

Student Name

Mathematics Stage 5 Paths

PROPERTIES OF GEOMETRICAL FIGURES

Book 2 Apply logical reasoning to proofs involving plane shapes

Version: 260207

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CONTENTS

Applying Proofs to Prove Properties2

OVERVIEW

SYLLABUS CONTENT

MA5-GEP-P-02 constructs proofs involving congruent triangles and similar triangles and proves properties of plane shapes

Apply logical reasoning to proofs involving plane shapes

- Recognise that a definition is the minimum amount of information needed to identify a particular figure
- Prove the properties of isosceles and equilateral triangles and special quadrilaterals from the formal definition of the shapes
- Prove and apply theorems and properties related to triangles and quadrilaterals
- Prove and apply tests for quadrilaterals
- Solve numerical and non-numerical problems in Euclidean geometry based on known assumptions and proven theorems

REVIEW OF PRIOR KNOWLEDGE

1 By drawing a line, make these quadrilaterals into two congruent triangles.

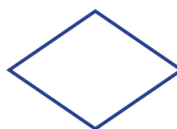
a



b



c

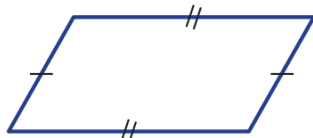


d

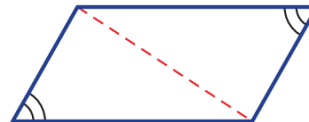


2 Which property of a parallelogram is illustrated in each diagram?

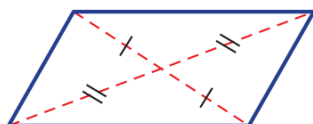
a



b



c

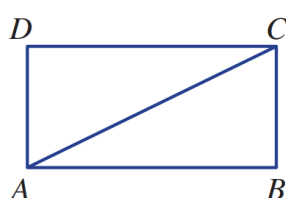


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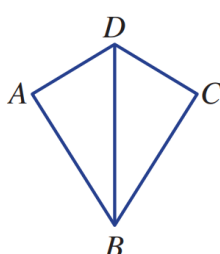


3 Name the side (e.g. AB) that is common to both triangles in each diagram.

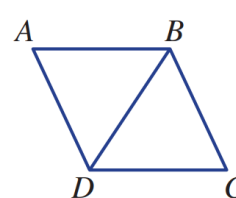
a



b

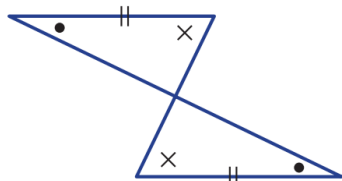


c

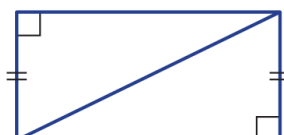


4 Which of the four tests would be used to prove each pair of triangles is congruent?

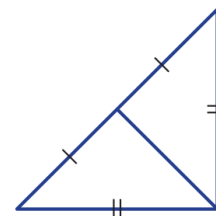
a



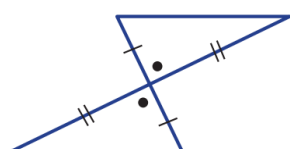
b



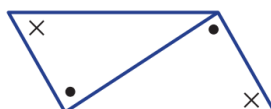
c



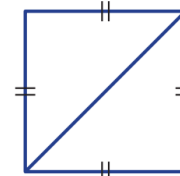
d



e



f



APPLYING PROOFS TO PROVE PROPERTIES

- We can use congruent triangles to prove properties of quadrilaterals.
 - Prove two triangles are congruent.
 - Use equal corresponding sides/angles to prove another property.

Example Prove geometrical properties.

Prove that the diagonals of a parallelogram bisect each other.

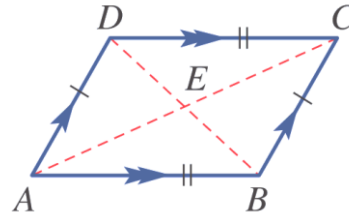
1.

A: $\angle BAE = \angle DCE$ (alternate angles in $AB \parallel DC$)

A: $\angle ABE = \angle CDE$ (alternate angles in $AB \parallel DC$)

S: $AB = CD$ (given)

$\therefore \triangle ABE \equiv \triangle CDE$ (AAS)



2.

$\therefore BE = DE$ and $AE = CE$ (corresponding sides in congruent triangles are equal)

$\therefore$ Diagonals bisect each other.

- a Given ABCD is a square, complete the proof to show that the diagonals are equal.

In $\triangle ABC$ and $\triangle BCD$:

$\angle ABC = \dots\dots\dots$ (angles in a $\dots\dots\dots$ are equal)

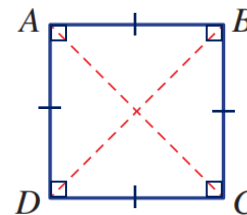
$AB = BC$ ($\dots\dots\dots$)

$\dots\dots\dots$ is common

$\therefore \triangle ABC \equiv \triangle BCD$ (S $\dots\dots\dots$)

$\therefore AC = \dots\dots\dots$ ($\dots\dots\dots$ sides in congruent triangles are equal)

$\therefore \dots\dots\dots$ of a square are equal.



- b ADCB is a parallelogram. Prove that opposite angles are equal.

In $\triangle ABD$ and $\triangle CDB$:

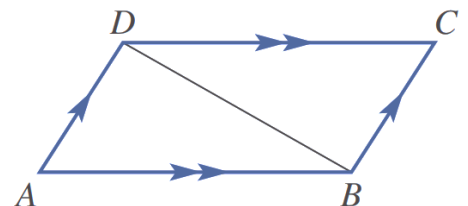
$\angle ABD = \angle CDB$ ($\dots\dots\dots$)

$\angle ADB = \angle CBD$ ($\dots\dots\dots$)

BD is common.

$\therefore \triangle ABD \equiv \triangle CDB$ ($\dots\dots\dots$)

$\therefore \angle DAB = \angle BCD$ ($\dots\dots\dots$)



In $\triangle ADC$ and $\triangle ABC$:

$\dots\dots\dots$

$\dots\dots\dots$

$\dots\dots\dots$

$\dots\dots\dots$

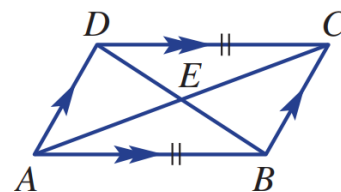
$\dots\dots\dots$

$\therefore$ Opposite angles of a parallelogram are equal.

1 In this question we will prove that the diagonals in a parallelogram bisect each other.

- a List two pairs of equal angles in $\triangle ABE$ and $\triangle CDE$, giving reasons why they are equal.

.....



- b List a pair of equal sides in $\triangle ABE$ and $\triangle CDE$, giving reasons why they are equal.

.....

- c Write $\triangle ABE \equiv \triangle CDE$ and give the reason SSS, SAS, AAS or RHS.

.....

- d State that $BE = DE$ and $AE = CE$ and give a reason.

.....

2 A parallelogram, ABCD, has two pairs of parallel sides.

- a Give a reason why $\angle ABD = \angle CDB$.

.....

- b Give a reason why $\angle BDA = \angle DBC$.

.....

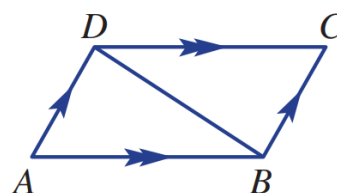
- c Which side is common to both $\triangle ABD$ and $\triangle CDB$?

.....

- d Which congruence test would be used to show that $\triangle ABD \equiv \triangle CDB$?

- e If $\triangle ABD \equiv \triangle CDB$, what can be said about the opposite sides of a parallelogram?

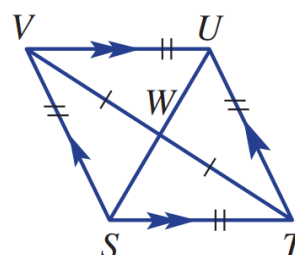
.....



3 For this rhombus assume that $VW = WT$.

- a Prove that $\triangle VWU \equiv \triangle TWU$.

.....



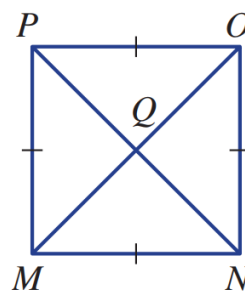
- b Give reasons why $\angle VWU = \angle TWU = 90^\circ$.

.....

4 For this square assume that $MQ = QO$ and $NQ = PQ$

a Prove that $\triangle MNQ \equiv \triangle ONQ$.

.....



b Give reasons why $\angle MQN = \angle OQN = 90^\circ$.

.....

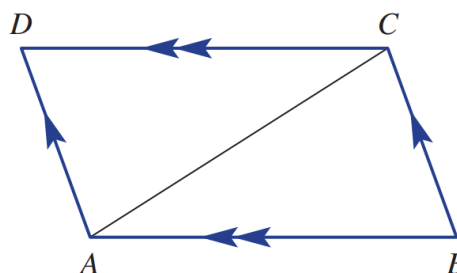
c Give reasons why $\angle QMN = 45^\circ$

.....

5 Complete these steps to prove that a parallelogram (with opposite sides parallel) has opposite sides equal in length.

a Prove $\triangle ABC \equiv \triangle CDA$.

.....



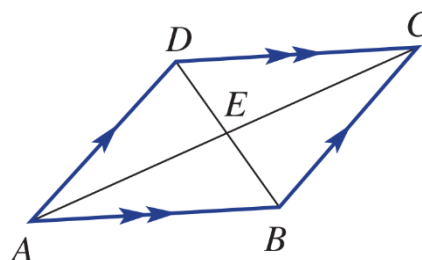
b Hence, prove opposite sides are equal in length.

.....

6 Complete these steps to prove that a parallelogram (with opposite sides parallel) has diagonals that bisect each other.

a Prove $\triangle ABE \equiv \triangle CDE$.

.....



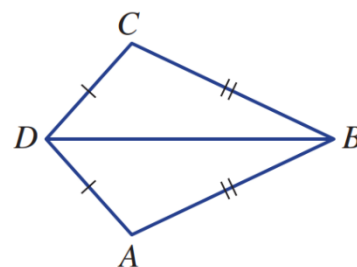
b Hence, prove $AE = CE$ and $BE = DE$.

.....

7 Use only the given information in this kite to prove these results.

a $\triangle ABD \equiv \triangle CBD$

.....



b $\angle DAB = \angle DCB$

.....

c $\angle ADB = \angle CDB$

.....

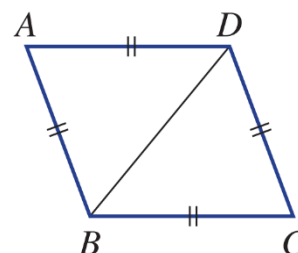
d What kite property have you proved?

.....

8 Prove that a rhombus (with sides of equal length) has diagonals that bisect the vertex angles.

a Prove $\triangle ABD \equiv \triangle CDB$.

.....



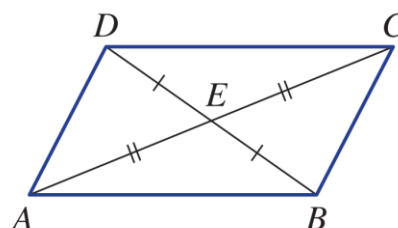
b Hence, prove BD bisects both $\angle ABC$ and $\angle CDA$.

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9 Prove that if the diagonals in a quadrilateral bisect each other, then it is a parallelogram.

a Prove $\triangle ABE \equiv \triangle CDE$.

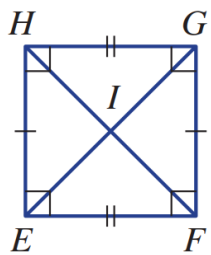
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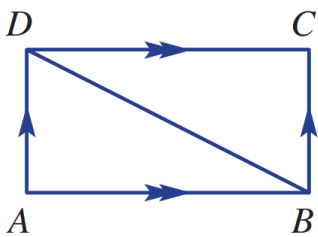
b Hence, prove $AB \parallel DC$ and $AD \parallel BC$.

.....

10 Prove that the diagonals in a square bisect each other.



11 Show that opposite sides of a rectangle are equal. Give all reasons.



12 A trapezium, ABCD, has one pair of parallel sides.

a Which angle is equal to $\angle BAE$?

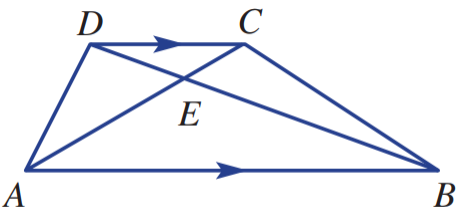
.....

b Which angle is equal to $\angle ABE$?

.....

c Explain why $\triangle ABE$ is not congruent to $\triangle CDE$.

.....



13 Use the information given for this kite to give reasons for the following properties.

a $\angle DAE = \angle DCE$

.....

b $\triangle AED \equiv \triangle CED$

.....

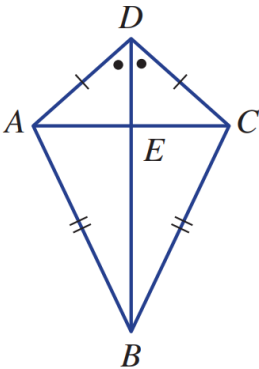
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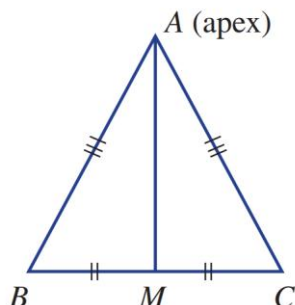
c $\angle AED = \angle CED = 90^\circ$

.....

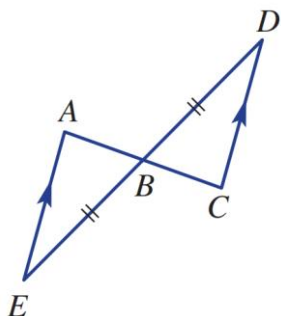


14 Write a proof as to why the following are true.

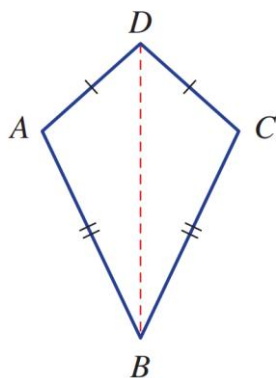
- a** The segment joining the apex of an isosceles triangle to the midpoint M of the base BC is at right angles to the base; that is, prove $\angle AMB = \angle AMC = 90^\circ$.



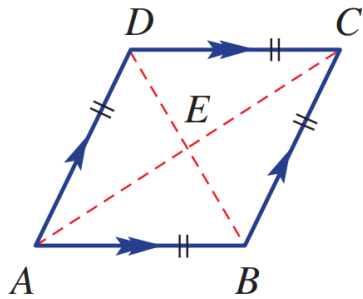
- b** The segment $AC = 2AB$ in this diagram.



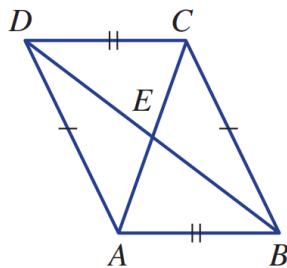
- c** A kite has one pair of equal opposite angles.



- d** The diagonals of a rhombus intersect at right angles.



- e** If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram; that is, show that the quadrilateral has two pairs of parallel sides.



- f** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

