Student Name

Mathematics Stage 5 Core

ALGEBRAIC TECHNIQUES A

Book 1 Apply the 4 operations to simplify algebraic fractions with numerical denominators

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-ALG-C-01 simplifies algebraic fractions with numerical denominators and expands algebraic expressions

Apply the 4 operations to simplify algebraic fractions with numerical denominators

- Simplify expressions that involve algebraic fractions with numerical denominators

REVIEW OF PRIOR KNOWLEDGE

1 Simplify these fractions.

a $\frac{6}{12}$	**b** $\frac{18}{24}$	**c** $\frac{9}{24}$	**d** $\frac{16}{16}$
e $\frac{14}{7}$	**f** $\frac{12}{18}$	**g** $\frac{8}{56}$	**h** $\frac{9}{27}$

2 Add and subtract these fractions.

a $\frac{5}{7}+\frac{1}{7}$	**b** $\frac{2}{15}+\frac{8}{15}+\frac{1}{15}$	**c** $\frac{6}{7}-\frac{2}{7}-\frac{4}{7}$
d $1-\frac{1}{4}$	**e** $1-\frac{3}{5}$	**f** $2-\frac{6}{7}$

3 Simplify these expressions.

a $7a+12a$	**b** $16u-3u$	**c** $3+b$
d $2x+3+5x$	**e** $7ab+b$	**f** $-4a+5a-7a$

4 Find the lowest common denominator (LCD) of these fractions.

a $\frac{}{3}$ and $\frac{}{5}$	**b** $\frac{}{4}$ and $\frac{}{5}$	**c** $\frac{}{7}$ and $\frac{}{6}$	**d** $\frac{}{3}$ and $\frac{}{6}$
e $\frac{}{3}$ and $\frac{}{5}$	**f** $\frac{}{4}$ and $\frac{}{5}$	**g** $\frac{}{12}$ and $\frac{}{6}$	**h** $\frac{}{10}$ and $\frac{}{20}$

5 Find the missing number to make these fractions equivalent.

a $\frac{2}{3}=\frac{\square}{6}$	**b** $\frac{4}{7}=\frac{\square}{21}$	**c** $\frac{1}{3}=\frac{\square}{12}$	**d** $\frac{6}{11}=\frac{\square}{55}$
e $\frac{x}{5}=\frac{\square}{10}$	**f** $\frac{2a}{7}=\frac{\square}{21}$	**g** $\frac{4z}{5}=\frac{\square}{20}$	**h** $\frac{3k}{10}=\frac{\square}{50}$
i $\frac{3c}{5}=\frac{\square}{10}$	**j** $\frac{v}{2}=\frac{\square}{16}$	**k** $\frac{7a}{8}=\frac{\square}{24}$	**l** $\frac{5n}{6}=\frac{\square}{42}$

6 Add and subtract these fractions.

a $\frac{7}{20}+\frac{5}{10}$	**b** $\frac{7}{10}-\frac{2}{5}$	**c** $\frac{11}{12}-\frac{1}{3}-\frac{1}{4}$
d $\frac{1}{7}+\frac{2}{5}$	**e** $\frac{1}{4}+\frac{2}{3}$	**f** $\frac{3}{7}-\frac{2}{9}$
g $\frac{1}{4}+\frac{2}{5}$	**h** $\frac{3}{9}+\frac{2}{5}$	**i** $\frac{7}{12}-\frac{2}{9}$

7 Simplify

a $5\times 2m$	**b** $3\times 6r$	**c** $8\times ab$
d $2\times -7y$	**e** $5\times b$	**f** $17y\times 3$

8 Multiply these fractions.

a $\frac{2}{5}\times\frac{1}{2}$	**b** $\frac{2}{5}\times\frac{3}{4}$	**c** $\frac{3}{5}\times\frac{6}{7}$	**d** $\left(\frac{3}{4}\right)^2$	**e** $\frac{2}{3}\times\frac{3}{5}\times\frac{4}{7}$

9 Simplify these algebraic fractions.

a $\frac{2x}{6}$	**b** $\frac{5xy}{2y}$	**c** $\frac{7ab}{14b}$	**d** $\frac{2bc}{5bc}$

10 Divide these fractions.

a $\frac{2}{3}\div\frac{4}{5}$	**b** $\frac{1}{2}\div\frac{1}{4}$	**c** $\frac{3}{4}\div\frac{2}{5}$	**d** $\frac{1}{3}\div 4$	**e** $4\div\frac{7}{3}$

11 Cancel these fractions before you multiply.

a	**b**	**c**	**d**	**e**
$\frac{12}{18} \times \frac{5}{7}$	$\frac{5}{7} \times \frac{13}{15}$	$\frac{2}{5} \times \frac{15}{24}$	$\frac{2}{9} \times \frac{3}{4}$	$\frac{8}{27} \times \frac{9}{16}$

12 Identify the reciprocal:

a	5	**k**	$\frac{1}{5}$
b	3	**l**	$\frac{1}{7}$
c	113	**m**	$\frac{1}{2754}$
d	1	**n**	$\frac{1}{m}$
e	a	**o**	$\frac{1}{2m}$
f	ab^2	**p**	$\frac{1}{2mn^2}$
g	$-ab^2$	**q**	$\frac{3}{2mn^2}$
h	$-a$	**r**	$\frac{5}{2mn^2}$
i	-3	**s**	$\frac{5a}{2m}$
j	-113	**t**	$-\frac{5a}{m}$

13 Divide these fractions.

a	**b**	**c**	**d**	**e**
$\frac{2}{3} \div \frac{4}{5}$	$\frac{1}{2} \div \frac{1}{4}$	$\frac{3}{4} \div \frac{2}{5}$	$\frac{1}{3} \div 4$	$4 \div \frac{7}{3}$

ALGEBRAIC FRACTION ADDITION: SAME DENOMINATORS

- **Algebraic fractions** have **algebraic expressions** in their **numerator** or **denominator**.

$\frac{2}{3}$ $\frac{5}{7}$ $\frac{20}{3}$

Fractions

$\frac{2x}{5}$ $\frac{3}{7a+4}$ $\frac{2x-4y}{7a+9b}$

Algebraic fractions

Rule for Adding and Subtracting Fractions	
The denominators must be the same	
Adding fractions	Subtracting fractions
$\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}$	$\frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}$

Example Add/subtract algebraic fractions with same denominator.

$\frac{\mathbf{5x}}{\mathbf{13}}+\frac{\mathbf{3x}}{\mathbf{13}}$ $=\frac{5x+3x}{13}=\frac{8x}{13}$	Why does $5x+3x$ become $8x$? *$5x$ and $3x$ are ....* Why can't we simplify $5x-3y$? *$5x$ and $-3y$ are not ....*
$\frac{\mathbf{5x}}{\mathbf{13}}-\frac{\mathbf{3y}}{\mathbf{13}}$ $=\frac{5x-3y}{13}$	

Simplify these expressions.

a $\frac{x}{1}+\frac{2x}{1}$

$3x$

b $\frac{x}{4}+\frac{2x}{4}$

$\frac{3x}{4}$

c $\frac{a}{3}+\frac{2a}{3}$

a

d $\frac{3y}{5}-\frac{y}{5}$

$\frac{2y}{5}$

e $\frac{4p}{6}-\frac{p}{6}$

$\frac{p}{2}$

f $\frac{2y}{15}+\frac{3y}{15}$

$\frac{y}{3}$

Key ideas

1. When adding or subtracting fractions with the same denominator, we add or subtract the ..... and keep the ..... the same.

1 Add and subtract these fractions.

a $\frac{3x}{10} + \frac{4x}{10} = \frac{\square}{10}$ $\frac{7x}{10}$

b $\frac{5a}{8} - \frac{2a}{8} = \frac{\square}{\square}$ $\frac{3a}{8}$

c $\frac{4c}{7} + \frac{2c}{7} - \frac{c}{7} = \frac{\square}{\square}$ $\frac{5c}{7}$

d $\frac{4k}{3} + \frac{k}{3}$ $\frac{5k}{3}$

e $\frac{3y}{5} - \frac{y}{5}$ $\frac{2y}{5}$

f $\frac{7p}{13} - \frac{2p}{13}$ $\frac{5p}{13}$

g $\frac{10r}{7} - \frac{2r}{7}$ $\frac{8r}{7}$

h $\frac{8q}{5} - \frac{2q}{5}$ $\frac{6q}{5}$

i $\frac{8q+5}{11} + \frac{2q}{11}$ $\frac{6q+5}{11}$

j $\frac{3x}{16} + \frac{5x}{16} + \frac{7x}{16}$ $\frac{15x}{16}$

k $\frac{9e}{11} - \frac{4e}{11} - \frac{e}{11}$ $\frac{4e}{11}$

l $\frac{5}{3k} - \frac{2}{3k} + \frac{4}{3k}$ $\frac{8}{3k}$

2 Add and subtract these fractions.

a $\frac{2y}{3} + \frac{5y}{3}$ $\frac{7y}{3}$

b $\frac{11d}{7} - \frac{5d}{7}$ $\frac{6d}{7}$

c $\frac{24a}{31} - \frac{13a}{31}$ $\frac{11a}{31}$

d $\frac{8b}{19} + \frac{5b}{19}$ $\frac{13b}{19}$

e $\frac{100k}{21} - \frac{99k}{21}$ $\frac{k}{21}$

f $\frac{5g^2}{47} + \frac{12g^2}{47}$ $\frac{17g^2}{47}$

g $\frac{a}{2} - \frac{a}{2}$ 0

h $\frac{7p}{9} + \frac{p}{9}$ $\frac{8p}{9}$

3 Add and subtract these fractions, then simplify the result.

a $\frac{5x}{10} + \frac{3x}{10} = \frac{8x}{10} = \frac{4x}{5}$ (÷ 2 above, ÷ 2 below)

b $\frac{a}{8} + \frac{a}{8} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{a}{4}$

c $\frac{3n}{4} + \frac{n}{4} = \frac{\square}{\square} = \_\_\_$ n

d $\frac{8}{9x} - \frac{2}{9x} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{2}{3x}$

e $\frac{3c}{2} + \frac{5c}{2} = \frac{\square}{\square} = \_\_\_$ $4c$

f $\frac{11h}{18} - \frac{h}{18} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{5h}{9}$

g $\frac{11}{3x} - \frac{2}{3x} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{3}{x}$

h $\frac{5y}{12} + \frac{3y}{12} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{2y}{3}$

i $\frac{13a}{18} - \frac{4a}{18} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{a}{2}$

ALGEBRAIC FRACTION ADDITION: DIFFERENT DENOMINATORS

- To add or subtract **algebraic fractions** with **different denominators**:
  1. Rewrite the fractions with the **lowest common multiple** of the denominators.
  2. Add or subtract the numerators.

Example Add/subtract algebraic fractions with different denominators.

$\frac{4x}{3} - \frac{x}{6}$ 1. Same denominators $\frac{\quad}{6} - \frac{\quad}{6}$ $\frac{8x}{6} - \frac{x}{6}$ 2. Subtract numerators $\frac{7x}{6}$	What needs to be the same for us to add or subtract fractions? *To add or subtract fractions, the ..... must be the same.* How do we determine the common denominator? *To make the denominators the same, we need to find the .... of the denominators.*
$\frac{5x}{3} + \frac{11x}{4}$ 1. Same denominators $\frac{\quad}{12} + \frac{\quad}{12}$ $\frac{20x}{12} + \frac{33x}{12}$ 2. Subtract numerators $\frac{53x}{12}$	

Simplify these expressions.

a $\frac{y}{4} + \frac{y}{2}$

$= \frac{y}{4} + \frac{\quad}{4}$

$=$

$\frac{3y}{4}$

b $\frac{3x}{6} - \frac{x}{2}$

0

c $\frac{y}{3} - \frac{a}{4}$

$\frac{4y - 3a}{12}$

d $\frac{x}{5} + \frac{2x}{3}$

$\frac{13x}{15}$

e $3x + \frac{x}{2}$

$\frac{7x}{2}$

f $\frac{4x}{3} + \frac{3x}{10}$

$\frac{49x}{30}$

Key ideas

1. To add or subtract fractions, the ..... must be the same.
2. To make the denominators the same, determine a .... of the two denominators.

1 Add and subtract these fractions.

a $\frac{a}{4} + \frac{a}{2}$

$= \frac{a}{4} + \frac{\square}{4} = \frac{\square}{\square}$

$\frac{3a}{4}$

b $\frac{r}{3} + \frac{r}{4}$

$= \frac{\square}{12} + \frac{\square}{12} = \frac{\square}{\square}$

$\frac{7r}{12}$

c $\frac{5t}{6} - \frac{2t}{3}$

$= \frac{\square}{6} - \frac{\square}{6} = \frac{\square}{\square}$

$\frac{t}{6}$

d $\frac{m}{4} + \frac{5m}{6}$

$= \frac{\square}{12} + \frac{\square}{12} = \frac{\square}{\square}$

$\frac{13m}{12}$

e $\frac{3a}{4} - \frac{2a}{5} + \frac{3a}{10}$

$= \frac{\square}{20} - \frac{\square}{20} + \frac{\square}{20} = \frac{\square}{\square}$

$\frac{13a}{20}$

f $\frac{g}{5} + \frac{2g}{3} - \frac{g}{6}$

$= \frac{\square}{30} + \frac{\square}{30} - \frac{\square}{30} = \frac{\square}{\square} = \frac{\square}{\square}$

$\frac{7g}{10}$

2 Complete these simplifications.

a $\frac{x}{4} + \frac{2x}{3}$

$= \frac{\square}{12} + \frac{\square}{12}$

$= \frac{\square}{12}$

$\frac{11x}{12}$

b $\frac{5x}{7} - \frac{2x}{5}$

$= \frac{\square}{35} - \frac{\square}{35}$

$= \frac{\square}{35}$

$\frac{11x}{35}$

c $\frac{x+1}{2} + \frac{2x+3}{4}$

$= \frac{\square(x+1)}{4} + \frac{2x+3}{4}$

$= \frac{\square + 2x + 3}{4}$

$= \frac{\square}{4}$

$\frac{4x+5}{4}$

3 Add and subtract these fractions.

a $\frac{5t}{6} - \frac{t}{2}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$ Simplify

$\frac{t}{3}$

b $\frac{4d}{5} + \frac{7d}{10}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$ Simplify

$\frac{3d}{2}$

c $\frac{n}{2} + \frac{5n}{12}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

$\frac{11n}{12}$

d $\frac{5c}{8} - \frac{c}{3}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$\frac{7c}{24}$

e $\frac{2q}{3} + \frac{q}{2} - \frac{3q}{4}$

$= \frac{\square}{\square} + \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$\frac{5q}{12}$

f $\frac{7e}{9} - \frac{e}{2} + \frac{5e}{6}$

$= \frac{\square}{\square} - \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$\frac{10e}{9}$

a $\frac{4v}{5} + \frac{v}{4} - \frac{3v}{10}$

$= \frac{\square}{\square} + \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$\frac{3v}{4}$

b $\frac{3p}{4} - \frac{2p}{3} + \frac{p}{6}$

$= \frac{\square}{\square} - \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$\frac{p}{4}$

4 Madeline and Angus are working out $\frac{11a}{4} - \frac{5a}{2}$

The answer is $\frac{a}{4}$

The answer is $\frac{1}{4}a$

Explain why they are both correct.

$\frac{a}{4}$ is the same as $a \div 4 = a \times \frac{1}{4}$

4 Simplify the following. You will need to rewrite the fractions with a common denominator.

a $\frac{p}{2} - \frac{p}{3}$ $\frac{p}{6}$	**b** $\frac{a}{2} + \frac{a}{3}$ $\frac{5a}{6}$	**c** $\frac{a}{4} + \frac{a}{5}$ $\frac{9a}{20}$	**d** $\frac{p}{2} + \frac{p}{5}$ $\frac{7p}{10}$
e $\frac{q}{4} + \frac{q}{2}$ $\frac{3q}{4}$	**f** $\frac{2k}{5} + \frac{3k}{7}$ $\frac{29k}{35}$	**g** $\frac{2t}{5} - \frac{t}{3}$ $\frac{t}{15}$	**h** $\frac{9u}{11} - \frac{u}{2}$ $\frac{7u}{22}$
i $\frac{8y}{3} - \frac{5y}{6}$ $\frac{11y}{6}$	**j** $\frac{2m}{5} + \frac{4m}{3}$ $\frac{26m}{15}$	**k** $\frac{7p}{6} + \frac{2p}{5}$ $\frac{47p}{30}$	**l** $\frac{x}{4} + \frac{3x}{8}$ $\frac{5}{8}$
m $\frac{r}{3} - \frac{r}{2}$ $-\frac{r}{6}$	**n** $\frac{6u}{7} - \frac{7u}{6}$ $-\frac{13u}{42}$	**o** $\frac{4h}{9} + \frac{2d}{5}$ $\frac{38d}{45}$	**p** $\frac{5m}{12} - \frac{3m}{8}$ $\frac{m}{24}$

5 Simplify the following. Write as a single fraction. Reminder that $4x$ is the same as $\frac{4x}{1}$

a $4x + \frac{x}{3}$ $\frac{13x}{3}$	**b** $3x + \frac{x}{2}$ $\frac{7x}{2}$	**a** $\frac{a}{5} + 2a$ $\frac{11x}{5}$	**c** $\frac{8p}{3} - 2p$ $\frac{2p}{3}$
d $\frac{10u}{3} + \frac{3v}{10}$ $\frac{100u + 9v}{30}$	**b** $\frac{7y}{10} - \frac{2x}{5}$ $\frac{7y - 4x}{10}$	**c** $2t + \frac{7p}{2}$ $\frac{4t + 7p}{2}$	**e** $\frac{x}{3} - y$ $\frac{x - 3y}{3}$

6 Complete each calculation, leaving your answer as a multiple of π.

a $\pi - \frac{\pi}{3}$ $\frac{2\pi}{3}$	**b** $\pi + \frac{\pi}{3}$ $\frac{4\pi}{3}$	**c** $\pi + \frac{\pi}{4}$ $\frac{5\pi}{4}$	**d** $\pi - \frac{\pi}{4}$ $\frac{3\pi}{4}$
e $\pi + \frac{\pi}{6}$ $\frac{7\pi}{6}$	**f** $\pi - \frac{\pi}{6}$ $\frac{5\pi}{6}$	**g** $2\pi - \frac{\pi}{3}$ $\frac{t\pi}{3}$	**h** $2\pi - \frac{\pi}{4}$ $\frac{7\pi}{4}$
i $2\pi - \frac{\pi}{6}$ $\frac{11\pi}{6}$	**j** $2\pi + \frac{\pi}{3}$ $\frac{7\pi}{3}$	**k** $\pi + \frac{\pi}{2}$ $\frac{3\pi}{2}$	**l** $\pi + \frac{2\pi}{3}$ $\frac{5\pi}{3}$
m $\pi + \frac{3\pi}{4}$ $\frac{7\pi}{4}$	**n** $2\pi - \frac{3\pi}{4}$ $\frac{5\pi}{4}$	**o** $\frac{3\pi}{2} + \frac{2\pi}{3}$ $\frac{13\pi}{6}$	**p** $\frac{\pi}{3} - \frac{\pi}{4}$ $\frac{\pi}{4}$
q $\frac{7\pi}{6} - \pi$ $\frac{\pi}{6}$	**r** $\frac{4\pi}{3} - \pi$ $\frac{\pi}{3}$	**s** $\frac{5\pi}{2} - 2\pi$ $\frac{\pi}{2}$	**t** $\frac{13\pi}{6} - 2\pi$ $\frac{\pi}{6}$

7 Cedric earns an amount $\$x$ every week. He spends $\frac{1}{3}$ of his income on rent and $\frac{1}{4}$ on groceries.

a Write an algebraic fraction for the amount of money he spends on rent.

$\frac{x}{3}$

b Write an algebraic fraction for the amount of money he spends on groceries.

$\frac{x}{4}$

c Write a simplified algebraic fraction for the total amount of money he spends on rent and groceries.

$\frac{7x}{12}$

8 Edgar fills the bathtub so it is a quarter full and then adds half a bucket of water.
A full bathtub can contain T litres and a bucket contains B litres.

a Write the total amount of water in the bathtub as the sum of two algebraic fractions.

$\frac{T}{4}+\frac{B}{2}$

b Simplify the expression in part **a** to get a single algebraic fraction.

$\frac{T+2B}{4}$

9 Annie's bank account is halved in value and then \$20 is removed.
If it initially had $\$A$ in it, write an algebraic fraction for the amount left.

$\frac{A}{2}-20$ or $\frac{A-40}{2}$

10 Simplify these expressions.

a $\frac{z}{4}+\frac{z}{3}+\frac{z}{12}$

$\frac{2z}{3}$

b $\frac{2x}{5}+\frac{x}{2}-\frac{x}{5}$

$\frac{7x}{10}$

c $\frac{7u}{2}+\frac{3u}{4}-\frac{5u}{8}$

$\frac{29u}{8}$

d $\frac{8k}{3}+\frac{k}{6}-\frac{5k}{12}$

$\frac{29k}{12}$

e $\frac{p}{4}+\frac{p}{2}-3$

$\frac{3p-12}{4}$

f $\frac{5j}{12}-\frac{j}{3}+2$

$\frac{j+24}{12}$

11 Find the missing algebraic fraction. The missing fraction is fully simplified.

a $\frac{x}{2}+\frac{\square}{\square}=\frac{5x}{6}$

$\frac{2x}{3}$

b $\frac{\square}{\square}+\frac{x}{4}=\frac{3x}{8}$

$\frac{x}{8}$

c $\frac{2x}{5}+\frac{\square}{\square}=\frac{9x}{10}$

$\frac{x}{2}$

d $\frac{2x}{3}-\frac{\square}{\square}=\frac{7x}{15}$

$\frac{x}{5}$

e $\frac{\square}{\square}-\frac{x}{3}=\frac{5x}{9}$

$\frac{8x}{9}$

f $\frac{2x}{3}-\frac{\square}{\square}=\frac{5x}{12}$

$\frac{x}{4}$

MULTIPLYING ALGEBRAIC FRACTIONS

- To multiply fractions, multiply the numerators and multiply the denominators.

Rule for Multiplying Fractions
$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Example Multiply algebraic fractions.

$\frac{2a}{5} \times \frac{3b}{7}$ $= \frac{2a \times 3b}{5 \times 7}$ $= \frac{6ab}{35}$	How do we multiply two fractions? *Multiply two fractions by multiplying the ..... and multiplying the .....*
$\frac{4x}{15} \times \frac{3x}{2}$ $= \frac{4x \times 3x}{15 \times 2}$ $= \frac{12x^2}{30}$ $= \frac{2x^2}{5}$	How do you know when a fraction can be simplified? *A fraction can be simplified if the numerator and denominator have ....*

Simplify these expressions.

a $\frac{x}{3} \times \frac{2}{5}$

$\frac{2x}{15}$

b $\frac{2}{3} \times \frac{4a}{5}$

$\frac{8a}{15}$

c $\frac{6x}{5} \times \frac{7y}{6}$

$\frac{7xy}{5}$

d $\frac{8a}{5} \times \frac{3b}{4a}$

$\frac{6b}{5a}$

e $\frac{3x}{2} \times 6$

$9x$

f $\frac{10x}{3} \times \frac{6x}{5x}$

$4x$

Key ideas

1. To multiply algebraic fractions, we multiply the ..... and multiply the .....

1 Multiply these fractions.

a $\frac{x}{2} \times \frac{y}{3} = \frac{\square}{\square}$ $\frac{xy}{6}$

b $\frac{e}{5} \times \frac{k}{y} = \frac{\square}{\square}$ $\frac{ek}{5y}$

c $\frac{n}{2} \times \frac{n}{3} = \frac{\square}{\square}$ $\frac{n^2}{6}$

d $\frac{3p}{5} \times \frac{4p}{7} = \frac{\square}{\square}$ $\frac{12p^2}{35}$

e $\frac{e}{2} \times \frac{t}{3} \times \frac{5e}{4} = \frac{\square}{\square}$ $\frac{5e^2t}{24}$

f $\frac{2}{k} \times \frac{m}{3} \times \frac{4m}{5} = \frac{\square}{\square}$ $\frac{8m^2}{15k}$

2 Simplify these products.

a $\frac{x}{3} \times \frac{2}{5}$ $\frac{2x}{15}$

b $\frac{1}{7} \times \frac{a}{9}$ $\frac{a}{63}$

c $\frac{2}{3} \times \frac{4a}{5}$ $\frac{8a}{15}$

d $\frac{4c}{5} \times \frac{1}{5}$ $\frac{4c}{25}$

e $\frac{4a}{3} \times \frac{2b}{5}$ $\frac{8ab}{15}$

f $\frac{3a}{2} \times \frac{7a}{5}$ $\frac{21a^2}{10}$

3 Multiply these fractions.

a $\frac{m}{2} \times \frac{x}{m} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{x}{2}$

b $\frac{a}{3} \times \frac{6}{a} = \frac{\square}{\square} = \_\_\_$ 2

c $\frac{2c}{3} \times \frac{x}{4} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{cx}{6}$

d $\frac{ux}{2} \times \frac{x}{u} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{x^2}{2}$

e $\frac{5n}{px} \times \frac{2}{nx} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{10}{px^2}$

f $\frac{3}{k} \times \frac{2}{k} \times \frac{ak}{2} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{3a}{k}$

g $\frac{3f}{2} \times 4 \times \frac{h}{6f} = \frac{\square}{\square} = \_\_\_$ h

h $\frac{2a}{b} \times \frac{b}{2c} \times \frac{2c}{a} = \frac{\square}{\square} = \_\_\_$ 2

4 Simplify these products. Remember to simplify before you multiply.

a $\frac{6x}{5} \times \frac{7y}{6}$ $\frac{7xy}{5}$

b $\frac{2b}{5} \times \frac{7d}{6}$ $\frac{7bd}{15}$

c $\frac{8a}{5} \times \frac{3b}{4c}$ $\frac{6ab}{5c}$

d $\frac{9d}{2} \times \frac{4e}{7}$ $\frac{18de}{7}$

e $\frac{3x}{2} \times \frac{1}{6x}$ $\frac{1}{4}$

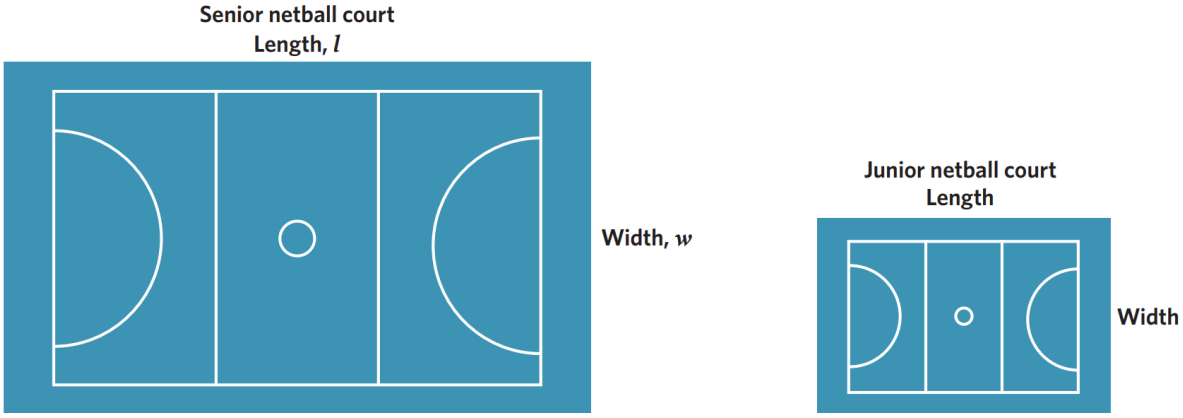
f $\frac{4}{9k} \times \frac{3k}{2}$ $\frac{2}{3}$

5 The width of a rectangle is $\frac{3x}{5}$ cm. Its length is four times its width.

Write an expression for the area of the rectangle.

$\frac{36x^2}{25}$ cm

6 The senior and junior netball courts have different dimensions.
The junior netball court has a length that is half of the senior netball court and a width that is two-thirds of the senior netball court.



a Write an expression for the area of the senior netball court.

lw

b Write an expression for the length and width of the junior netball court.

$\frac{l}{2}, \frac{2w}{3}$

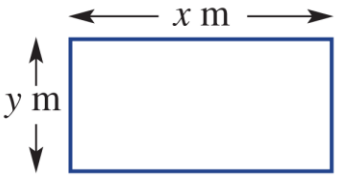
c Write an expression for the area of the junior netball court.

$\frac{lw}{3}$

d Write an expression for the difference between the senior netball court and the junior netball court.

$\frac{2lw}{3}$

7 The rectangular field below has length x metres and breadth y metres.



a Write an expression for the area of the field.

xy

b A smaller section is fenced off. It is $\frac{1}{2}$ the length and $\frac{3}{4}$ the breadth.

i Write an expression for the length and breadth of the smaller section.

$\frac{x}{2}, \frac{3y}{4}$

ii Hence, write an expression for the area of the smaller section.

$\frac{3xy}{8}$

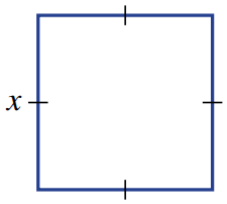
c Find the percentage of the smaller area compared to the larger area.

37.5%

8 Consider a square with side length x.

a Write an expression for the area of the square.

x^2



b The length of each side is now halved.
Give an expression for the area of the new square.

$\frac{x^2}{4}$

c If each side of the original square is multiplied by 0.7, find an expression for the area of the square, as an algebraic fraction. Recall that $0.7 = \frac{7}{10}$

$\frac{49x^2}{100}$

d If each side of the original square is three-fifths of the original length, would the new area be more or less than half of the original area? Show your reasoning.

The new area is $\frac{9x^2}{25}$, which is less than $\frac{1}{2}$ of x^2

e If you want to halve the area of the original square, by what factor would you have to divide each length?

Divide by $\sqrt{2}$

f If you continued to reduce each side of the original square by a factor of $\frac{3}{5}$, show that the area after 2 reductions is $\frac{81}{625}x^2$

g If you continued to reduce each side of the original square by a factor of $\frac{3}{5}$, write an expression for the area after n reductions.

$\left(\frac{3}{5}\right)^{2n} x^2$

DIVIDING ALGEBRAIC FRACTIONS

- If we are dividing by a fraction, then we multiply by the **reciprocal** instead.

Rule for Dividing Fractions
$\frac{a}{b} \div \frac{c}{d}$ $= \frac{a}{b} \times \frac{d}{c}$

Example Divide algebraic fractions.

$\frac{3a}{8} \div \frac{b}{5}$

$= \frac{3a}{8} \times \frac{5}{b}$

$= \frac{15a}{8b}$

$\frac{u}{4} \div \frac{15p}{2}$

$= \frac{u}{4} \times \frac{2}{15p}$

$= \frac{2u}{60p} = \frac{u}{30p}$

Which fraction do you take the reciprocal of when changing division to multiplication?

To change division to ……., you take the reciprocal of the fraction you are ……. by.

In the second example, you could have "cancelled" before multiplying. Show this.

$\frac{u}{4} \times \frac{2}{15p}$

Simplify these expressions.

a $\frac{3a}{5} \div \frac{1}{5}$ $3a$

b $\frac{9a}{10} \div \frac{1}{4}$ $\frac{18a}{5}$

c $\frac{4}{5} \div \frac{2y}{3}$ $\frac{6}{5y}$

d $\frac{4a}{7} \div \frac{2}{5}$ $\frac{10a}{7}$

e $\frac{4x}{5} \div 3$ $\frac{4x}{15}$

f $2 \div \frac{x}{5}$ $\frac{10}{x}$

Key ideas

1. To multiply two fractions, we multiply the …… and multiply the ……
2. Dividing by a fraction is the same as multiplying by the ……

1 Divide these fractions.

a $\frac{c}{2} \div \frac{3}{v} = \frac{c}{2} \times \frac{v}{3} = \frac{cv}{6}$ (÷ → ×; reciprocal)

b $\frac{a}{c} \div \frac{d}{b} = \square \times \square = \square$ $\frac{ab}{cd}$

c $\frac{2}{u} \div \frac{u}{2} = \square \times \square = \square$ $\frac{4}{u^2}$

d $\frac{d}{3} \div \frac{4}{d} = \square \times \square = \square$ $\frac{d^2}{12}$

e $\frac{y}{3} \div \frac{1}{2} = \square \times \square = \square$ $\frac{2y}{3}$

f $\frac{2n}{3} \div \frac{5}{x} = \square \times \square = \square$ $\frac{2nx}{15}$

g $\frac{3c}{m} \div 5 = \square \times \square = \square$ $\frac{3c}{5m}$

h $\frac{4k}{3} \div \frac{8}{3} = \square \times \square = \square = \square$ $\frac{k}{2}$

i $\frac{6y}{5} \div \frac{3}{4g} = \square \times \square = \square = \square$ $\frac{8gy}{5}$

2 Simplify these divisions. Remember to simplify before you multiply.

a $\frac{2}{3} \div \frac{4x}{7}$ $\frac{7}{6x}$

b $\frac{4}{5} \div \frac{2y}{3}$ $\frac{6}{5y}$

c $\frac{1}{7} \div \frac{2}{x}$ $\frac{x}{14}$

d $\frac{4a}{7} \div \frac{2}{5}$ $\frac{10x}{7}$

e $\frac{4b}{7} \div \frac{2c}{5}$ $\frac{10}{7c}$

f $\frac{2x}{5} \div \frac{4y}{3}$ $\frac{3}{10y}$

g $\frac{2y}{x} \div \frac{3}{y}$ $\frac{2y^2}{3x}$

h $\frac{5}{12x} \div \frac{7x}{2}$ $\frac{5}{42x^2}$

i $\frac{4a}{5} \div \frac{2b}{7a}$ $\frac{14a}{5b}$

3 Simplify the following.

a $3 \times \frac{4x}{5}$ $\frac{12x}{5}$

b $\frac{4x}{5} \div 3$ $\frac{4x}{15}$

c $2 \div \frac{x}{5}$ $\frac{10}{x}$

d $4 \times \frac{a}{3}$ $\frac{4a}{3}$

e $5 \times \frac{7}{10x}$ $\frac{7}{2x}$

f $1 \div \frac{x}{2}$ $\frac{2}{x}$

4 Simplify these expressions. Only invert the fractions being divided.

a $\frac{k}{3} \div \frac{2}{3} \times \frac{4}{k}$

$\frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \_\_$ 2

b $\frac{4}{e} \times \frac{2e}{3} \div \frac{1}{e}$

$\frac{\square}{\square} \times \frac{\square}{\square} \times \square = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{8e}{3}$

c $\frac{ch}{2} \div \frac{1}{2} \times \frac{a}{h}$

$\frac{\square}{\square} \times \square \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$ ac

d $\frac{2v}{5} \times \frac{3}{2} \div \frac{v^2}{3}$

$\frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{9}{5v}$

e $\frac{1}{d} \times \frac{3d}{4} \div \frac{1}{d}$

$\frac{\square}{\square} \times \frac{\square}{\square} \times \square = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{3d}{4}$

f $\frac{q}{p} \times \frac{aq}{2p} \div \frac{4q}{5p}$

$\frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$ $\frac{5aq}{8p}$

5 Krystal's family goes to dinner with Helen's family.
The bill comes to a total of $\$x$ and each family pays half.

a Write an algebraic fraction for the amount Krystal's family pays. $\frac{x}{2}$

b Krystal says she will pay one third of her family's bill.
Write an algebraic fraction for the amount that she pays. $\frac{x}{6}$

6 Write an algebraic fraction for the result of the following operations.

a A number q is halved and then the result is tripled. $\frac{3q}{2}$

b A number x is multiplied by $\frac{2}{3}$ and the result is divided by $1\frac{1}{3}$. $\frac{x}{2}$

c A fraction $\frac{a}{b}$ is multiplied by its reciprocal. 1

d The number x is reduced by 25% and then halved. $\frac{3x}{8}$

7 Simplify the expressions.

a

$$5mn \times \frac{4d}{n} \times \frac{1}{15mn}$$

$\frac{3d}{3n}$

b

$$\frac{6}{r} \times \frac{5r}{9} \div \frac{15}{xy}$$

$\frac{2xy}{9}$

c

$$\frac{6cf}{q} \div \frac{5}{qf} \div \frac{10c}{a}$$

$\frac{3a}{25}$

d

$$\frac{4ab}{5} \div \frac{a}{2b} \div \frac{8b}{15}$$

$3b$

8 Expand and simplify the following.

a

$$\frac{1}{2}(x+4)$$

$\frac{x}{2}+2$

b

$$\frac{2}{3}(y-3)$$

$\frac{2y}{3}-2$

c

$$\frac{3}{4}(2a+4b)$$

$\frac{3a}{2}+3b$

d

$$\frac{1}{2}(6x+8)+\frac{1}{3}(12x-3)$$

$7x+3$

e

$$\frac{2}{3}(12y-6)+\frac{1}{4}(2y+3)$$

$\frac{17y}{2}-\frac{13}{4}$

f

$$\frac{x}{3}(6+9y)+y(x+4)$$

$2x+4xy+4y$

CHECK YOUR UNDERSTANDING

Adding and Subtracting Algebraic Fractions

1 Simplify fully. $\frac{x}{3} + \frac{2x}{3}$

a $\frac{3x}{6}$ **b** x **c** $\frac{3x}{3}$ **d** $\frac{x}{2}$

2 Simplify fully. $\frac{x}{4} + \frac{3x}{4}$

a $\frac{4x}{8}$ **b** $\frac{x}{2}$ **c** $\frac{4x}{4}$ **d** x

3 Simplify fully. $\frac{2x}{3} + \frac{2x}{9}$

a $\frac{x}{6}$ **b** $\frac{7x}{9}$ **c** $\frac{4x}{9}$ **d** $\frac{x}{3}$

4 Simplify fully. $\frac{2x}{6} + \frac{5x}{12}$

a $\frac{18x}{24}$ **b** $\frac{9x}{12}$ **c** $\frac{5x}{12}$ **d** $\frac{3x}{4}$

5 Simplify fully. $\frac{x}{8} + \frac{3}{4}$

a $\frac{6x}{8}$ **b** $\frac{7x}{8}$ **c** $\frac{x+6}{8}$ **d** $\frac{3x}{8}$

6 Simplify fully. $\frac{x}{3} + \frac{x}{4}$

a $\frac{x^2}{7}$ **b** $\frac{2x}{7}$ **c** $\frac{7x}{12}$ **d** $\frac{x}{6}$

7 Simplify fully. $\frac{2x}{5} - \frac{3x}{4}$

a $-\frac{x}{20}$ **b** $-x$ **c** $-\frac{7x}{20}$ **d** $\frac{6x^2}{20}$

Multiplying and Dividing Algebraic Fractions

1 Simplify fully: $\frac{t}{s} \times \frac{2}{t}$

a $\frac{2s+t}{s+t}$ **b** $\frac{t+2s}{st}$ **c** 2 **d** $\frac{t^2+2s^2}{st}$

2 Simplify fully: $\frac{1}{s} \div \frac{2}{t}$

a $\frac{2s}{t}$ **b** $\frac{2t}{s}$ **c** $\frac{s}{2t}$ **d** $\frac{t}{2s}$

3 Simplify fully: $\frac{t}{s} \div \frac{t}{2s}$

a $\frac{2s^2}{t^2}$ **b** $\frac{1}{2}$ **c** $\frac{t^2}{2s^2}$ **d** 2

4 Simplify fully: $\frac{7}{p} \div \frac{14}{p}$

a $\frac{98}{p^2}$ **b** $\frac{1}{2}$ **c** $\frac{98}{p}$ **d** $\frac{7p}{14p}$

5 Simplify fully: $\frac{3}{x} \times \frac{4}{x}$

a $\frac{12}{x}$ **b** $\frac{7}{x}$ **c** $\frac{12}{2x} = \frac{6}{x}$ **d** $\frac{12}{x^2}$

6 Simplify fully: $\frac{6}{x} \times \frac{x^2}{3}$

a $2x$ **b** $\frac{18}{x^3}$ **c** $\frac{x^3}{18}$ **d** $\frac{6x^2}{3x}$

7 Simplify fully: $\frac{3}{x} \div \frac{x^2}{4}$

a $\frac{3x}{4}$ **b** $\frac{12}{x}$ **c** $\frac{3}{4x}$ **d** $\frac{12}{x^3}$

8 Simplify fully: $\frac{3g}{2} \times \frac{6g}{9}$

a 1 **b** $\frac{18g^2}{18}$ **c** g^2 **d** $\frac{g}{2}$

9 Simplify fully: $\frac{5n}{2} \div \frac{6n}{10}$

a $\frac{30n^2}{20}$ **b** $\frac{3n^2}{2}$ **c** $\frac{25}{6}$ **d** $\frac{50n}{12n}$