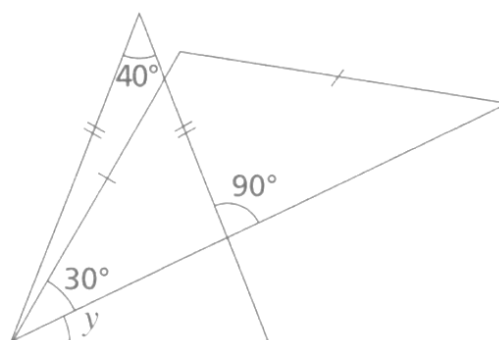


286°

e This is a pair of overlapping equilateral triangles. Find every angle.

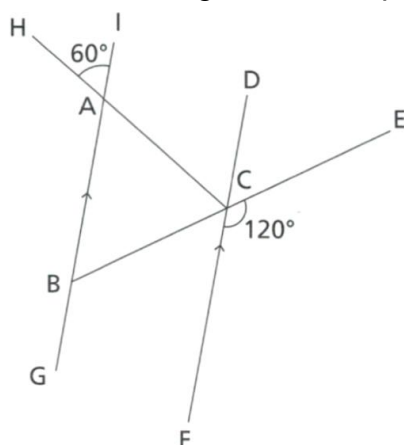


f This is a pair of isosceles triangles. Work out the size of angle y .



20°

15 Show that triangle ABC is equilateral. Give full reasoning for each stage of your working.



$\angle BAC = 60^\circ$ (vertically opposite)

$\angle BCF = 60^\circ$ (angles on straight line)

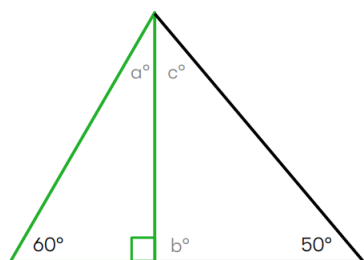
$\angle ABC = \angle BCF$ (alternate angles on parallel lines)
 $= 60^\circ$

$\angle ACB = 180^\circ - 60^\circ - 60^\circ$ (angle sum of $\triangle$)
 $= 60^\circ$

$\therefore \triangle ABC$ is equilateral.

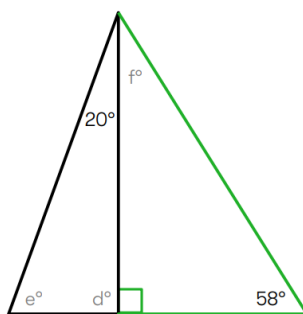
16 Determine the value of the pronumerals in these diagrams.

a



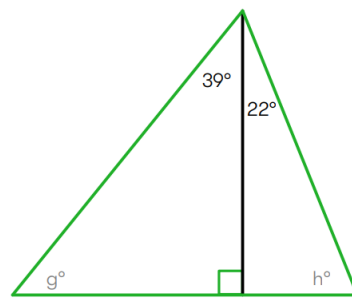
$$a = 30, b = 90, c = 40$$

b



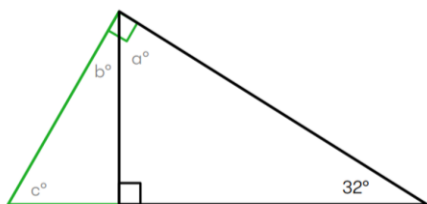
$$d = 90, e = 70, f = 32$$

c



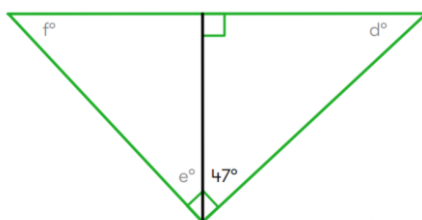
$$g = 51, h = 68$$

d



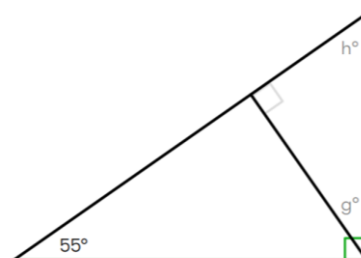
$$a = 58, b = 22, c = 58$$

e



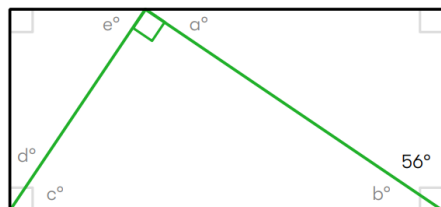
$$d = 43, e = 43, f = 47$$

f



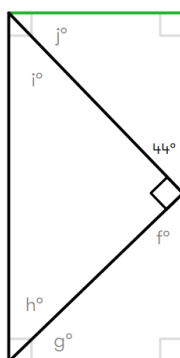
$$g = 55, h = 35$$

g



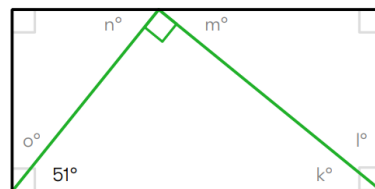
$$a = 34, b = 34, c = 56, d = 34, e = 56$$

h



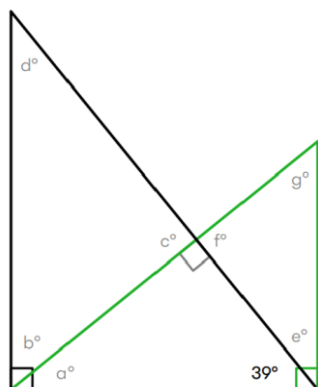
$$f = 46, g = 44, h = 46, i = 44, j = 46$$

i



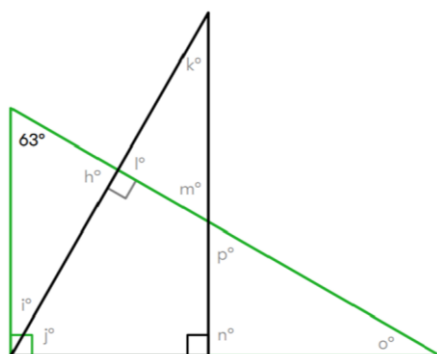
$$k = 39, l = 51, m = 39, n = 51, o = 39$$

j



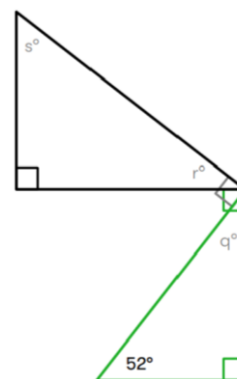
$$a = 51, b = 39, c = 90 \\ d = 51, e = 51, f = 90, g = 39$$

k



$$h = 90, i = 27, j = 63, k = 27, l = 90, \\ m = 63, n = 90, o = 27, p = 63$$

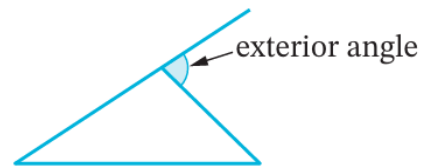
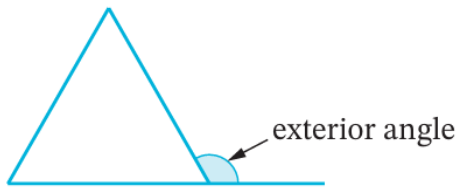
l



$$q = 38, r = 38, s = 52$$

EXTERIOR ANGLE THEOREM

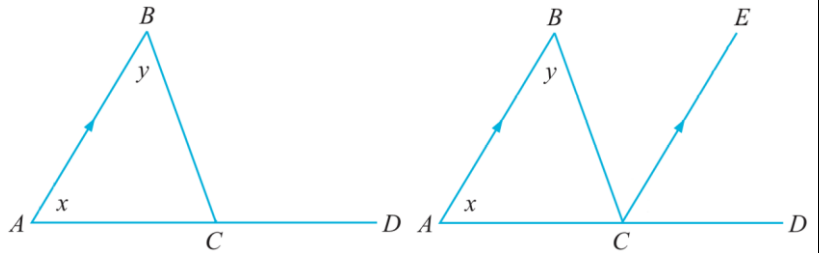
- If we extend one side of a triangle, we make an **exterior angle**.



Investigation Exterior angle theorem.

Consider triangle $\triangle ABC$ with exterior angle $\angle BCD$.

Construct line CE parallel to AB

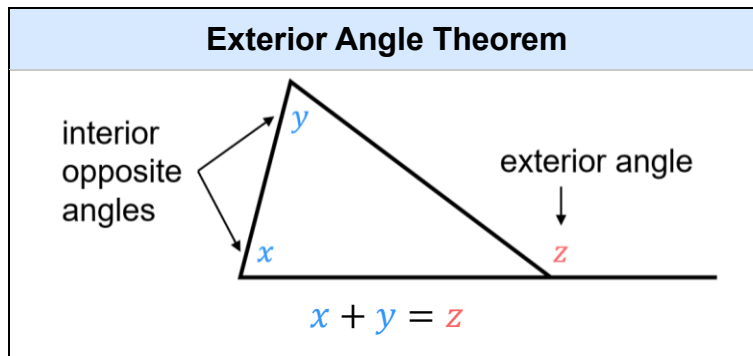


$\angle BAC = \angle ECD$ (.....)

$\angle ABC = \angle \dots\dots$ (alternate angles on $\parallel$ lines)

$\therefore \angle BCD = \dots\dots + \dots\dots$

The exterior angle of a triangle is equal to the of the two opposite interior angles.



Example Write an equation using exterior angle theorem.

Indicate the two interior angles opposite the exterior angle x in these diagrams

a	b	c
d	e	f

For each triangle, write an equation using exterior angle theorem.

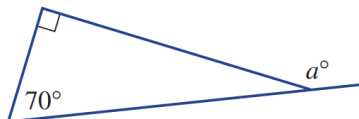
a $a + b = \dots\dots\dots$	b $\dots\dots\dots + \dots\dots\dots = \dots\dots\dots$	c $\dots\dots\dots + \dots\dots\dots = \dots\dots\dots$
---	---	---

Key ideas

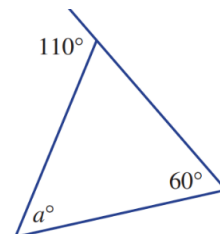
1. The exterior angle of a triangle is the of the two interior opposite angles.

- To solve angle relationship problems:
 1. Write an equation using angle relationships.
 2. Simplify the equation, if needed.
 3. Solve for the unknown using inverse operations.

Example Solve problems involving the exterior angle theorem.



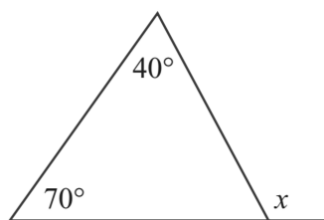
1. $90 + 70 = a$ (exterior angle theorem)
3. $160^\circ = a$



1. $a + 60 = 110$ (exterior angle theorem)
3. $a = 50^\circ$

Find the value of the pronumerals in these diagrams.

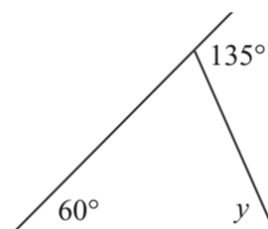
a



$$x = \dots + \dots$$

$$= \dots$$

b



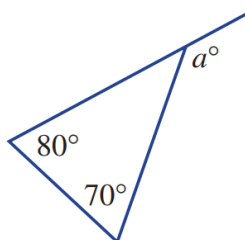
$$\dots + y = \dots$$

$$y = \dots$$

$$x = 110^\circ$$

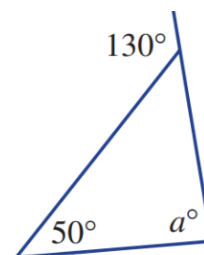
$$y = 75^\circ$$

c



$$a = 150^\circ$$

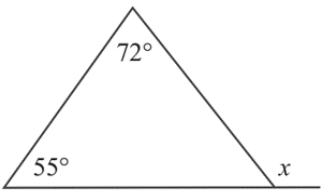
d



$$a = 80^\circ$$

1 Calculate the value of the missing exterior angle using exterior angle theorem.

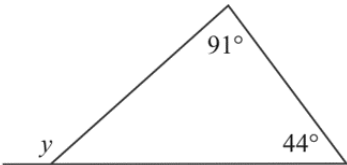
a



$x = \dots + \dots$
 $= \dots$

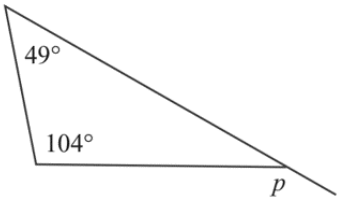
127°

b



135°

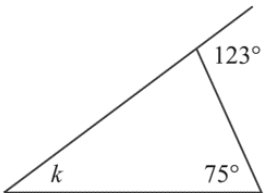
c



153°

2 Calculate the value of the missing interior angle using exterior angle theorem.

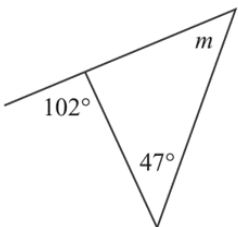
a



$\dots = k + \dots$
 $\dots = k$

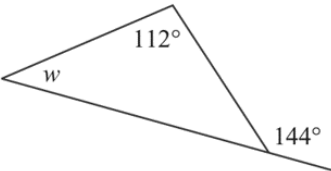
48°

b



55°

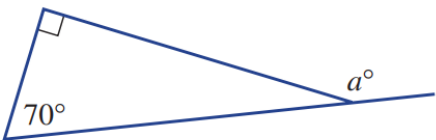
c



32°

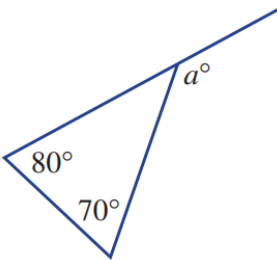
3 Calculate the value of the pronumeral. You do not need to show reasoning.

a



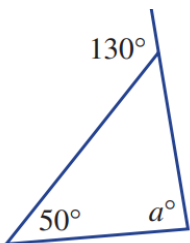
160°

b



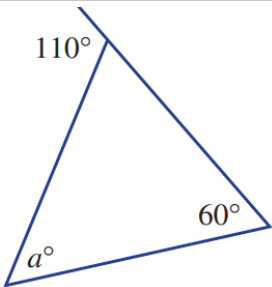
150°

c



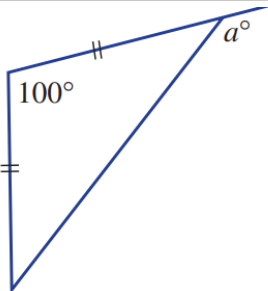
80°

d



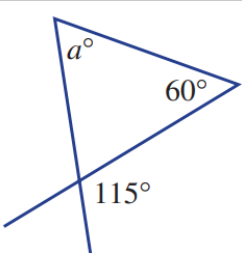
50°

e



140°

f



55°

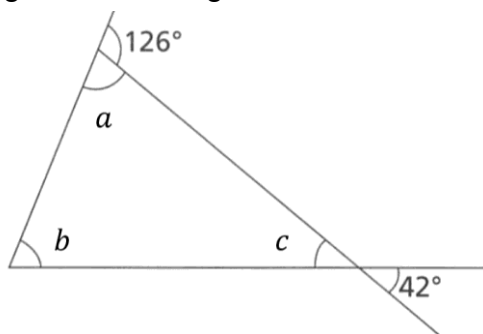
4 Find the value of each pronumeral, you do not need to show reasoning.

a 104	b 16	c $d = 85, e = 95$	d $a = 50, b = 116$
e 135	f $x = 66, y = 114$	g 67	h 29
i 90	j 70	k 130	l 88

5 Determine the value of the pronumeral, using all the angle relationships you know. You do not need to show reasoning.

a 60	b 231	c 18
d 91	e 65.5	f 60

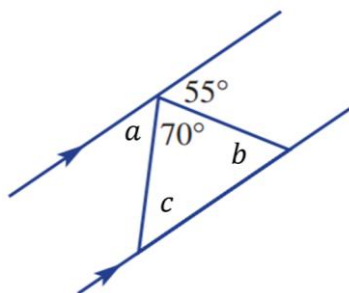
- 6 Determine all the missing angles in this diagram.



$$\begin{aligned} a &= 54^\circ \\ b &= 84^\circ \\ c &= 42^\circ \end{aligned}$$

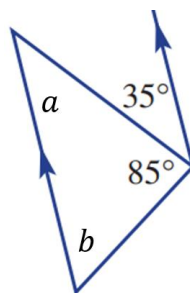
- 7 Use your knowledge of angle relationships on parallel lines to calculate the unknown angles.

a



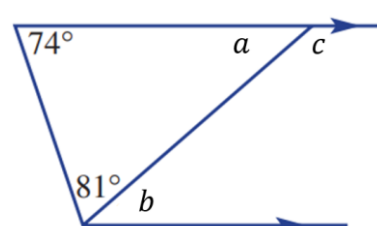
$$a = 55^\circ, b = 55^\circ, c = 55^\circ$$

b



$$a = 35^\circ, b = 60^\circ$$

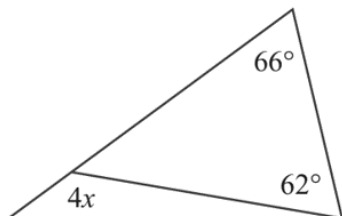
c



$$a = 25^\circ, b = 25^\circ, c = 155^\circ$$

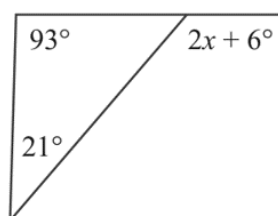
- 8 By writing an equation, solve for the pronumeral.

a



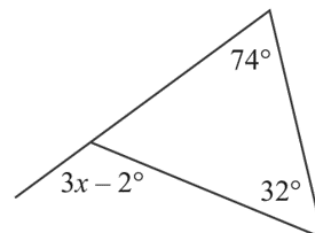
$$\begin{aligned} 4x + 52 &= 180 \\ 32^\circ \end{aligned}$$

b



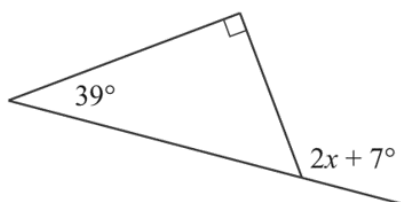
$$\begin{aligned} 2x + 72 &= 180 \\ 54^\circ \end{aligned}$$

c



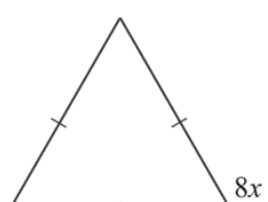
$$\begin{aligned} 3x + 72 &= 180 \\ 36^\circ \end{aligned}$$

d



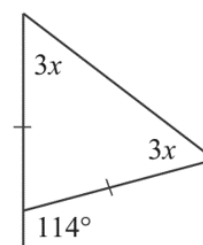
$$\begin{aligned} 2x + 7 &= 129 \\ 61^\circ \end{aligned}$$

e



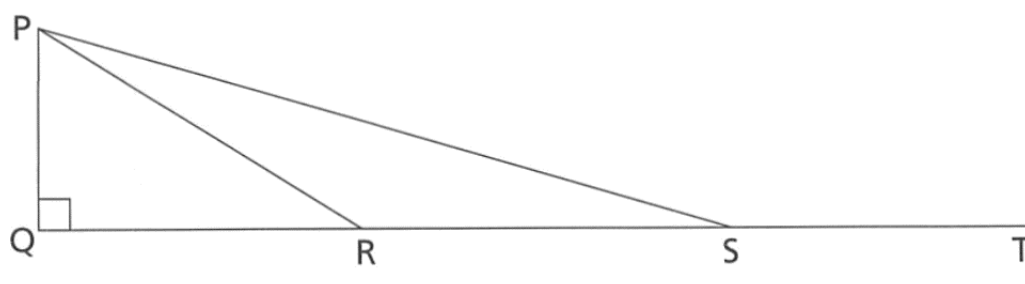
$$\begin{aligned} 8x + 60 &= 180 \\ 15^\circ \end{aligned}$$

f



$$\begin{aligned} 6x &= 114 \\ 14^\circ \end{aligned}$$

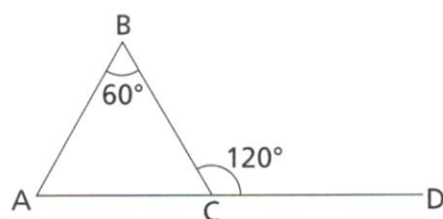
- 9 $\angle PRQ = 48^\circ$ and $\angle TSP = 141^\circ$. Determine the sizes of every angle.



$$\begin{aligned}\angle QPR &= 42^\circ \\ \angle PRS &= 132^\circ \\ \angle RSP &= 39^\circ \\ \angle RPS &= 9^\circ\end{aligned}$$

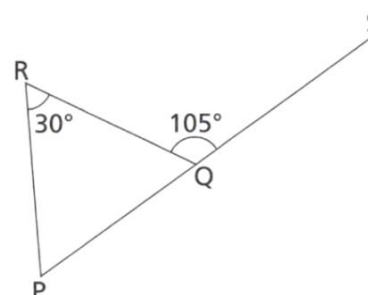
- 10 Show these properties, using full reasoning.

a Show that triangle ABC is equilateral.



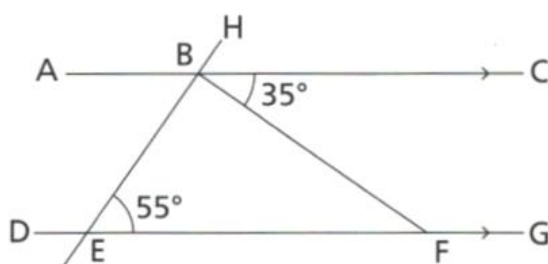
$$\begin{aligned}\angle BAC &= 120 - 60 \text{ (exterior angle theorem)} \\ &= 60^\circ \\ \angle BCA &= 180 - 120 \text{ (angles on a straight line)} \\ &= 60^\circ \\ \therefore \triangle ABC &\text{ is equilateral.}\end{aligned}$$

b Show that triangle PQR is isosceles.



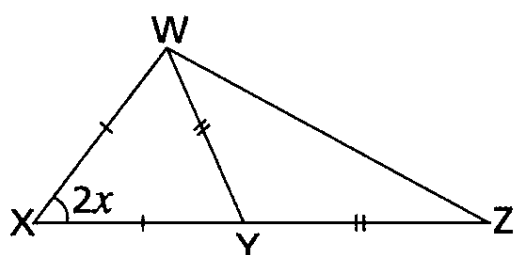
$$\begin{aligned}\angle RPQ &= 105 - 30 \text{ (exterior angle theorem)} \\ &= 75^\circ \\ \angle RQP &= 180 - 105 \text{ (angles on a straight line)} \\ &= 75^\circ \\ \angle RPQ &= \angle RQP \\ \therefore \triangle PQR &\text{ is isosceles.}\end{aligned}$$

- 11 Prove that $\triangle BEF$ is a right-angled triangle, giving full geometrical reasoning.



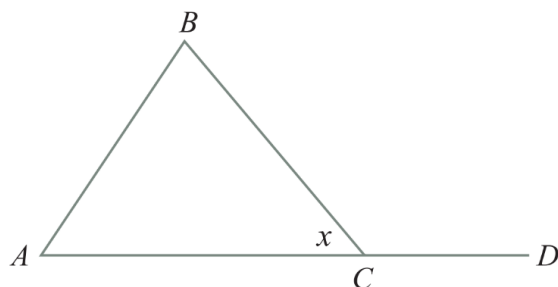
$$\begin{aligned}\angle CBF &= \angle BFE = 35^\circ \text{ (alternate angles on parallel lines)} \\ \angle EBF &= 180 - 55 - 35 \\ &= 90^\circ \text{ (angle sum of triangle)} \\ \therefore \triangle BEF &\text{ is right-angled}\end{aligned}$$

- 12 Find expressions, in terms of x , for all the angles in this triangle.
(Hint: simplify the expressions as you go)



$$\begin{aligned}\angle XWY &= \angle XYW = 90 - x \\ \angle WYZ &= 90 + x \\ \angle YWZ &= \angle WZY = 45 - \frac{x}{2}\end{aligned}$$

- 13 Another proof of the alternate angle theorem is possible by using the angle sum of a triangle.



By considering $\triangle ABC$, $x + \angle BAC + \angle ABC = 180$ (.....)

Angle sum of $\triangle$

$$\angle BAC + \angle ABC = 180 - \dots\dots\dots$$

By considering $\angle ACD$, $x + \angle BCD = \dots\dots\dots$ (.....)

x

180 (angles on straight lines)

$$\angle BCD = \dots\dots\dots$$

$180 - x$

To summarise:

$$\angle BAC + \angle ABC = 180 - x$$

$$\angle BCD = 180 - x$$

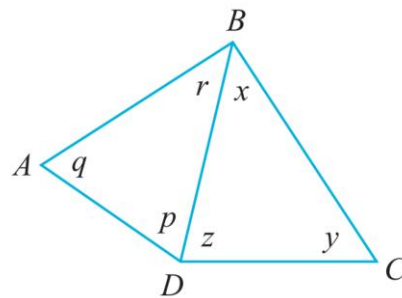
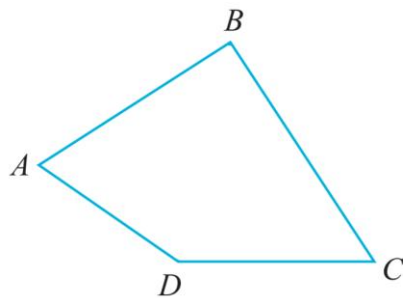
So $\dots\dots\dots = \dots\dots\dots$

$\angle BAC + \angle ABC = \angle BCD$

ANGLE SUM OF A QUADRILATERAL

Investigation Angle sum of a quadrilateral.

Every quadrilateral can be split into two triangles.

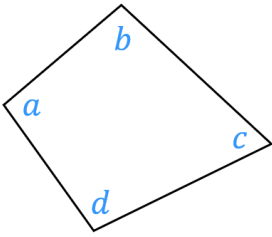


By considering $\triangle ABD$, $q + p + r = 180^\circ$ (.....)

By considering $\triangle BCD$, $x + y + z = \dots\dots\dots$ (.....)

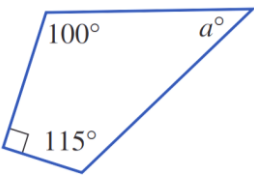
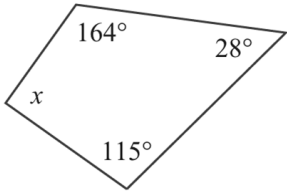
So, the sum of all angles in a quadrilateral is

$$\begin{aligned} & (q + p + r) + (x + y + z) \\ &= \dots\dots\dots + \dots\dots\dots \\ &= \dots\dots\dots \end{aligned}$$

Angle Sum of a Quadrilateral	
$a + b + c + d = 360^\circ$	

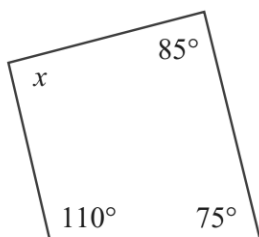
1. Write an equation using angle relationships.
2. Simplify the equation, if needed.
3. Solve for the unknown using inverse operations.

Example Solve problems involving the angle sum of a quadrilateral.

	
1. $a + 90 + 100 + 115 = 360$ (angle sum of quad)	1. $x + 164 + 115 + 28 = 360$ (angle sum of quad)
2. $a + 305 = 360$	2. $x + 307 = 360$
3. $a = 55$	3. $x = 53$

Find the value of the pronumerals in these quadrilaterals.

a



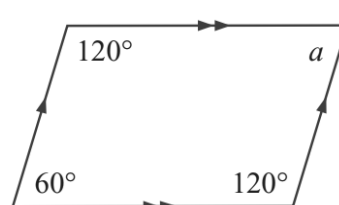
$$x + \dots + \dots + \dots = \dots$$

$$x + \dots = \dots$$

$$x = \dots$$

$$x = 90^\circ$$

b



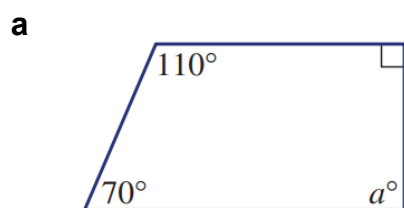
$$a + \dots + \dots + \dots = \dots$$

$$x = 60^\circ$$

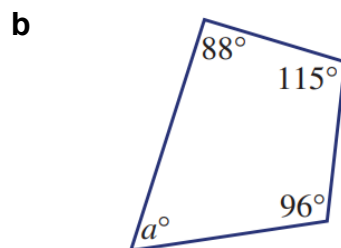
Key ideas

1. All the interior angles of a quadrilateral add up to
2. The properties of quadrilaterals may be required to find unknown angles; they can also be used to check your answer.

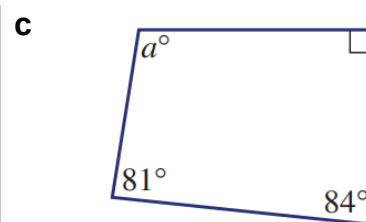
1 Determine the size of each unknown angle in these quadrilaterals.



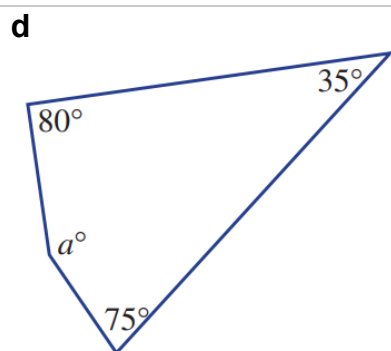
90



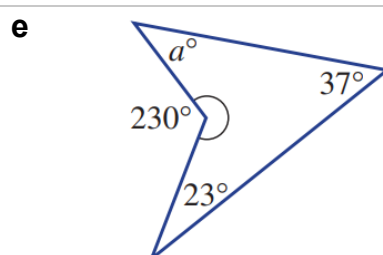
61



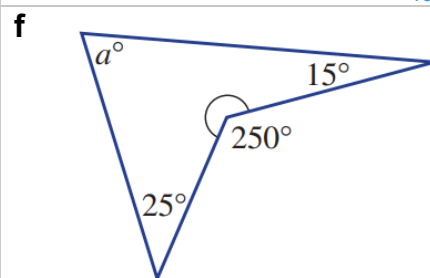
105



170

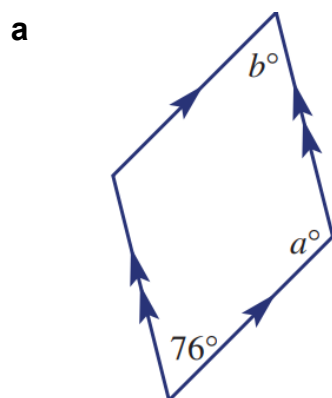


70

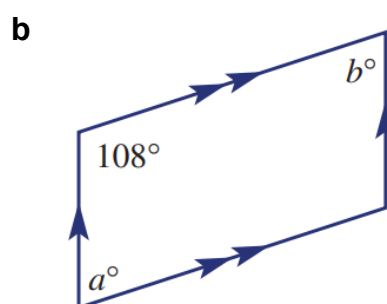


70

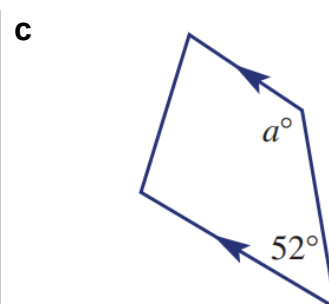
2 Using your knowledge of co-interior angles, determine the value of these pronumerals.



$a = 104, b = 76$

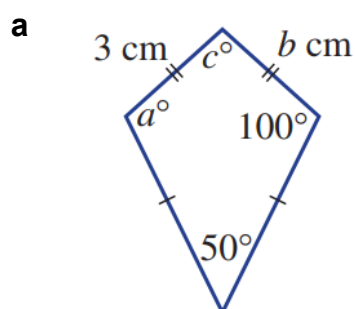


$a = 72, b = 72$

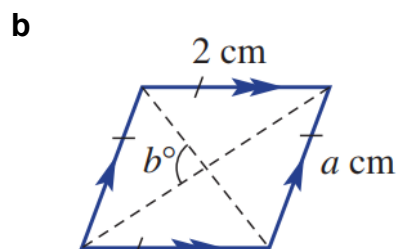


$a = 128$

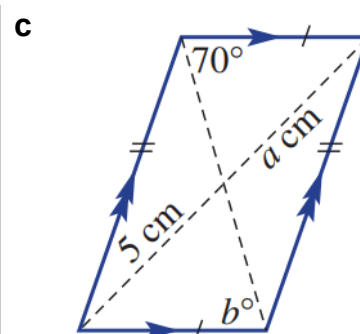
3 Using properties of special quadrilaterals, determine the value of these pronumerals.



$a = 100, b = 3, c = 110$



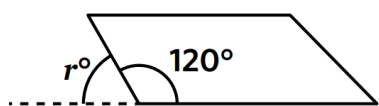
$a = 2, b = 90$



$a = 5, b = 70$

4 Find the value represented by the pronumeral.

a



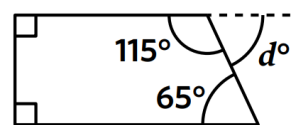
60

b



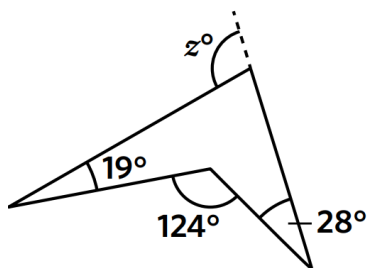
90

c



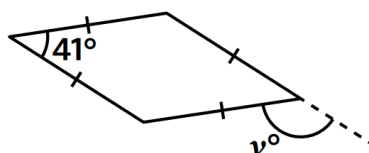
65

d



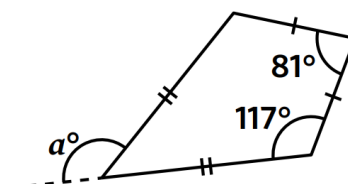
103

e



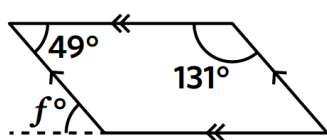
139

f



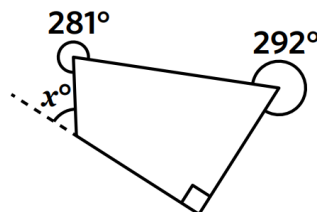
135

g



49

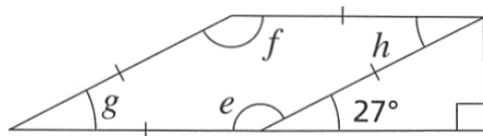
h



57

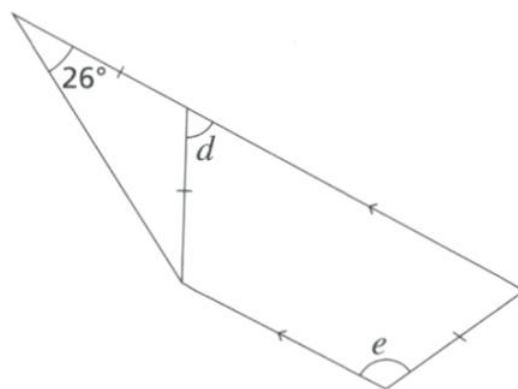
5 Find the values of the pronumerals.

a



$e = 153, f = 153, g = 27, h = 27$

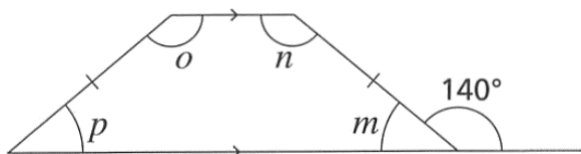
b



$d = 52, e = 128$

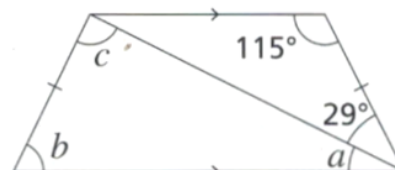
6 These two questions involve **isosceles trapeziums**. Adjacent angles in isosceles trapeziums are equal. Work out the size of each angle labelled with a letter.

a



$m = p = 40^\circ$
 $o = n = 140^\circ$

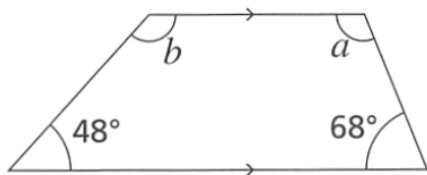
b



$a = 36, b = 65, c = 79$

7 Work out the values of the letters in these diagrams.

a



$$a = 112^\circ$$

$$b = 132^\circ$$

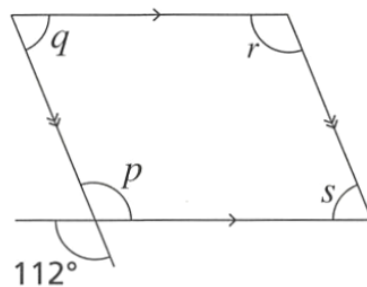
b



$$x = 125^\circ$$

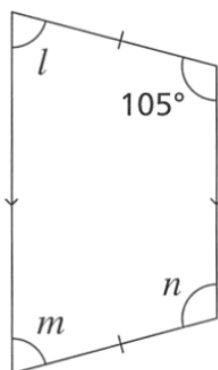
$$y = 64^\circ$$

c



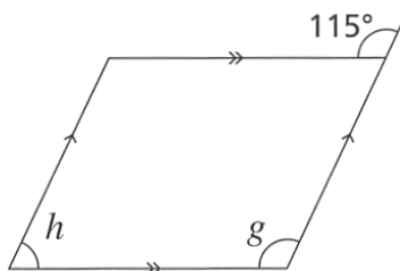
$$p = 112^\circ, q = 68^\circ, r = 112^\circ, s = 68^\circ$$

d



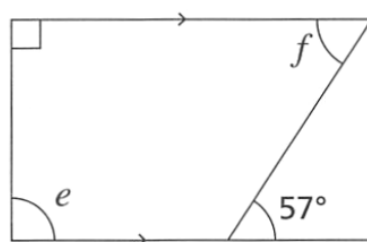
$$l = 75^\circ, m = 75^\circ, n = 105^\circ$$

e



$$g = 115^\circ, h = 65^\circ$$

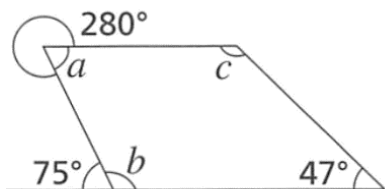
f



$$e = 90^\circ, f = 57^\circ$$

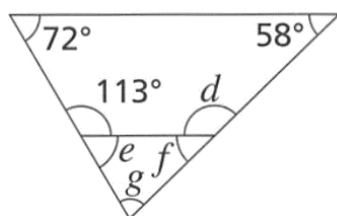
8 Determine the value of these pronumerals using your angle relationships knowledge.

a



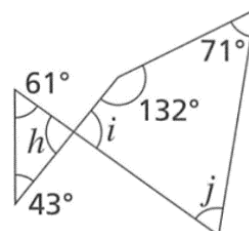
$$a = 80^\circ, b = 105^\circ, c = 128^\circ$$

b



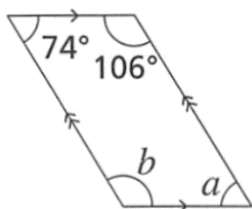
$$d = 117^\circ, e = 67^\circ, f = 63^\circ, g = 50^\circ$$

c



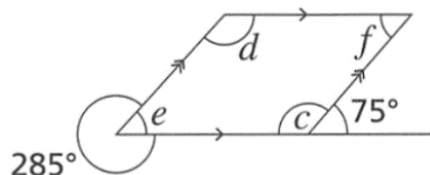
$$h = i = 76^\circ, j = 81^\circ$$

d



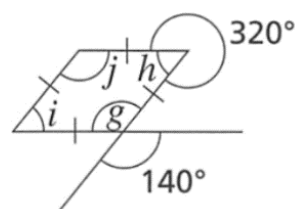
$$a = 74^\circ, b = 106^\circ$$

e



$$c = d = 105^\circ, e = f = 75^\circ$$

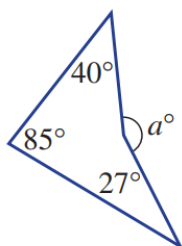
f



$$i = h = 40^\circ, j = g = 140^\circ$$

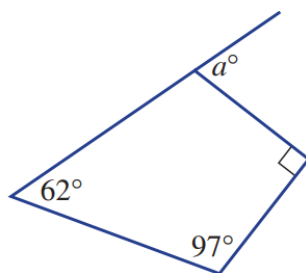
9 Determine the value of these pronumerals using your angle relationships knowledge.

a



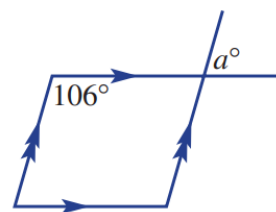
152

b



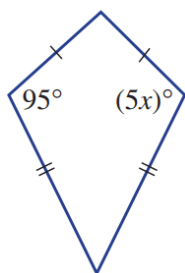
69

c



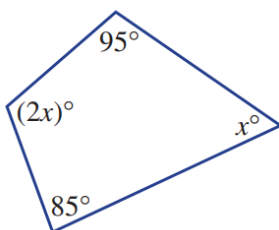
74

d



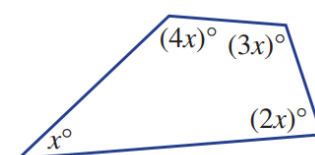
19

e



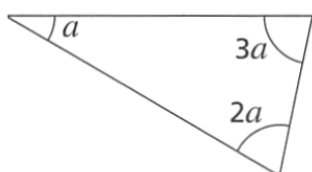
60

f



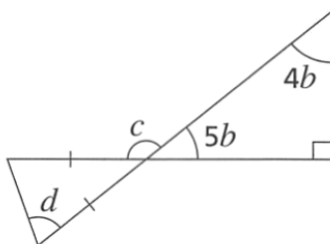
36

g



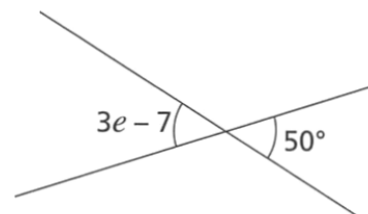
30

h



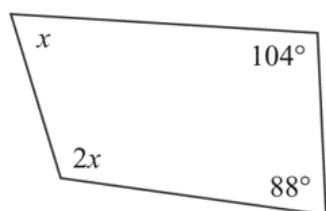
$b = 10, c = 130, d = 65$

i



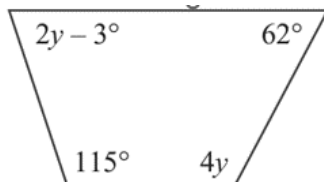
19

j



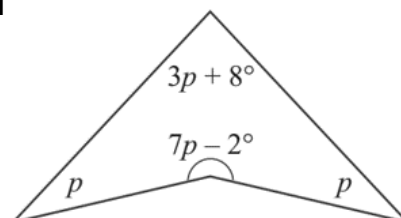
56

k



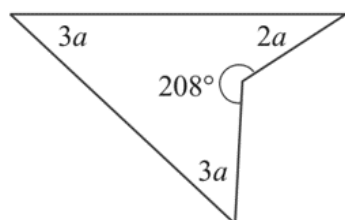
31

l



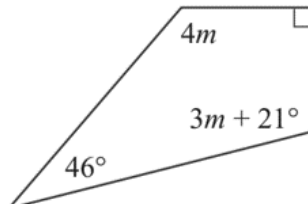
29.5

m



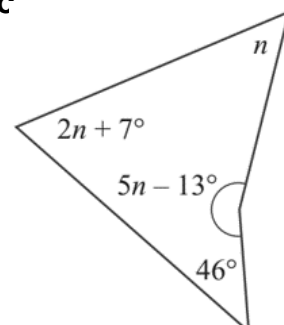
19

n



29

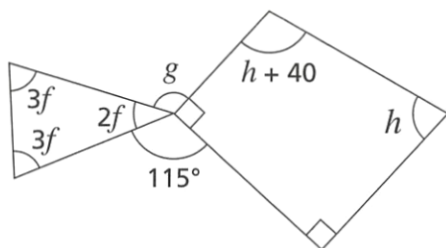
c



40

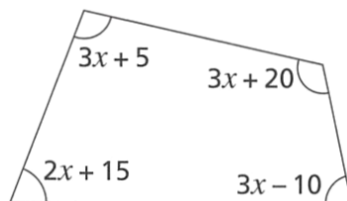
10 Determine the value of these pronumerals using your angle relationships knowledge.

a



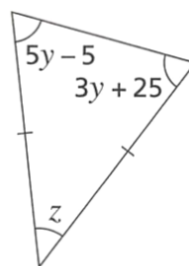
$$h = 70, f = 22.5, g = 110$$

b



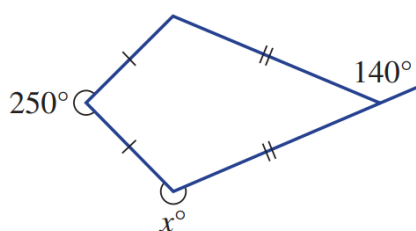
$$30$$

c



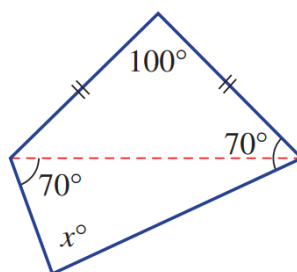
$$y = 15, z = 40$$

d



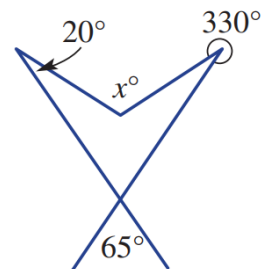
$$255$$

e



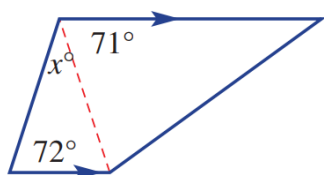
$$80$$

f



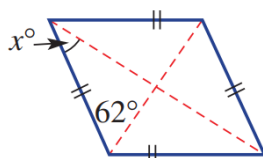
$$115$$

g



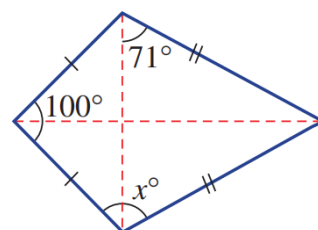
$$37$$

h



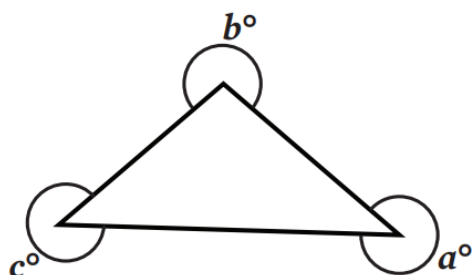
$$28$$

i



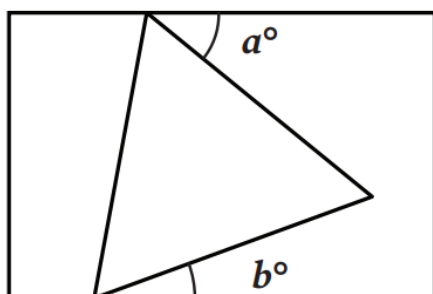
$$111$$

- 11 What is the value of $a + b + c$? Explain your reasoning.



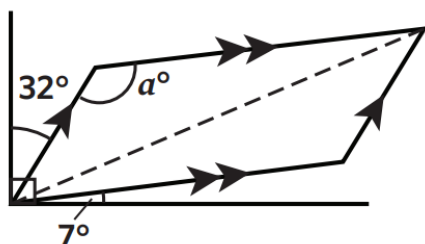
$$\begin{aligned} c + x &= 360 \\ b + y &= 360 \\ a + z &= 360 \\ c + x + b + y + a + z &= 1080 \\ a + b + c &= 1080 - (x + y + z) \\ x + y + z &= 180 \\ \therefore a + b + c &= 1080 - 180 = 900^\circ \end{aligned}$$

- 12 The diagram has an equilateral triangle inside a rectangle. Determine the value of $a + b$. Explain your reasoning.



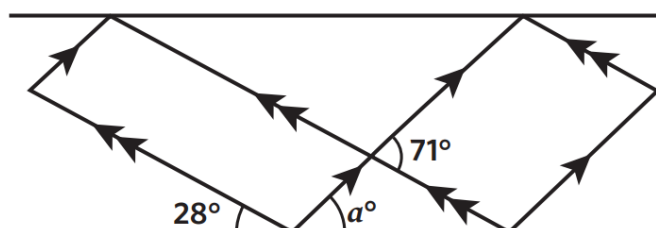
Construct line AB at C parallel to sides of rectangle
 $x = a$ and $y = b$ (alternate angles on parallel lines)
 $x + y = 60^\circ$ (angles in equilateral $\triangle$)
 $\therefore a + b = 60^\circ$

- 13 Determine the value of a in this diagram. Explain your reasoning.



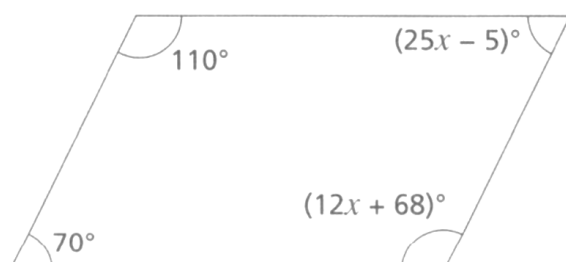
$$\begin{aligned} b &= 90 - 32 - 7 \text{ (angles in right-angle)} \\ &= 51^\circ \\ 2a + 2b &= 360 \text{ (angle sum of quad)} \\ 2a &= 360 - 2b = 360 - 2(51) \\ 2a &= 258^\circ \\ \therefore a &= 129^\circ \end{aligned}$$

- 14 Determine the value of a in this diagram. Explain your reasoning.



$$\begin{aligned} 2x + 71 + 71 &= 360 \text{ (angle sum of quad)} \\ 2x &= 218 \\ x &= 109 \\ a + 28 + 109 &= 180 \text{ (angles on straight line)} \\ a &= 43^\circ \end{aligned}$$

- 15 Is this shape a parallelogram? Explain your reasoning.

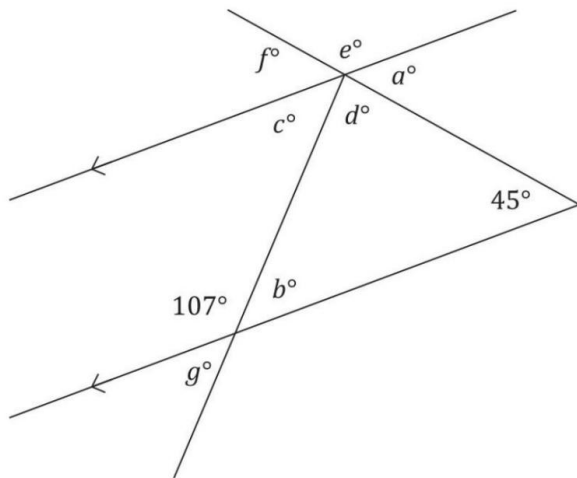


If parallelogram, then co-interior angles equal
 $110 + 25x - 5 = 180$
 $25x = 75$
 $x = 3$
 check using the other pair of co-interior angles $70 + 12x + 68 = 180$
 $70 + 12(3) + 68 = 174 \neq 180$
 $\therefore$ not parallelogram

MIXED PRACTICE

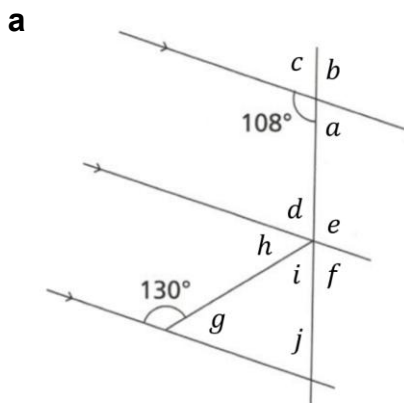
FOUNDATION

- 1 Determine the value represented by each pronumeral.

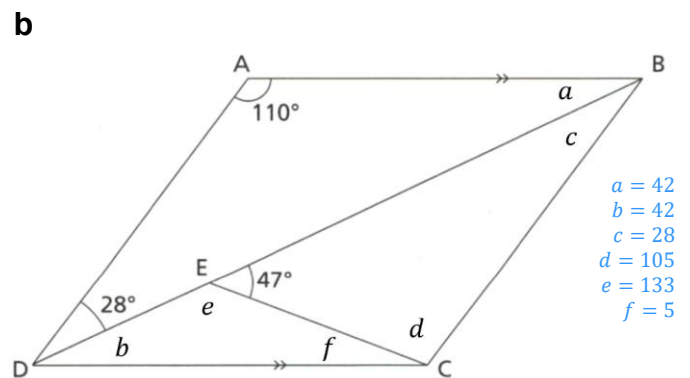


$$a = 45, b = 73, c = 73, d = 62, e = 135, f = 45, g = 73$$

- 2 Determine the sizes of all the unknown angles.

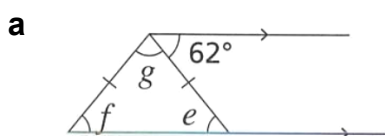


$$\begin{aligned} a &= 72 \\ b &= 108 \\ c &= 72 \\ d &= 72 \\ e &= 108 \\ f &= 72 \\ g &= 50 \\ h &= 50 \\ i &= 58 \\ j &= 72 \end{aligned}$$

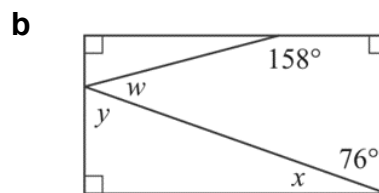


$$\begin{aligned} a &= 42 \\ b &= 42 \\ c &= 28 \\ d &= 105 \\ e &= 133 \\ f &= 5 \end{aligned}$$

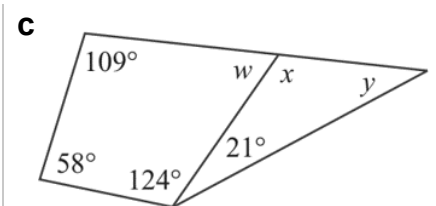
- 3 Work out the values of the pronumerals.



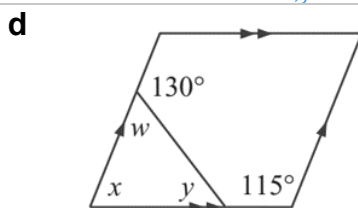
$$e = 62, f = 62, g = 56$$



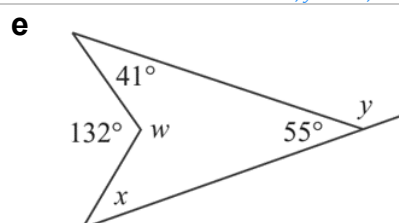
$$x = 14, y = 76, w = 36$$



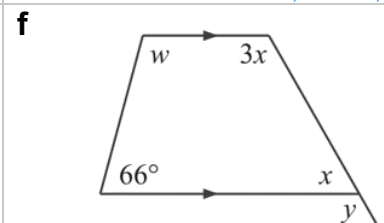
$$w = 69, x = 111, y = 48$$



$$w = 50, x = 65, y = 65$$



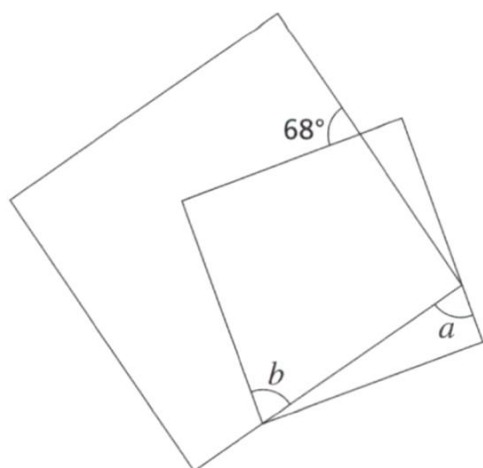
$$w = 228, x = 36, y = 125$$



$$x = 45, w = 114, y = 135$$

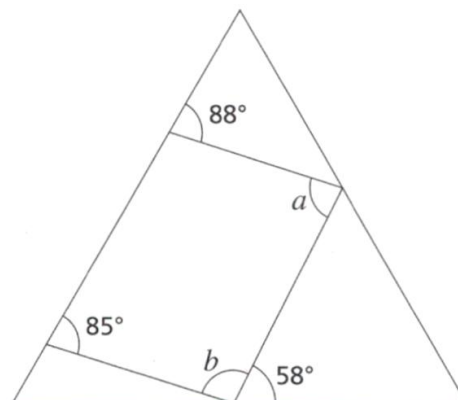
4 Work out the size of angles a and b .

a Two overlapping squares



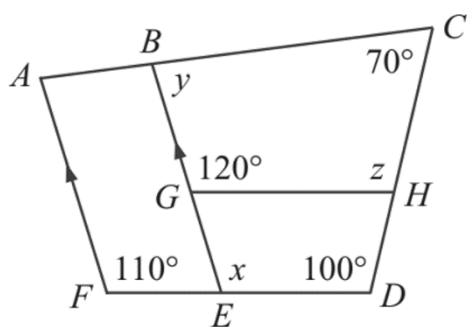
$$a = b = 68^\circ$$

b Quadrilateral inside an equilateral triangle



$$a = 86^\circ, b = 97^\circ$$

5 Find the values of the pronumerals. Provide full reasoning.



$$x = 110^\circ (\text{corresponding angles on parallel lines})$$

$$\angle EGH = 60^\circ (\text{angle sum of straight line})$$

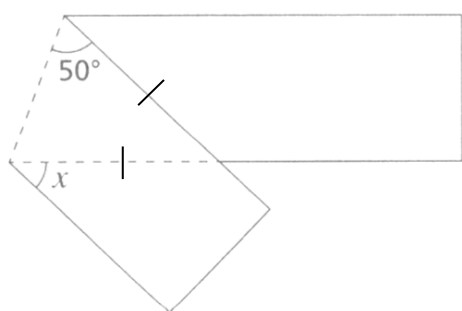
$$\angle GHD = 90^\circ (\text{angle sum of quadrilateral})$$

$$z = 90^\circ (\text{angle sum of straight line})$$

$$y = 80^\circ (\text{angle sum of quadrilateral})$$

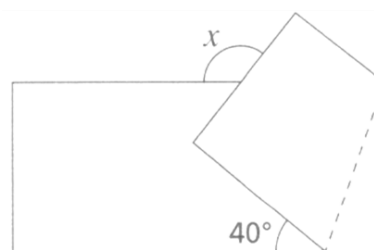
6 Juna is exploring angles by folding rectangular strips of paper. Work out the size of each angle marked x .

a



$$x = 80$$

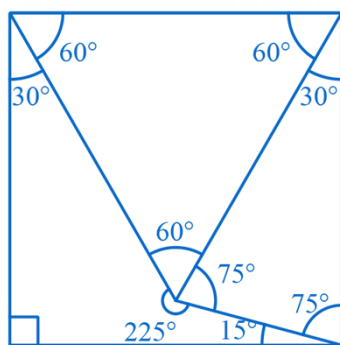
b



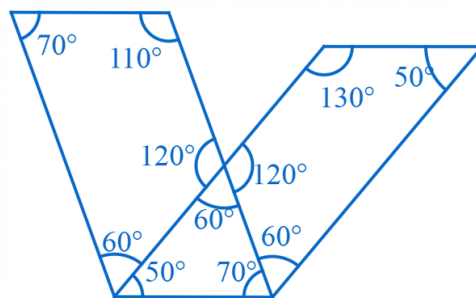
$$x = 130$$

7 Find every angle you can.

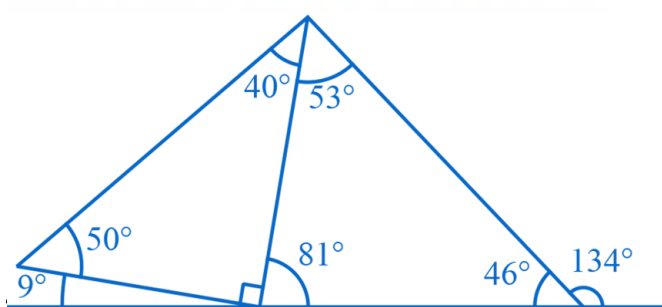
a PQRS is a square and PQT is an equilateral triangle.



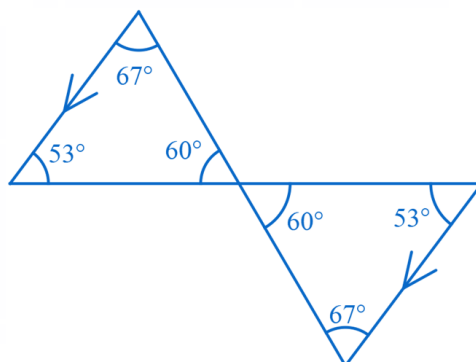
b This diagram shows two overlapping parallelograms.



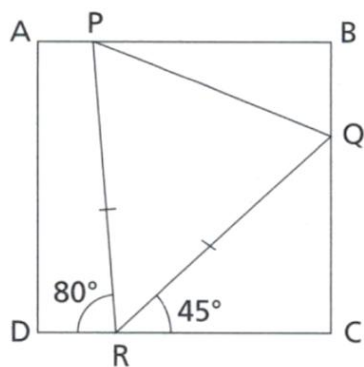
c



d



8 The diagram shows a triangle in a square. Find the size of $\angle BQP$. Provide full reasoning.



$$\angle PRQ = 55^\circ \text{ (angles on a straight line add up to } 180^\circ)$$

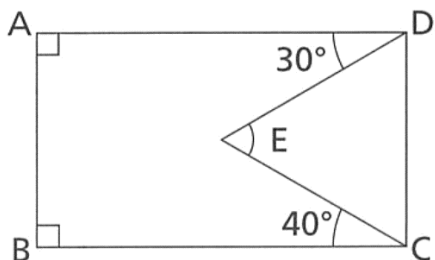
$$\angle RQP = 62.5^\circ \text{ (angles in a triangle add up to } 180^\circ \text{ and two angles in an isosceles triangle are equal)}$$

$$\angle CQR = 45^\circ \text{ (angles in a triangle add up to } 180^\circ)$$

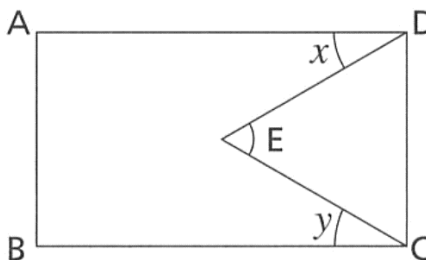
$$\angle BQP = 72.5^\circ \text{ (angles on a straight line add up to } 180^\circ)$$

9 In both diagrams, ABCD is a rectangle.

i



ii



a Using diagram i, prove that $\angle DEC = 70^\circ$. Provide full reasoning.

$$\angle EDC = 90^\circ - 30^\circ = 60^\circ \text{ (angles in a rectangle are right angles)}$$

$$\angle ECD = 90^\circ - 40^\circ = 50^\circ \text{ (angles in a rectangle are right angles)}$$

$$\angle DEC = 180^\circ - (60^\circ + 50^\circ) = 70^\circ \text{ (angles in a triangle add up to } 180^\circ)$$

b Using diagram ii, prove that $\angle DEC = (x + y)$

$$\angle EDC = 90^\circ - x \text{ (angles in a rectangle are right angles)}$$

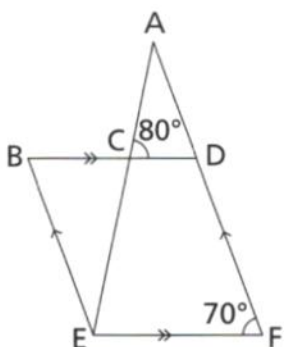
$$\angle ECD = 90^\circ - y \text{ (angles in a rectangle are right angles)}$$

$$\text{So } \angle EDC + \angle ECD = 180^\circ - (x + y)$$

$$\angle DEC = x + y \text{ (angles in a triangle add up to } 180^\circ)$$

10

a Prove that $\angle BEC = 30^\circ$

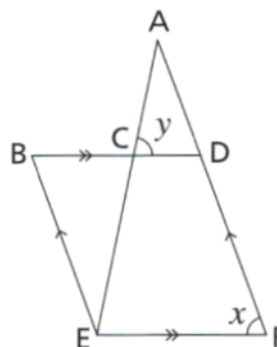


$$\angle BEF = 110^\circ \text{ (co-interior angles add up to } 180^\circ)$$

$$\angle AEF = 80^\circ \text{ (corresponding angles are equal)}$$

$$\angle BEC = 110^\circ - 80^\circ = 30^\circ$$

b Prove that $\angle BEC = 180^\circ - (x + y)$

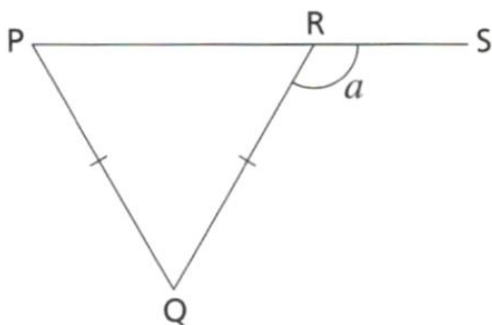


$$\angle BEF = 180^\circ - x \text{ (co-interior angles add up to } 180^\circ)$$

$$\angle AEF = y \text{ (corresponding angles are equal)}$$

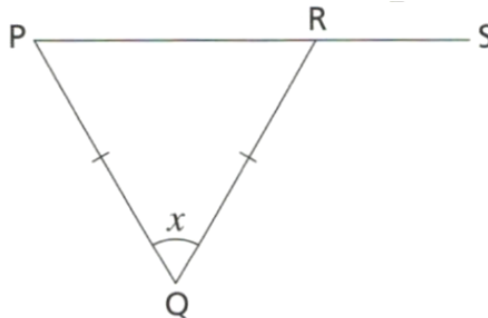
$$\angle BEC = (180^\circ - x) - y = 180^\circ - (x + y)$$

a Prove that that $\angle PQR = (2a - 180)$



$$\begin{aligned}\angle PRQ &= 180^\circ - a \text{ (angles on a straight line add up to } 180\text{)} \\ \angle QPR &= 180^\circ - a \text{ (base angles in an isosceles triangle are equal)} \\ \angle PQR &= 180^\circ - (180^\circ - a + 180^\circ - a) \\ &= (2a - 180)^\circ \text{ (angles in a triangle add up to } 180^\circ\text{)}\end{aligned}$$

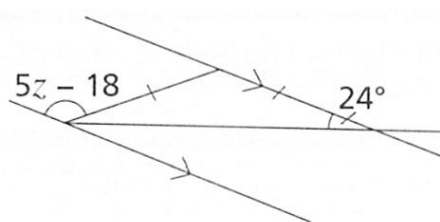
b Show that $\angle QRS = \left(90 + \frac{x}{2}\right)$



$$\begin{aligned}\angle PRQ &= \frac{180-x}{2} \text{ (base angles in an isosceles triangle are equal)} \\ \angle QRS &= 180 - \frac{180-x}{2} \text{ (angles on a straight line add up to } 180^\circ\text{)} \\ &= 90 + \frac{x}{2}\end{aligned}$$

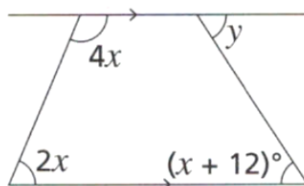
12 In each diagram, work out the value of each pronumeral.

a



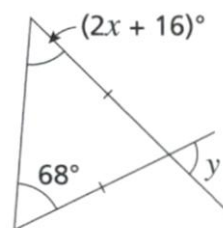
$$z = 30$$

b



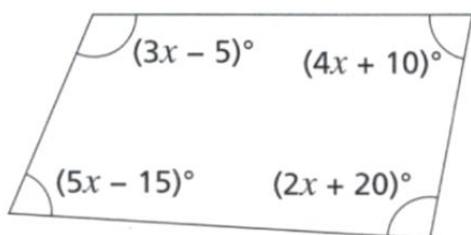
$$x = 30^\circ, y = 42^\circ$$

c



$$x = 26^\circ, y = 44^\circ$$

- 13 Show that this shape is a parallelogram. Justify your answer clearly.



Using angle sum of quadrilateral, $x = 25$

If this is a parallelogram, then $5x - 15 + 2x + 20 = 180$

$$\text{And } 3x - 5 + 4x + 10 = 180$$

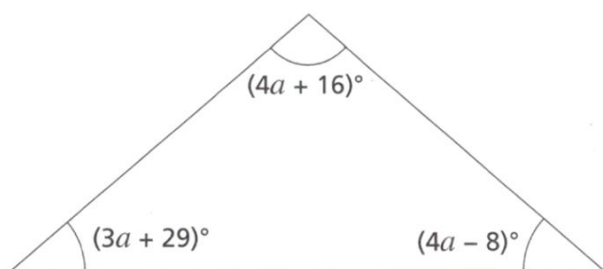
$$\text{And } 3x - 5 + 5x - 15 = 180$$

$$\text{And } 4x + 10 + 2x + 20 = 180$$

Because they would be co-interior.

By substituting $x = 25$, all these equations are true, so there are two pairs of parallel sides, and therefore the quadrilateral is a parallelogram.

- 14 Show that this triangle is isosceles.



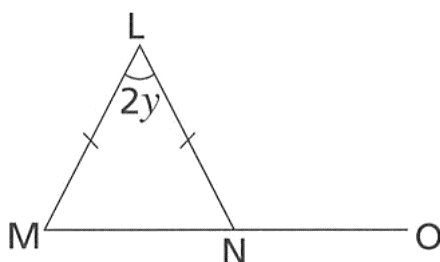
Using angle sum of triangle, $a = 13$

$$3a + 29 = 3(13) + 29 = 68$$

$$4a + 16 = 4(13) + 16 = 68$$

Therefore isosceles as it has two equal angles.

- 15 Prove that $\angle LNO = 90^\circ + y$



$$\angle LNM = \frac{180 - 2y}{2} \text{ (angles at the base of an isosceles triangle are equal)}$$

$$\text{So } \angle LNM = 90^\circ - y$$

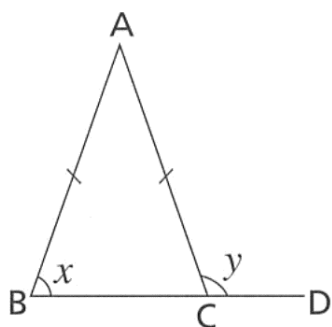
$$\angle LNM + \angle LNO = 180^\circ$$

(adjacent angles on a straight line add up to 180°)

$$\text{So } 90^\circ - y + \angle LNO = 180^\circ,$$

$$\text{so } \angle LNO = 90^\circ + y$$

16 Consider the diagram to the right.



a Prove that $y = 180^\circ - x$

$\angle ACB = x$ (angles at the base of an isosceles triangle are equal)

$\angle ACB + \angle ACD = 180^\circ$

(adjacent angles on a straight line add up to 180°)

So $x + y = 180^\circ$, so $y = 180^\circ - x$

b Prove that $\angle BAC = 180^\circ - 2x$

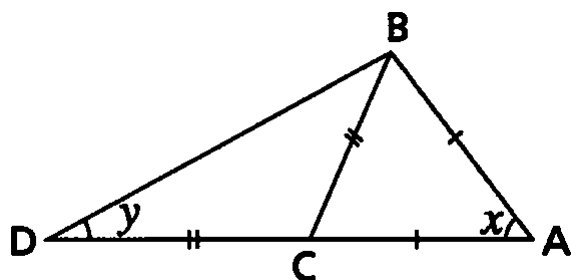
$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

(angles in a triangle add up to 180°)

So $\angle BAC + x + x = 180^\circ$,

so $\angle BAC = 180^\circ - 2x$

17 Prove that $x + 4y = 180^\circ$



$\angle CBD = y$ (angles at the base of an isosceles triangle are equal)

$\angle BCD = 180^\circ - 2y$ (angles in a triangle add up to 180°)

$\angle BCA = 2y$ (adjacent angles on a straight line add up to 180°)

$\angle ABC = 2y$ (angles at the base of an isosceles triangle are equal)

In triangle ABC, $\angle BCA + \angle ABC +$

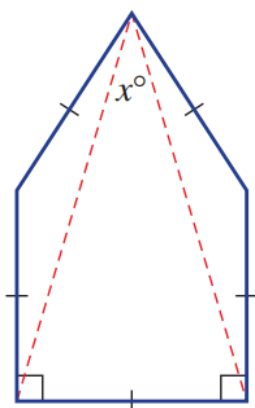
$\angle CAB = 180^\circ$ (angles in a triangle add up to 180°)

So $2y + 2y + x = 180^\circ$,

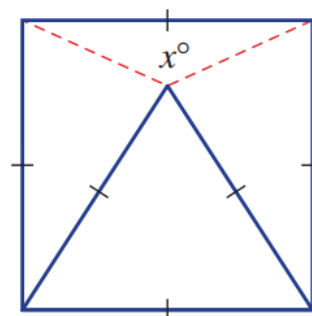
hence $x + 4y = 180^\circ$

18 Find the value of x in these diagrams.

a



b

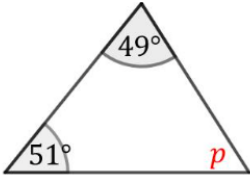
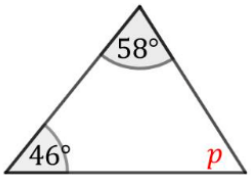
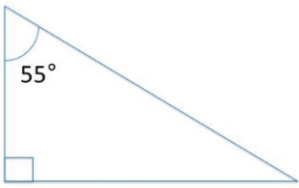
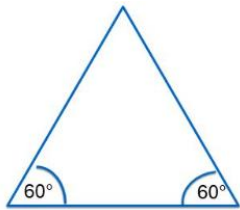
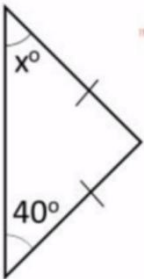
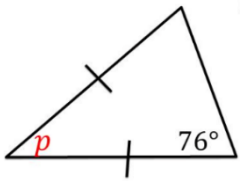
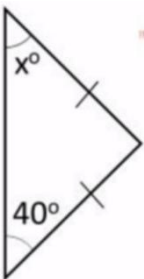
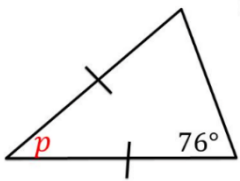
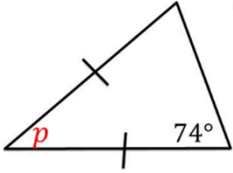
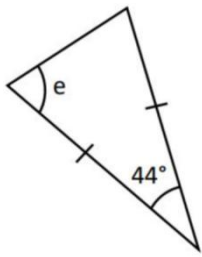


30°

150°

CHECK YOUR UNDERSTANDING

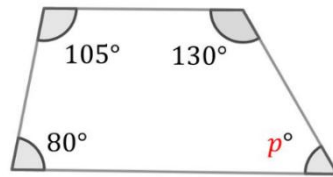
Angle sum of a triangle

1 What is the value of p? a 51° b 70° c 80° d Not enough information 	2 What is the value of p? a 76° b 66° c 46° d Not enough information 
3 What is the missing angle? a 215° b 45° c 55° d 35° 	4 What type of triangle is this? a Equilateral b Right-angled c Isosceles d Scalene 
5 What is the value of x? a 40° b 140° c 50° d 100° 	6 What is the value of p? a 28° b 76° c 104° d Not enough information 
7 What is the value of x? a 40° b 140° c 50° d 100° 	8 What is the value of p? a 28° b 76° c 104° d Not enough information 
9 What is the value of x? a 32° b 74° c 106° d Not enough information 	10 What is the value of e? a 92° b 136° c 68° d 44° 

Angle sum of a quadrilateral

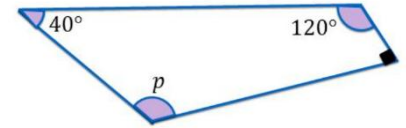
1 What is the value of p ?

- a 50°
- b 80°
- c 45°
- d 55°



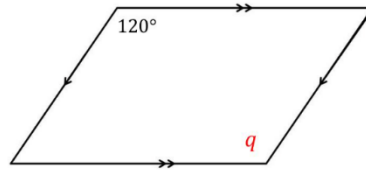
2 What is the value of p ?

- a 200°
- b 110°
- c 120°
- d Not enough information.



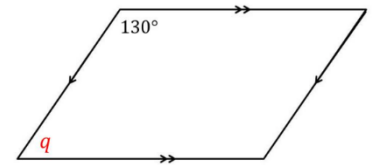
3 What is the value of q ?

- a 60°
- b 80°
- c 120°
- d Not enough information



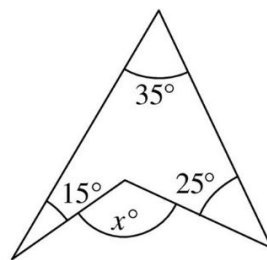
4 What is the value of q ?

- a 50°
- b 130°
- c 70°
- d Not enough information



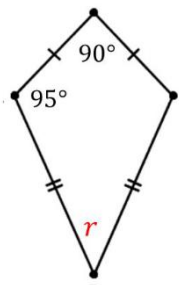
5 What is the value of x ?

- a 75°
- b 95°
- c 105°
- d 115°



6 What is the value of r ?

- a 90°
- b 85°
- c 80°
- d 95°



7 What is the value of p ?

- a 75°
- b 60°
- c 30°
- d Not enough information

