Student Name

Mathematics Stage 5 Paths

INDICES C

Book 2 Apply knowledge of surds to solve problems

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-IND-P-02 describes and performs operations with surds and fractional indices

Apply knowledge of surds to solve problems

- Establish and apply the following results for $x > 0$ and $y > 0$: $\left(\sqrt{x}\right)^2 = x = \sqrt{x^2}$, $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$ and $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$
- Apply the 4 operations to simplify expressions involving surds
- Expand and simplify expressions involving surds
- Rationalise the denominators of surds of the form $\frac{a\sqrt{b}}{c\sqrt{d}}$

REVIEW OF PRIOR KNOWLEDGE

1 Evaluate the following:

a	2^2	**b**	3^2	**c**	4^2	**d**	5^2
e	6^2	**f**	7^2	**g**	8^2	**h**	9^2

2 Evaluate the following:

a	$\sqrt{4}$	**b**	$\sqrt{25}$	**c**	$\sqrt{49}$	**d**	$-\sqrt{64}$
e	$\sqrt{81}$	**f**	$\sqrt{100}$	**g**	$-\sqrt{121}$	**h**	$\sqrt{9}$

3 Factorise these numbers based on their **highest square factor**.
(highest factor that is also a square number)

a	20 $= 4 \times 5$	**b**	18	**c**	125	**d**	24
e	48	**f**	96	**g**	72	**h**	108

4 Simplify these expressions.

a	$2x + 5 + 6x$	**b**	$4y + 2x - 12y$	**c**	$-3x + 2 + 4x - 7$

5 Expand these expressions.

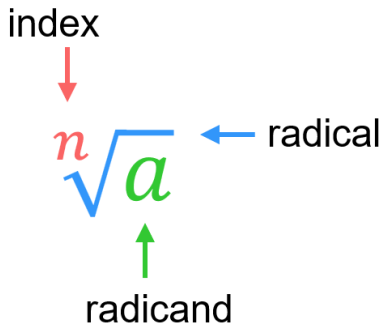
a	$4(3x + 5)$	**b**	$7(2x - 5)$	**c**	$3x(x + 5)$
d	$-4(x + 8)$	**e**	$-4(y - 12)$	**f**	$-4x(2x - 9)$

6 Expand these brackets.

a	$(x + 2)(x + 5)$	**b**	$(x + 3)(x - 4)$	**c**	$(x + 3)(2x + 5)$
d	$(5x + 2)(2x - 7)$	**e**	$(2x - 2)(3x - 1)$	**f**	$(x + 5)(x - 5)$

SURD LAWS

Language of Surds



Example Identify parts of a surd.

Surd	Index	Radicand
$\sqrt[3]{7}$		
$\sqrt{7}$		
$\sqrt[3]{a}$		
$3\sqrt{a}$		
$\sqrt[9]{n^2}$		
$\sqrt[9]{n+5}$		
$2\sqrt[6]{n^2+5}$		
	5	z^3

SURD LAWS

Inverse Relationship	Surd Law of Multiplication	Surd Law of Division
$(\sqrt{x})^2 = x$ $\sqrt{x^2} = x$	$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$	$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

Example Multiply and divide surds.

$(\sqrt{11})^2$ $= 11$	$\sqrt{3} \times \sqrt{10}$ $= \sqrt{30}$	$\sqrt{24} \div \sqrt{8}$ $= \sqrt{3}$

Simplify these expressions.

a $\sqrt{3} \times \sqrt{7}$ $\sqrt{21}$	**b** $\sqrt{8} \times \sqrt{2}$ 4	**c** $\sqrt{2} \times \sqrt{11}$ $\sqrt{22}$	**d** $(\sqrt{12})^2$ 12
e $\sqrt{20} \div \sqrt{2}$ $\sqrt{10}$	**f** $\sqrt{45} \div \sqrt{5}$ 3	**g** $\sqrt{18} \div \sqrt{6}$ $\sqrt{3}$	**h** $\sqrt{49} \div \sqrt{7}$ $\sqrt{7}$

Check your understanding

Determine whether you can use the surd laws to simplify the expression.

a	$\sqrt{7} \times \sqrt{5}$	Yes | No		**e**	$\sqrt{6} \times \sqrt{0.5}$	Yes | No	
b	$\sqrt{7} + \sqrt{5}$	Yes | No		**f**	$\sqrt{6} \div \sqrt{10}$	Yes | No	
c	$\sqrt{7} \div \sqrt{5}$	Yes | No		**g**	$\sqrt{6} + \sqrt{12}$	Yes | No	
d	$\sqrt{7} - \sqrt{7}$	Yes | No		**h**	$\sqrt{6} - \sqrt{3}$	Yes | No	

Key ideas

1. Raising to a power and rooting are …… operations; they undo each other.
2. $\sqrt{x^2} = (\sqrt{x})^2 =$ ……
3. The surd law of multiplication says that $\sqrt{a} \times \sqrt{b} =$ ……
4. The surd law of division says that $\frac{\sqrt{a}}{\sqrt{b}} =$ ……

1 Evaluate the following.

a $(\sqrt{11})^2$ — 11

b $\sqrt{11^2}$ — 11

c $(\sqrt{8})^2$ — 8

d $\sqrt{8^2}$ — 8

e $(\sqrt{6})^2$ — 6

2

a Write down the value of

i $\sqrt{4}$ — 2

ii $\sqrt{25}$ — 5

iii $\sqrt{100}$ — 10

ii $\sqrt{400}$ — 20

b Hence show that

i $\sqrt{4} \times \sqrt{100} = \sqrt{400}$ — $2 \times 10 = 20$

ii $\sqrt{100} \div \sqrt{4} = \sqrt{25}$ — $10 \div 2 = 5$

3

a Write down the value of

i $\sqrt{9}$ — 3

ii $\sqrt{16}$ — 4

b Hence show that

i $\sqrt{9} + \sqrt{16} \neq \sqrt{25}$ — $3 + 4 \neq 5$

ii $\sqrt{16} - \sqrt{9} \neq \sqrt{7}$ — $4 - 3 \neq \sqrt{7}$

4 Use the fact that $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ to write each expression as a single surd.

a $\sqrt{5} \times \sqrt{3}$ — $\sqrt{15}$	**b** $\sqrt{7} \times \sqrt{11}$ — $\sqrt{77}$	**c** $\sqrt{51} \times \sqrt{2}$ — $\sqrt{102}$	**d** $\sqrt{15} \times \sqrt{21}$ — $\sqrt{315}$
e $\sqrt{2} \times \sqrt{11}$ — $\sqrt{22}$	**f** $\sqrt{3} \times \sqrt{7}$ — $\sqrt{21}$	**g** $\sqrt{10} \times \sqrt{3}$ — $\sqrt{30}$	**h** $\sqrt{10} \times \sqrt{7}$ — $\sqrt{70}$

5 Use the fact that $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$ to write each expression as a single surd.

a $\sqrt{21} \div \sqrt{3}$ — $\sqrt{7}$	**b** $\sqrt{26} \div \sqrt{2}$ — $\sqrt{13}$	**c** $\sqrt{51} \div \sqrt{17}$ — $\sqrt{3}$	**d** $\sqrt{3} \div \sqrt{21}$ — $\sqrt{\frac{1}{7}}$
e $\frac{\sqrt{20}}{\sqrt{2}}$ — $\sqrt{10}$	**f** $\frac{\sqrt{45}}{\sqrt{5}}$ — 3	**g** $\frac{\sqrt{32}}{\sqrt{8}}$ — 2	**h** $\frac{\sqrt{35}}{\sqrt{5}}$ — $\sqrt{7}$

6

a Using your knowledge of index laws, explain why $(3\sqrt{5})^2 = 3^2(\sqrt{5})^2$

Index law of power of a product: $(ab)^n = a^n b^n$

b Therefore, explain why $(3\sqrt{5})^2 = 45$

$3^2\sqrt{5}^2 = 9(5) = 45$

7 Evaluate the following.

a $(3\sqrt{2})^2$

18

b $(2\sqrt{3})^2$

12

c $(5\sqrt{2})^2$

50

d $(10\sqrt{7})^2$

700

e $(4\sqrt{5})^2$

80

8 Simplify these expressions. Write your answer as a whole number.

a $(\sqrt{2})^2$

2

b $(\sqrt{3})^2$

3

c $(\sqrt{5})^2$

5

d $(\sqrt{10})^2$

10

e $(2\sqrt{10})^2$

40

f $(3\sqrt{10})^2$

90

g $3(\sqrt{10})^2$

30

h $5(\sqrt{10})^2$

50

i $10(\sqrt{10})^2$

100

SIMPLIFYING SURDS

1. Factorise the **radicand** (number inside the root) according to its **highest square factor**.
   $4,\quad 9,\quad 16,\quad 25,\quad 36,\quad 49,\quad 64,\quad 81,\quad 100,\quad 121,\quad 144\ldots$
2. Split the surd, then simplify perfect squares.

Example Simplify surds.

$\sqrt{45}$ 1. $\sqrt{9\times5}$ 2. $\sqrt{9}\times\sqrt{5}$ $=3\sqrt{5}$	What is meant by highest square factor. *The ..... factor that is a ..... number.* Which law says $\sqrt{9\times5}=\sqrt{9}\times\sqrt{5}$? *Surd law of .....*
$3\sqrt{40}$ 1. $3\sqrt{4\times10}$ 2. $3\sqrt{4}\sqrt{10}$ $=6\sqrt{10}$	Why did we split $\sqrt{40}$ into $\sqrt{4}\times\sqrt{10}$, rather than $\sqrt{8}\times\sqrt{5}$? *Neither 8 nor 5 are ..... numbers.* Why is $3\sqrt{4}=6$? $3\sqrt{4}=3\times\ldots\ldots=6$
$\frac{\sqrt{80}}{3}$ 1. $\frac{\sqrt{16\times5}}{3}$ 2. $\frac{4\sqrt{5}}{3}$	Why is this not fully simplified? $\sqrt{80}=\sqrt{4}\times\sqrt{20}$ $=2\sqrt{20}$ $\sqrt{20}$ *can be further ..... to .....*

Simplify these surds.

a $\sqrt{12}$ $2\sqrt{3}$	**b** $\sqrt{28}$ $2\sqrt{7}$	**c** $\sqrt{54}$ $3\sqrt{6}$	**d** $3\sqrt{8}$ $6\sqrt{2}$
e $\frac{\sqrt{48}}{2}$ $2\sqrt{3}$	**f** $\frac{5\sqrt{40}}{6}$ $\frac{5\sqrt{10}}{3}$	**g** $\sqrt{\frac{75}{9}}$ $\frac{5}{\sqrt{3}}$ or $\frac{5\sqrt{3}}{3}$	**h** $\sqrt{\frac{50}{16}}$ $\frac{5}{2\sqrt{2}}$ or $\frac{5\sqrt{2}}{4}$

Key ideas

1. To simplify surds, we factorise the number inside the root (called the .....) according to the highest ..... factor.
2. Then we split the surd using the surd law of .....

1 Simplify these surds

a $\sqrt{12}$ $= \sqrt{4 \times 3}$ $=$ $2\sqrt{3}$	**b** $\sqrt{45}$ $3\sqrt{5}$	**c** $\sqrt{24}$ $2\sqrt{6}$	**d** $\sqrt{48}$ $4\sqrt{3}$
e $\sqrt{75}$ $5\sqrt{3}$	**f** $\sqrt{500}$ $10\sqrt{5}$	**g** $\sqrt{98}$ $7\sqrt{2}$	**h** $\sqrt{90}$ $3\sqrt{10}$
i $\sqrt{128}$ $8\sqrt{2}$	**j** $\sqrt{360}$ $6\sqrt{10}$	**k** $\sqrt{162}$ $9\sqrt{2}$	**l** $\sqrt{80}$ $4\sqrt{5}$

2 There are many ways to factorise $\sqrt{600}$. 1 is given below. Find 2 more different ways.

$\sqrt{600} = 2\sqrt{150}$

$5\sqrt{24}, 10\sqrt{6}$

What is the simplest form of $\sqrt{600}$?

$10\sqrt{6}$

3 Work out each value of x

a $\sqrt{150} = 5\sqrt{x}$ 6	**b** $\sqrt{300} = x\sqrt{3}$ 10	**c** $\sqrt{x} = 7\sqrt{13}$ 637

4 Samira has simplified $\sqrt{200}$: $\sqrt{200} = \sqrt{4} \times \sqrt{50} = 2 \times \sqrt{50} = 2\sqrt{50}$

Explain how Samira can improve her answer.

Further simplify $\sqrt{50}$; $2\sqrt{50} = 2 \times 5\sqrt{2} = 10\sqrt{2}$

5 Consider the working out to simplify $\sqrt{72}$

$$\begin{aligned}\sqrt{72} &= \sqrt{9 \times 8} \\ &= 3\sqrt{8} \\ &= 3\sqrt{4 \times 2} \\ &= 3 \times 2 \times \sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

a Explain why the working out for simplifying $\sqrt{72}$ is longer than it needs to be.

You can split $\sqrt{72}$ to $\sqrt{36}\sqrt{2}$;

9 was not the largest square factor of 72.

b Therefore, simplify $\sqrt{72}$ using as few steps as possible.

$\sqrt{72} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$

6 Simplify these surds. Try to write as little working out as possible.

a $\sqrt{8}$

$2\sqrt{2}$

b $\sqrt{12}$

$2\sqrt{3}$

c $\sqrt{32}$

$4\sqrt{2}$

d $\sqrt{64}$

8

e $\sqrt{128}$

$8\sqrt{2}$

f $\sqrt{192}$

$8\sqrt{3}$

g $\sqrt{320}$

$8\sqrt{5}$

h $\sqrt{80}$

$4\sqrt{5}$

i $\sqrt{40}$

$2\sqrt{10}$

j $\sqrt{90}$

$3\sqrt{10}$

k $\sqrt{160}$

$4\sqrt{10}$

l $\sqrt{1600}$

40

m $\sqrt{200}$

$10\sqrt{2}$

n $\sqrt{250}$

$5\sqrt{10}$

o $\sqrt{175}$

$5\sqrt{7}$

7 Simplify these surds.

a $2\sqrt{18}$

$= 2 \times 3\sqrt{2}$

$= 6\sqrt{2}$

b $4\sqrt{48}$

$= 4 \times 4\sqrt{3}$

$16\sqrt{3}$

c $3\sqrt{98}$

$21\sqrt{2}$

d $4\sqrt{125}$

$20\sqrt{5}$

8 Simplify these surds.

a $\sqrt{8}$

$2\sqrt{2}$

b $2\sqrt{8}$

$4\sqrt{2}$

c $3\sqrt{8}$

$6\sqrt{2}$

d $\sqrt{72}$

$6\sqrt{2}$

f $\sqrt{72}$

$6\sqrt{2}$

g $5\sqrt{72}$

$30\sqrt{2}$

h $5\sqrt{144}$

60

i $5\sqrt{1440}$

$60\sqrt{10}$

k $5\sqrt{720}$

$60\sqrt{5}$

l $5\sqrt{360}$

$30\sqrt{10}$

m $5\sqrt{36}$

30

n $5\sqrt{25}$

25

9 Determine the exact side length, in simplest form, of a square with the given area.

a 32 m^2

$4\sqrt{2}$ m

b 120 cm^2

$2\sqrt{30}$ cm

c 240 mm^2

$4\sqrt{15}$ mm

10 Determine the exact radius, in simplest form, of a circle with the given area. $A = \pi r^2$

a 24π m^2

$2\sqrt{6}$ m

b 54π cm^2

$3\sqrt{6}$ cm

c 128π mm^2

$8\sqrt{2}$ mm

11 Simplify.

a $\sqrt{675}$

$15\sqrt{3}$

b $\sqrt{1183}$

$13\sqrt{7}$

c $\sqrt{2883}$

$31\sqrt{3}$
Sorry :)

12 Express these surds as a square root of a positive integer.

a $2\sqrt{3}$
$= \sqrt{4} \times \sqrt{3}$
$= \sqrt{12}$

b $4\sqrt{2}$
$= \sqrt{16} \times \sqrt{2}$
$= \ldots\ldots\ldots\ldots$
$\sqrt{32}$

c $3\sqrt{3}$
$\sqrt{27}$

d $3\sqrt{5}$
$\sqrt{45}$

e $6\sqrt{3}$
$\sqrt{108}$

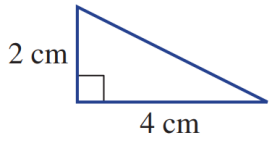
f $8\sqrt{2}$
$\sqrt{128}$

g $10\sqrt{7}$
$\sqrt{700}$

h $9\sqrt{10}$
$\sqrt{810}$

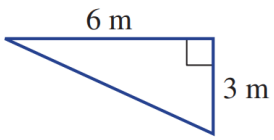
13 Use Pythagoras' Theorem to find the unknown length in simplest form.

a



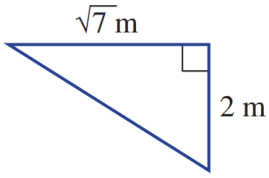
$2\sqrt{5}$ cm

b



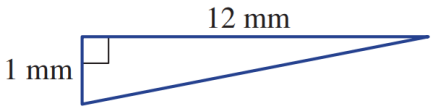
$3\sqrt{5}$ m

c



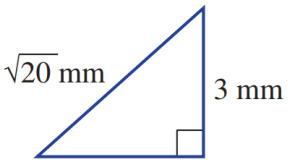
$\sqrt{11}$ m

d



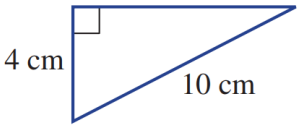
$\sqrt{145}$ mm

e



$\sqrt{11}$ mm

f



$2\sqrt{21}$ cm

14 Simplify the following.

a $\sqrt[3]{125} - \sqrt[3]{8}$
3

b $\sqrt[3]{24}$
$2\sqrt[3]{3}$

c $\sqrt[3]{2000}$
$10\sqrt[3]{2}$

d $\sqrt[5]{64}$
$2\sqrt[5]{2}$

15 Simplify the following.

a $\sqrt{a^2b}$
$a\sqrt{b}$

b $\sqrt{a^4b^2}$
a^2b

c $\sqrt[3]{a^6b^3}$
a^2b

d $\sqrt{a^5b^3}$
$a^2b\sqrt{ab}$

ADDITION AND SUBTRACTION OF SURDS

- We can only add and subtract "**like**" surds; like surds have the same radicand and index.

1. Simplify all surds.
2. Collect like surds.

Example Add and subtract surds.

$3\sqrt{7} - \sqrt{6} + 2\sqrt{7}$ 2. $3\sqrt{7} - \sqrt{6} + 2\sqrt{7}$ $= 5\sqrt{7} - \sqrt{6}$	How is this similar to simplifying $3x - y + 2x$? *We collect …… surds.*
$\sqrt{44} + \sqrt{99}$ 1. $\sqrt{4}\sqrt{11} + \sqrt{9}\sqrt{11}$ $2\sqrt{11} + 3\sqrt{11}$ 2. $5\sqrt{11}$	How did we add $\sqrt{44}$ and $\sqrt{99}$ even though they aren't like surds? *We …… the surds to make like surds.*

Simplify these expressions.

a $2\sqrt{11} + 6\sqrt{11}$ $8\sqrt{11}$	**b** $3\sqrt{6} + \sqrt{6}$ $4\sqrt{6}$	**c** $5\sqrt{7} + 3\sqrt{5} - 4\sqrt{7}$ $\sqrt{7} + 3\sqrt{5}$
d $\sqrt{8} + \sqrt{2}$ $3\sqrt{2}$	**e** $\sqrt{12} - \sqrt{3}$ $\sqrt{3}$	**f** $\sqrt{54} + \sqrt{24}$ $5\sqrt{6}$
g $2\sqrt{8} - \sqrt{18}$ $\sqrt{2}$	**h** $2\sqrt{5} + \sqrt{45}$ $5\sqrt{5}$	**i** $\sqrt{72} - \sqrt{50} + \sqrt{12}$ $2\sqrt{3} + \sqrt{2}$

Key ideas

1. We can only add or subtract …… surds; surds with the same …… and ……
2. Sometimes we can create like surds by …… them.

1 Decide whether these pairs of numbers are like or unlike surds.

a	$3\sqrt{4},\quad 4\sqrt{2}$	**b**	$5\sqrt{3},\quad 2\sqrt{3}$	**c**	$4\sqrt{2},\quad 4\sqrt{7}$	**d**	$\sqrt{3},\quad 3\sqrt{5}$
	unlike		like		unlike		unlike
e	$6\sqrt{6},\quad 3\sqrt{3}$	**f**	$\sqrt{8},\quad -\sqrt{8}$	**g**	$19\sqrt{2},\quad -2\sqrt{2}$	**h**	$-3\sqrt{6},\quad 3\sqrt{5}$
	unlike		like		like		unlike

2 Complete the following:

a $11x - 5x =$ ....., so $11\sqrt{2} - 5\sqrt{2} =$ .....

$6x,\quad 6\sqrt{2}$

b $8x - x =$ ....., so $8\sqrt{2} - \sqrt{2} =$ .....

$7x,\quad 7\sqrt{2}$

c $8x - 7x =$ ....., so $8\sqrt{2} - 7\sqrt{2} =$ .....

$x,\quad \sqrt{2}$

3 Simplify these expressions.

a	$2\sqrt{5} + 4\sqrt{5}$	**b**	$5\sqrt{3} - 2\sqrt{3}$	**c**	$7\sqrt{2} - 3\sqrt{2}$
	$6\sqrt{5}$		$3\sqrt{3}$		$4\sqrt{2}$
d	$8\sqrt{2} - 5\sqrt{2}$	**e**	$4\sqrt{10} + 3\sqrt{10} - \sqrt{10}$	**f**	$\sqrt{21} - 5\sqrt{21} + 2\sqrt{21}$
	$3\sqrt{2}$		$6\sqrt{10}$		$-2\sqrt{21}$

4 Simplify the surd $\sqrt{48}$ .....

$4\sqrt{3}$

Hence, simplify the following:

a	$\sqrt{3} + \sqrt{48}$	**b**	$\sqrt{48} - 7\sqrt{3}$	**c**	$5\sqrt{48} - 3\sqrt{3}$
	$5\sqrt{3}$		$-3\sqrt{3}$		$17\sqrt{3}$

5 Simplify all surds, then collect like surds.

a	$\sqrt{8} - \sqrt{2}$ $= 2\sqrt{2} - \sqrt{2}$ $=$	**b**	$\sqrt{8} + 3\sqrt{2}$	**c**	$\sqrt{27} + \sqrt{3}$
	$\sqrt{2}$		$5\sqrt{2}$		$4\sqrt{3}$
d	$\sqrt{20} - \sqrt{5}$	**e**	$4\sqrt{18} - 5\sqrt{2}$	**f**	$2\sqrt{75} + 2\sqrt{3}$
	$\sqrt{5}$		$7\sqrt{2}$		$12\sqrt{3}$
g	$3\sqrt{44} + 2\sqrt{11}$	**h**	$3\sqrt{8} - \sqrt{18}$	**i**	$2\sqrt{125} - 3\sqrt{45}$
	$8\sqrt{11}$		$3\sqrt{2}$		$\sqrt{5}$

6 Simplify these expressions.

a $2\sqrt{3} + 3\sqrt{2} - \sqrt{3} + 2\sqrt{3}$

$3\sqrt{3} + 3\sqrt{2}$

b $3\sqrt{5} - 4\sqrt{2} + \sqrt{5} - 3\sqrt{2}$

$4\sqrt{5} - 7\sqrt{2}$

c $5\sqrt{2} + 2\sqrt{5} - 7\sqrt{2} - \sqrt{5}$

$-2\sqrt{2} + \sqrt{5}$

d $2\sqrt{3} + 2\sqrt{7} + 2\sqrt{3} - 2\sqrt{7}$

$4\sqrt{3}$

e $5\sqrt{11} + 3\sqrt{6} - 3\sqrt{11}$

$2\sqrt{11} + 3\sqrt{6}$

f $-4\sqrt{10} - 5\sqrt{2} + \sqrt{10}$

$-3\sqrt{10} - 5\sqrt{2}$

7 Simplify these expressions.

a $2\sqrt{3} + 5\sqrt{3}$

$7\sqrt{3}$

b $2\sqrt{3} + 5\sqrt{6}$

Can't simplify

c $2\sqrt{3} + 5\sqrt{12}$

$12\sqrt{3}$

d $2\sqrt{3} + 5\sqrt{27}$

$17\sqrt{3}$

e $2\sqrt{3} + 6\sqrt{27}$

$20\sqrt{3}$

f $2\sqrt{30} + 6\sqrt{27}$

$2\sqrt{30} + 18\sqrt{3}$

g $2\sqrt{30} + 6\sqrt{120}$

$14\sqrt{30}$

h $6\sqrt{30} + 2\sqrt{120}$

$10\sqrt{30}$

i $6\sqrt{60} + 2\sqrt{240}$

$20\sqrt{15}$

j $6\sqrt{6} + 2\sqrt{24}$

$10\sqrt{6}$

k $6\sqrt{6} + 2\sqrt{24} + 5\sqrt{36}$

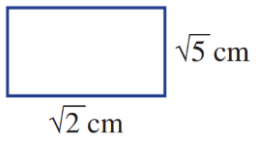
$10\sqrt{6} + 30$

l $6\sqrt{16} + 2\sqrt{24} + 5\sqrt{36}$

$54 + 4\sqrt{6}$

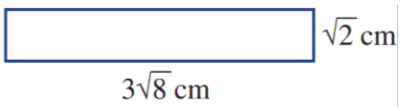
8 Calculate the perimeter in simplest form.

a



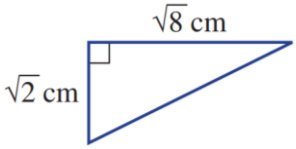
$2\sqrt{2} + 2\sqrt{5}$

b



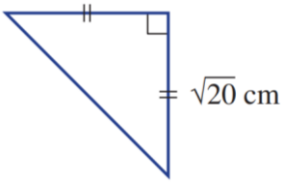
$14\sqrt{2}$

c



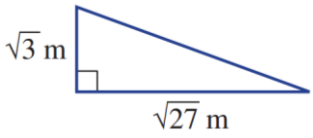
$3\sqrt{2} + \sqrt{10}$

d



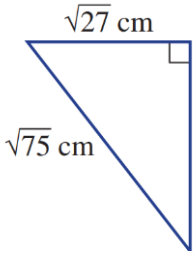
$4\sqrt{5} + 2\sqrt{10}$

e



$4\sqrt{3} + \sqrt{30}$

f



$12\sqrt{3}$

9 Prove that each of the following simplifies to zero by showing all steps.

a $5\sqrt{3} - \sqrt{108} + \sqrt{3}$

b $6\sqrt{2} - 2\sqrt{32} + 2\sqrt{2}$

c $2\sqrt{20} - 7\sqrt{5} + \sqrt{45}$

d $3\sqrt{2} - 2\sqrt{27} - \sqrt{50} + 6\sqrt{3} + \sqrt{8}$

10 Simplify the surds and combine the fractions using a common denominator.

a $\frac{\sqrt{8}}{3} - \frac{\sqrt{2}}{5}$ $\frac{7\sqrt{2}}{15}$

b $\frac{\sqrt{12}}{4} + \frac{\sqrt{3}}{6}$ $\frac{2\sqrt{3}}{3}$

c $\frac{3\sqrt{5}}{4} - \frac{\sqrt{20}}{3}$ $\frac{\sqrt{5}}{12}$

d $\frac{\sqrt{98}}{4} - \frac{5\sqrt{2}}{2}$ $-\frac{3\sqrt{2}}{4}$

e $\frac{2\sqrt{75}}{5} - \frac{3\sqrt{3}}{2}$ $\frac{\sqrt{3}}{2}$

f $\frac{\sqrt{63}}{9} - \frac{4\sqrt{7}}{5}$ $\frac{-21\sqrt{7}}{45}$

g $\frac{2\sqrt{18}}{3} - \frac{\sqrt{72}}{2}$ $-\sqrt{2}$

h $\frac{\sqrt{54}}{4} + \frac{\sqrt{24}}{7}$ $\frac{29\sqrt{6}}{28}$

i $\frac{\sqrt{27}}{5} - \frac{\sqrt{108}}{10}$ 0

j $\frac{5\sqrt{48}}{6} + \frac{2\sqrt{147}}{3}$ $8\sqrt{3}$

k $\frac{2\sqrt{96}}{5} - \frac{\sqrt{600}}{7}$ $\frac{34\sqrt{6}}{35}$

l $\frac{3\sqrt{125}}{14} - \frac{2\sqrt{80}}{21}$ $\frac{29\sqrt{5}}{42}$

16 Simplify fully.

a $\sqrt{600} + \sqrt{300} - \sqrt{216}$

$4\sqrt{6} + 10\sqrt{3}$

b $4\sqrt{18} + 3\sqrt{12} - 2\sqrt{50}$

$2\sqrt{2} + 6\sqrt{3}$

c $2\sqrt{175} - 5\sqrt{140} - 3\sqrt{28}$

$4\sqrt{7} - 10\sqrt{35}$

17 Find the value of x.

a $\sqrt{63} - \sqrt{28} = \sqrt{x}$

7

b $\sqrt{80} - \sqrt{20} = \sqrt{x}$

20

c $2\sqrt{150} - 3\sqrt{24} = \sqrt{x}$

96

MULTIPLYING SURDS

Multiplying Surds
$a\sqrt{x} \times b\sqrt{y} = ab\sqrt{xy}$

1. Multiply the coefficients and the radicands separately.
2. Simplify, if possible.

Example Multiply surds.

$\sqrt{15} \times \sqrt{5}$ 1. $\sqrt{75}$ 2. $5\sqrt{3}$	Why is $\sqrt{75} = 5\sqrt{3}$? *We can split $\sqrt{75}$ into $\sqrt{\ }\sqrt{\ }$, which is $5\sqrt{3}$*
$5\sqrt{6} \times 7\sqrt{10}$ 1. $35\sqrt{60}$ 2. $70\sqrt{15}$	How is this similar to simplifying $5x \times 7y$? *We multiply the ..... and ..... separately.* Why is $35\sqrt{60} = 70\sqrt{15}$? *$35\sqrt{60}$ can be split into $35\sqrt{\ }\sqrt{\ }$, which is $70\sqrt{15}$.*

Simplify these expressions.

a $\sqrt{7} \times \sqrt{3}$ $\sqrt{21}$	**b** $\sqrt{5} \times \sqrt{20}$ 10	**c** $\sqrt{12} \times \sqrt{8}$ $4\sqrt{6}$
d $2 \times 3\sqrt{2}$ $6\sqrt{2}$	**e** $2\sqrt{6} \times 5\sqrt{3}$ $30\sqrt{2}$	**f** $-4\sqrt{5} \times 3\sqrt{11}$ $-12\sqrt{55}$
g $2\sqrt{3} \times 3\sqrt{15}$ $18\sqrt{5}$	**h** $(2\sqrt{5})^2$ 20	**i** $3\sqrt{2} \times -4\sqrt{6}$ $-24\sqrt{3}$

Key ideas

1. $x\sqrt{m} \times y\sqrt{n} =$ .....
2. Remember to ..... the result as much as possible.

1 Simplify these expressions.

a $\sqrt{3} \times \sqrt{5}$

$\sqrt{15}$

b $\sqrt{7} \times \sqrt{3}$

$\sqrt{21}$

c $\sqrt{2} \times \sqrt{13}$

$\sqrt{26}$

d $\sqrt{2} \times (-\sqrt{15})$

$-\sqrt{30}$

e $-\sqrt{6} \times \sqrt{5}$

$-\sqrt{30}$

f $-\sqrt{6} \times (-\sqrt{11})$

$\sqrt{66}$

g $\sqrt{20} \div \sqrt{2}$

$\sqrt{10}$

h $\sqrt{18} \div \sqrt{3}$

$\sqrt{6}$

i $\sqrt{30} \div (-\sqrt{6})$

$-\sqrt{5}$

j $\frac{\sqrt{15}}{\sqrt{5}}$

$\sqrt{3}$

k $\frac{\sqrt{30}}{\sqrt{3}}$

$\sqrt{10}$

l $-\frac{\sqrt{26}}{\sqrt{2}}$

$-\sqrt{13}$

2 Simplify the following. Ensure your final answer is fully simplified.

a $\sqrt{9} \times \sqrt{9}$

9

b $\sqrt{14} \times \sqrt{7}$

$7\sqrt{2}$

c $\sqrt{2} \times \sqrt{22}$

$2\sqrt{11}$

d $\sqrt{3} \times \sqrt{18}$

$3\sqrt{6}$

e $\sqrt{10} \times \sqrt{5}$

$5\sqrt{2}$

f $\sqrt{12} \times \sqrt{8}$

$4\sqrt{6}$

g $(2\sqrt{3})^2$

12

h $(7\sqrt{3})^2$

147

i $(9\sqrt{2})^2$

162

3 Simplify $\sqrt{8} \times \sqrt{27}$ by:

a Simplifying each surd then multiplying

$2\sqrt{2} \times 3\sqrt{3}$
$= 6\sqrt{6}$

b Multiplying the surds then simplifying.

$\sqrt{216}$
$= \sqrt{36}\sqrt{6}$
$= 6\sqrt{6}$

Why is simplifying before multiplying easier in the example above?

You don't need to find square factors of a large number.

Simplify these expressions by simplifying each surd before multiplying.

c $\sqrt{8} \times \sqrt{18}$
$= 2\sqrt{\ } \times 3\sqrt{\ }$
$=$

12

d $\sqrt{54} \times \sqrt{8}$

$12\sqrt{3}$

e $\sqrt{75} \times \sqrt{3}$

15

f $\sqrt{12} \times \sqrt{18}$

$6\sqrt{6}$

4 Simplify these expressions.

a	$\sqrt{2} \times \sqrt{3}$	$\sqrt{6}$	**f**	$\sqrt{12} \times \sqrt{3}$	6	**k**	$\sqrt{20} \times \sqrt{4}$	$4\sqrt{5}$
b	$\sqrt{4} \times \sqrt{3}$	$2\sqrt{3}$	**g**	$\sqrt{12} \times \sqrt{4}$	$4\sqrt{3}$	**l**	$\sqrt{20} \times \sqrt{5}$	10
c	$\sqrt{5} \times \sqrt{3}$	$\sqrt{15}$	**h**	$\sqrt{11} \times \sqrt{4}$	$2\sqrt{11}$	**m**	$\sqrt{20} \times \sqrt{10}$	$10\sqrt{2}$
d	$\sqrt{6} \times \sqrt{3}$	$3\sqrt{2}$	**i**	$\sqrt{10} \times \sqrt{4}$	$2\sqrt{10}$	**n**	$\sqrt{20} \times \sqrt{20}$	20
e	$\sqrt{24} \times \sqrt{3}$	$6\sqrt{2}$	**j**	$\sqrt{10} \times \sqrt{8}$	$4\sqrt{5}$	**o**	$\sqrt{20} \times \sqrt{40}$	$20\sqrt{2}$

5 Simplify these expressions.

a	$2\sqrt{2} \times \sqrt{3}$	$2\sqrt{6}$	**f**	$2\sqrt{12} \times 3\sqrt{3}$	36	**k**	$\sqrt{20} \times 2\sqrt{4}$	$8\sqrt{5}$
b	$2\sqrt{4} \times \sqrt{3}$	$4\sqrt{3}$	**g**	$2\sqrt{12} \times 3\sqrt{4}$	$24\sqrt{3}$	**l**	$5\sqrt{20} \times 3\sqrt{5}$	150
c	$\sqrt{5} \times 2\sqrt{3}$	$2\sqrt{15}$	**h**	$2\sqrt{11} \times 3\sqrt{4}$	$12\sqrt{11}$	**m**	$5\sqrt{20} \times 3\sqrt{10}$	$150\sqrt{2}$
d	$\sqrt{6} \times 2\sqrt{3}$	$6\sqrt{2}$	**i**	$\sqrt{10} \times 2\sqrt{4}$	$4\sqrt{10}$	**n**	$5\sqrt{20} \times 3\sqrt{20}$	300
e	$\sqrt{24} \times 3\sqrt{3}$	$18\sqrt{2}$	**j**	$\sqrt{10} \times 2\sqrt{8}$	$8\sqrt{5}$	**o**	$2\sqrt{20} \times 3\sqrt{40}$	$120\sqrt{2}$

6 Consider the method of **factorising** a surd before multiplying:

Worked Example	Explanation
$\sqrt{7} \times \sqrt{14}$ $= \sqrt{7} \times \sqrt{2}\sqrt{7}$ $= 7\sqrt{2}$	The "normal" method would have been to multiply 7 and 14, then find the largest square factor. Why is this method faster? This method avoids working with large numbers.

Use the method of factorising to simply these surd multiplications.

a $\sqrt{8} \times \sqrt{24}$

$= \sqrt{8} \times \sqrt{\ }\sqrt{8}$

$=$

$8\sqrt{3}$

b $\sqrt{12} \times \sqrt{18}$

$= \sqrt{\ }\sqrt{6} \times \sqrt{\ }\sqrt{6}$

$=$

$6\sqrt{6}$

c $2\sqrt{14} \times \sqrt{21}$

$14\sqrt{6}$

d $\sqrt{11} \times \sqrt{55}$

$11\sqrt{5}$

7 Simplify the following. Ensure your final answer is fully simplified.
For some questions, it is useful to simplify before multiplying or factorise the surd.

a $2\sqrt{5} \times \sqrt{15}$

$10\sqrt{3}$

b $3\sqrt{7} \times \sqrt{14}$

$21\sqrt{2}$

c $-5\sqrt{10} \times \sqrt{30}$

$-50\sqrt{3}$

d $3\sqrt{6} \times (-\sqrt{18})$

$-18\sqrt{3}$

e $3\sqrt{14} \times 2\sqrt{21}$

$42\sqrt{6}$

f $-2\sqrt{7} \times (-3\sqrt{14})$

$42\sqrt{2}$

g $\sqrt{24} \times \sqrt{20}$

$4\sqrt{30}$

h $\sqrt{54} \times \sqrt{75}$

$45\sqrt{2}$

i $2\sqrt{72} \times 3\sqrt{80}$

$144\sqrt{10}$

j $\sqrt{108} \times (-2\sqrt{125})$

$-60\sqrt{15}$

k $-4\sqrt{27} \times (-\sqrt{28})$

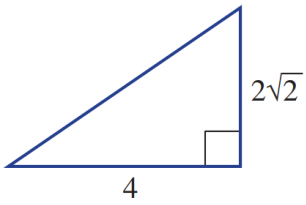
$24\sqrt{21}$

l $\sqrt{98} \times \sqrt{300}$

$70\sqrt{6}$

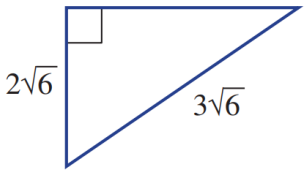
8 Determine the unknown side of the following right-angled triangles.

a



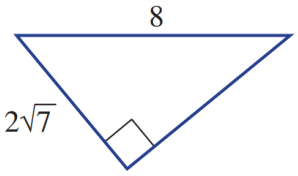
$2\sqrt{6}$

b



$\sqrt{30}$

c



6

9 Find the length of the diagonal of a square, which has an area of 12 cm^2.

A square is a rhombus. The diagonals are equal.

$A = \frac{xy}{2}$

$12 = \frac{x^2}{2}$

$x^2 = 24$

$x = 2\sqrt{6}$

Alternate method, each side of the square is $\sqrt{12}$

Using Pythaogras, diagonal is $\sqrt{(\sqrt{12})^2 + (\sqrt{12})^2}$

$= \sqrt{12 + 12}$

$= \sqrt{24}$

$= 2\sqrt{6}$

DIVIDING SURDS

Dividing Surds
$a\sqrt{x} \div b\sqrt{y} = \frac{a}{b}\sqrt{\frac{x}{y}}$

1. Divide the coefficients and the radicands separately.
2. Simplify, if possible.

Example Divide surds.

$\sqrt{10} \div \sqrt{15}$	$18\sqrt{48} \div 6\sqrt{2}$
1. $\sqrt{\frac{10}{15}}$	1. $\frac{18}{6}\sqrt{\frac{48}{2}}$
2. $= \sqrt{\frac{2}{3}}$	2. $= 3\sqrt{24}$
$= \frac{\sqrt{2}}{\sqrt{3}}$	$= 3\sqrt{4}\sqrt{6}$
	$= 6\sqrt{6}$

Simplify

a $\sqrt{15} \div \sqrt{3}$	**b** $3\sqrt{10} \div \sqrt{5}$	**c** $6\sqrt{21} \div 3\sqrt{7}$
$\sqrt{5}$	$3\sqrt{2}$	$2\sqrt{3}$
d $8\sqrt{11} \div 6\sqrt{25}$	**e** $3\sqrt{80} \div 8\sqrt{5}$	**f** $\sqrt{2\frac{1}{4}}$
$\frac{4}{15}\sqrt{11}$	$\frac{3}{2}$	$\frac{3}{2}$

Key ideas

1. $x\sqrt{m} \div y\sqrt{n} =$ .....
2. Remember to ..... the result as much as possible.

1 Simplify these surds.

a $\sqrt{20} \div \sqrt{2}$ $\sqrt{10}$

b $\sqrt{15} \div \sqrt{3}$ $\sqrt{5}$

c $\sqrt{33} \div (-\sqrt{11})$ $-\sqrt{3}$

d $-\sqrt{30} \div -\sqrt{6}$ $\sqrt{5}$

e $\dfrac{\sqrt{15}}{\sqrt{5}}$ $\sqrt{3}$

f $\dfrac{-\sqrt{40}}{\sqrt{8}}$ $-\sqrt{5}$

2 Simplify these surds.

a $\sqrt{\dfrac{8}{9}}$ $\dfrac{2\sqrt{2}}{3}$

b $\sqrt{\dfrac{12}{49}}$ $\dfrac{2\sqrt{3}}{7}$

c $\sqrt{\dfrac{18}{25}}$ $\dfrac{3\sqrt{2}}{5}$

d $\sqrt{\dfrac{10}{9}}$ $\dfrac{\sqrt{10}}{3}$

e $\sqrt{\dfrac{21}{144}}$ $\dfrac{\sqrt{21}}{12}$

f $\sqrt{\dfrac{26}{32}}$ $\dfrac{\sqrt{13}}{4}$

g $\sqrt{\dfrac{28}{50}}$ $\dfrac{2\sqrt{7}}{5\sqrt{2}}$ or $\dfrac{\sqrt{14}}{5}$

h $\sqrt{\dfrac{15}{27}}$ $\dfrac{\sqrt{5}}{3}$

i $\sqrt{\dfrac{45}{72}}$ $\dfrac{\sqrt{5}}{2\sqrt{2}}$ or $\dfrac{\sqrt{10}}{4}$

3 Simplify the following expressions. Ensure your final answer is fully simplified.

a $\dfrac{\sqrt{45}}{3}$ $\sqrt{5}$

b $\dfrac{\sqrt{24}}{4}$ $\dfrac{\sqrt{6}}{2}$

c $\dfrac{\sqrt{80}}{20}$ $\dfrac{\sqrt{5}}{5}$

d $\dfrac{\sqrt{99}}{18}$ $\dfrac{\sqrt{11}}{6}$

e $\dfrac{3\sqrt{44}}{2}$ $3\sqrt{11}$

f $\dfrac{2\sqrt{98}}{7}$ $2\sqrt{2}$

g $\dfrac{6\sqrt{75}}{20}$ $\dfrac{3\sqrt{3}}{2}$

h $\dfrac{2\sqrt{108}}{18}$ $\dfrac{2\sqrt{3}}{3}$

4 Simplify $\dfrac{\sqrt{12}}{\sqrt{3}}$ by:

a Simplifying each surd then dividing $\dfrac{2\sqrt{3}}{\sqrt{3}} = 2$

b Dividing the surds then simplifying. $\sqrt{4} = 2$

5 Simplify the following. Ensure your final answer is fully simplified.
For some questions, it is useful to simplify before dividing.

a $\frac{\sqrt{27}}{\sqrt{3}}$ 3

b $\frac{\sqrt{20}}{\sqrt{5}}$ 2

c $-\frac{\sqrt{162}}{\sqrt{2}}$ -9

d $-\frac{2\sqrt{2}}{5\sqrt{8}}$ $-\frac{1}{5}$

e $\frac{2\sqrt{45}}{15\sqrt{5}}$ $\frac{2}{5}$

f $\frac{5\sqrt{27}}{\sqrt{75}}$ 3

6 Simplify the following.

a $\frac{6\sqrt{14}}{3\sqrt{7}}$ $2\sqrt{2}$

b $\frac{15\sqrt{12}}{5\sqrt{2}}$ $3\sqrt{6}$

c $\frac{4\sqrt{30}}{8\sqrt{6}}$ $\frac{\sqrt{5}}{2}$

d $-\frac{8\sqrt{2}}{2\sqrt{26}}$ $-\frac{4}{\sqrt{13}}$

e $-\frac{3\sqrt{3}}{9\sqrt{21}}$ $-\frac{1}{3\sqrt{7}}$

f $\frac{12\sqrt{70}}{18\sqrt{14}}$ $\frac{2\sqrt{5}}{3}$

7 Simplify fully.

a $\frac{\sqrt{40}}{\sqrt{10}}$ 2

b $\frac{\sqrt{18}}{\sqrt{50}}$ $\frac{3}{5}$

c $\frac{2\sqrt{6}\times\sqrt{5}}{\sqrt{10}}$ $2\sqrt{3}$

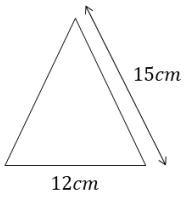
d $\frac{5\sqrt{7}\times\sqrt{3}}{\sqrt{28}}$ $\frac{5\sqrt{3}}{2}$

e $\frac{\sqrt{15}\times\sqrt{20}}{\sqrt{12}}$ 5

f $\frac{6\sqrt{3}\times8\sqrt{2}}{\sqrt{32}\times\sqrt{27}}$ 4

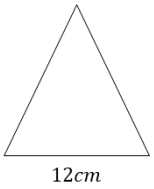
8 Consider the isosceles triangles below.

a Calculate the area of the triangle.



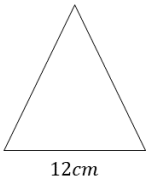
$18\sqrt{21}$ cm²

b The area of the triangle is $18\sqrt{5}$ cm². Calculate the height.



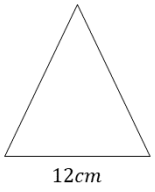
$3\sqrt{5}$ cm

c The area of this triangle is $30\sqrt{5}$ cm². Calculate the perimeter.



$12 + 2\sqrt{161}$ cm

d The perimeter of the triangle is 50 cm. Calculate the area.



$30\sqrt{13}$ cm²

9 Show that $\frac{\sqrt{18y}}{\sqrt{2y}}$ is an integer.

$\frac{\sqrt{18}\sqrt{y}}{\sqrt{2}\sqrt{y}} = \sqrt{9} = 3$

10 Simplify the following.

a $\sqrt{45} + \sqrt{80} - \sqrt{72}$

$7\sqrt{5} - 6\sqrt{2}$

b $3\sqrt{24} + 6\sqrt{150} - 5\sqrt{54}$

$21\sqrt{6}$

c $\frac{4+\sqrt{12}}{2}$

$2 + \sqrt{3}$

d $\frac{2\sqrt{6}\times 3\sqrt{7}}{12\sqrt{21}}$

$\frac{\sqrt{2}}{2}$

11 Simplify the following.

a $(3\sqrt{2})^3$

$= 3^3\sqrt{2}^3$

$= 27 \times 2\sqrt{2}$

$=$

$54\sqrt{2}$

b $(5\sqrt{3})^3$

$375\sqrt{3}$

c $2(3\sqrt{3})^3$

$162\sqrt{3}$

d $(\sqrt{5})^4$

25

e $(-\sqrt{3})^4$

9

f $(2\sqrt{2})^5$

$128\sqrt{2}$

g $-3(2\sqrt{5})^3$

$-120\sqrt{5}$

h $2(-3\sqrt{2})^3$

$-32\sqrt{2}$

i $5(2\sqrt{3})^4$

720

j $\frac{(2\sqrt{7})^3}{4}$

$14\sqrt{7}$

k $\frac{(3\sqrt{2})^3}{4}$

$\frac{27\sqrt{2}}{2}$

l $\frac{(3\sqrt{2})^4}{4}$

81

m $\frac{(5\sqrt{2})^2}{4} \times \frac{(2\sqrt{3})^3}{3}$

$100\sqrt{3}$

n $\frac{(2\sqrt{3})^2}{9} \times \frac{(-3\sqrt{2})^4}{3}$

144

o $\frac{(2\sqrt{5})^3}{5} \times \frac{(-2\sqrt{3})^5}{24}$

$-96\sqrt{15}$

p $\frac{(3\sqrt{3})^3}{2} \div \frac{(5\sqrt{2})^2}{4}$

$\frac{81\sqrt{3}}{25}$

q $\frac{(2\sqrt{5})^4}{50} \div \frac{(2\sqrt{3})^3}{5}$

$\frac{5}{3\sqrt{3}}$

r $\frac{(2\sqrt{2})^3}{9} \div \frac{(2\sqrt{8})^2}{(\sqrt{27})^3}$

$\frac{9\sqrt{6}}{2}$

DISTRIBUTIVE LAW WITH SURDS

Distributive Law

$$a(b+c) = ab + ac$$

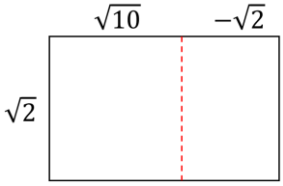
1. Expand brackets.
2. Simplify.

Example Expand brackets involving surds.

$\sqrt{3}(3\sqrt{5}+\sqrt{6})$	$3\sqrt{6}(2\sqrt{10}-4\sqrt{6})$
1. $3\sqrt{15}+\sqrt{18}$	1. $6\sqrt{60}-12\sqrt{36}$
2. $3\sqrt{15}+3\sqrt{2}$	2. $12\sqrt{15}-72$

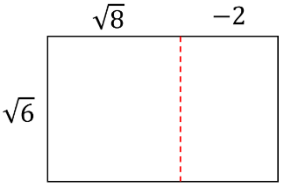
Expand these expressions.

a $\sqrt{2}(\sqrt{10}-\sqrt{2})$



$2\sqrt{5}-2$

b $\sqrt{6}(\sqrt{8}-2)$



$4\sqrt{3}-2\sqrt{6}$

c $\sqrt{3}(\sqrt{2}+\sqrt{5})$

$\sqrt{6}+\sqrt{15}$

d $2\sqrt{3}(\sqrt{5}+\sqrt{7})$

$2\sqrt{15}+2\sqrt{21}$

e $3\sqrt{2}(2\sqrt{13}-\sqrt{11})$

$6\sqrt{26}-3\sqrt{22}$

f $4\sqrt{5}(\sqrt{5}-\sqrt{10})$

$20-20\sqrt{2}$

Key ideas

1. The distributive law states that we can expand brackets using $a(b+c) =$ ……
2. The distributive law applies to …… as well.

1 Use the distributive law to expand the following and simplify the surds where necessary.

a $\sqrt{2}(\sqrt{10}-\sqrt{2})$

$2\sqrt{5}-2$

b $\sqrt{6}(3+\sqrt{3})$

$3\sqrt{6}+3\sqrt{2}$

c $\sqrt{5}(\sqrt{15}+4)$

$5\sqrt{3}+4\sqrt{5}$

d $\sqrt{6}(\sqrt{8}-2)$

$4\sqrt{3}-2\sqrt{6}$

e $3\sqrt{3}(9-\sqrt{21})$

$27\sqrt{3}-9\sqrt{7}$

f $3\sqrt{7}(\sqrt{14}-2\sqrt{7})$

$21\sqrt{2}-42$

g $-2\sqrt{6}(3\sqrt{2}-2\sqrt{3})$

$-12\sqrt{3}+12\sqrt{2}$

h $3\sqrt{7}(2\sqrt{7}+3\sqrt{14})$

$42+63\sqrt{2}$

i $-2\sqrt{8}(2\sqrt{2}-3\sqrt{20})$

$-16+24\sqrt{10}$

j $2\sqrt{3}(7\sqrt{6}+5\sqrt{3})$

$42\sqrt{2}+30$

2 Expand and simplify the following.

a $\sqrt{2}(3-\sqrt{3})-\sqrt{8}$

$\sqrt{2}-\sqrt{6}$

b $\sqrt{8}(\sqrt{6}+\sqrt{2})-\sqrt{3}$

$3\sqrt{3}+4$

c $\sqrt{5}(\sqrt{2}+1)-\sqrt{40}$

$\sqrt{5}-\sqrt{10}$

d $\sqrt{44}-2(\sqrt{11}-1)$

2

e $\sqrt{24}-2\sqrt{2}(\sqrt{3}-4)$

$8\sqrt{2}$

f $2\sqrt{3}(\sqrt{6}-\sqrt{3})-\sqrt{50}$

$\sqrt{2}-6$

EXPANDING BINOMIALS INVOLVING SURDS

Distributive Law for Binomial Products

$$(a+b)(c+d) = ac + ad + bc + bd$$

1. Expand brackets.
2. Simplify.

Example Expand binomials involving surds.

$(\sqrt{15}+2)(\sqrt{3}-3)$

$= \sqrt{45} - 3\sqrt{15} + 2\sqrt{3} - 6$

$= 3\sqrt{5} - 3\sqrt{15} + 2\sqrt{3} - 6$

	$\sqrt{3}$	-3
$\sqrt{15}$	$\sqrt{45}$	$-3\sqrt{15}$
2	$2\sqrt{3}$	-6

$(\sqrt{15}-\sqrt{6})^2$

$= 15 - 2\sqrt{90} + 6$

$= 21 - 2\sqrt{90}$

$= 21 - 6\sqrt{10}$

	$\sqrt{15}$	$-\sqrt{6}$
$\sqrt{15}$	15	$-\sqrt{90}$
$-\sqrt{6}$	$-\sqrt{90}$	6

Simplify these expressions.

a $(\sqrt{3}+1)(\sqrt{2}-1)$

	$\sqrt{3}$	1
$\sqrt{2}$		
-1		

$\sqrt{6} - \sqrt{3} + \sqrt{2} - 1$

b $(\sqrt{5}+1)(\sqrt{5}-1)$

	$\sqrt{5}$	1
$\sqrt{5}$		
-1		

4

c $(\sqrt{7}-\sqrt{5})^2$

	$\sqrt{7}$	$-\sqrt{5}$
$\sqrt{7}$		
$-\sqrt{5}$		

$12 - 2\sqrt{35}$

d $(2\sqrt{6}-1)(\sqrt{6}-2)$

$14 - 5\sqrt{6}$

e $(5\sqrt{2}-1)(3\sqrt{2}+3)$

$27 + 12\sqrt{2}$

f $(\sqrt{13}+\sqrt{9})^2$

$22 + 6\sqrt{13}$

Key ideas

1. The rule for expanding binomials is $(a+b)(c+d) =$ .....

1 Expand these binomial expressions.

a $(\sqrt{3}+1)(\sqrt{2}-1)$

$\sqrt{6}+\sqrt{2}-\sqrt{3}-1$

b $(\sqrt{5}-2)(\sqrt{7}+3)$

$\sqrt{35}+3\sqrt{5}-2\sqrt{7}-6$

c $(\sqrt{5}+\sqrt{2})(\sqrt{3}+\sqrt{2})$

$\sqrt{15}+\sqrt{10}+\sqrt{6}+2$

d $(\sqrt{6}-1)(\sqrt{6}-2)$

$8-3\sqrt{6}$

e $(\sqrt{7}-2)(2\sqrt{7}+5)$

$4+\sqrt{7}$

f $(3\sqrt{2}-1)(\sqrt{6}-\sqrt{3})$

$7\sqrt{3}-4\sqrt{6}$

2 Expand these expressions.

a $(3-2\sqrt{7})(\sqrt{21}-4\sqrt{3})$

$11\sqrt{21}-26\sqrt{3}$

b $(2\sqrt{6}+5)(\sqrt{30}-2\sqrt{5})$

$2\sqrt{5}+\sqrt{30}$

c $(3\sqrt{5}+1)(\sqrt{7}+2\sqrt{35})$

$30+\sqrt{7}+5\sqrt{35}$

d $(4\sqrt{2}+\sqrt{7})(3\sqrt{14}-5)$

$19\sqrt{7}+\sqrt{2}$

e $(3\sqrt{3}+4)(\sqrt{6}-2\sqrt{2})$

$\sqrt{2}-2\sqrt{6}$

f $(5-3\sqrt{2})(2\sqrt{10}+3\sqrt{5})$

$\sqrt{10}+3\sqrt{5}$

3 Expand these expressions.

a $(\sqrt{5}+2)(\sqrt{5}+1)$

$7+3\sqrt{5}$

b $(2+\sqrt{5})(1+\sqrt{5})$

$7+3\sqrt{5}$

c $(2-\sqrt{5})(1+\sqrt{5})$

$-3+\sqrt{5}$

d $(\sqrt{5}-2)(\sqrt{5}-1)$

$7-3\sqrt{5}$

e $(2+2\sqrt{5})(1+\sqrt{5})$

$12+4\sqrt{5}$

f $(\sqrt{5}+2)(\sqrt{5}-1)$

$3+\sqrt{5}$

g $(\sqrt{7}+2)(\sqrt{7}-1)$

$5+\sqrt{7}$

h $(2\sqrt{7}+2)(3\sqrt{7}-1)$

$40+4\sqrt{7}$

i $(2\sqrt{7}+2)(3\sqrt{7}-6)$

$30-6\sqrt{7}$

j $(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})$

1

k $(\sqrt{7}-\sqrt{6})(\sqrt{7}-\sqrt{6})$

$13-2\sqrt{42}$

l $(\sqrt{7}+\sqrt{6})^2$

$13+2\sqrt{2}$

4 Expand these perfect squares.

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

a $(\sqrt{3}+1)^2$

$4 + 2\sqrt{3}$

b $(2-\sqrt{6})^2$

$10 - 4\sqrt{6}$

c $(4+\sqrt{7})^2$

$23 + 8\sqrt{7}$

d $(2\sqrt{3}+4)^2$

$28 + 16\sqrt{3}$

e $(2\sqrt{7}+\sqrt{5})^2$

$33 + 4\sqrt{35}$

f $(3\sqrt{2}-2\sqrt{3})^2$

$30 - 12\sqrt{6}$

g $(5\sqrt{3}-2\sqrt{8})^2$

$107 - 40\sqrt{6}$

h $(3\sqrt{5}+4\sqrt{7})^2$

$157 + 24\sqrt{35}$

i $(5\sqrt{6}+3\sqrt{5})^2$

$195 + 30\sqrt{30}$

5 Expand these using the difference of two squares.

$$(a+b)(a-b) = a^2 - b^2$$

a $(3-\sqrt{2})(3+\sqrt{2})$

7

b $(5-\sqrt{6})(5+\sqrt{6})$

19

c $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$

1

d $(3\sqrt{2}-\sqrt{11})(3\sqrt{2}+\sqrt{11})$

7

e $(2\sqrt{6}+3)(2\sqrt{6}-3)$

15

f $(7-2\sqrt{5})(7+2\sqrt{5})$

29

g $(5\sqrt{8}-10\sqrt{2})(5\sqrt{8}+10\sqrt{2})$

0

h $(3\sqrt{7}-4\sqrt{6})(3\sqrt{7}+4\sqrt{6})$

-33

i $(2\sqrt{10}+4\sqrt{5})(2\sqrt{10}-4\sqrt{5})$

-40

j $(4\sqrt{10}-5\sqrt{6})(4\sqrt{10}+5\sqrt{6})$

10

6 What do you notice about the result of a difference of two squares above?

The answer has no surds.

7 Simplify the following expressions.

a 5×5

25

b $5 \times \sqrt{5}$

$5\sqrt{5}$

c $\sqrt{5} \times \sqrt{5}$

5

d $5 \times 5\sqrt{5}$

$25\sqrt{5}$

e $\sqrt{5} \times 5\sqrt{5}$

25

f $5\sqrt{5} \times 5\sqrt{5}$

125

g $5(5 + \sqrt{5})$

$25 + 5\sqrt{5}$

h $\sqrt{5}(5 + \sqrt{5})$

$5 + 5\sqrt{5}$

i $5(5 + 5\sqrt{5})$

$25 + 25\sqrt{5}$

j $5\sqrt{5}(5 + \sqrt{5})$

$25 + 25\sqrt{5}$

k $5\sqrt{5}(\sqrt{5} + 5\sqrt{5})$

150

l $(5 + \sqrt{5})(5 + \sqrt{5})$

$30 + 10\sqrt{5}$

m $(5 + \sqrt{5})(5 - \sqrt{5})$

20

n $(5 + \sqrt{5})(5 + 5\sqrt{5})$

$50 + 30\sqrt{5}$

o $(5 + 5\sqrt{5})(5 + 5\sqrt{5})$

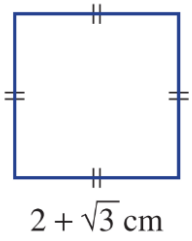
$150 + 50\sqrt{5}$

p $(\sqrt{5} + 5\sqrt{5})(\sqrt{5} + \sqrt{5})$

60

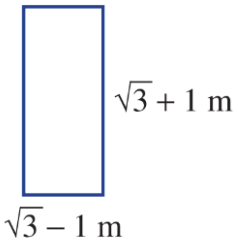
8 Find the area of these shapes in exact form.

a



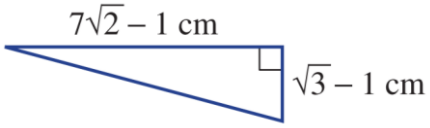
$7 + 4\sqrt{3}$

b



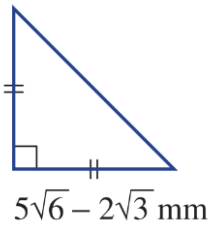
2

c



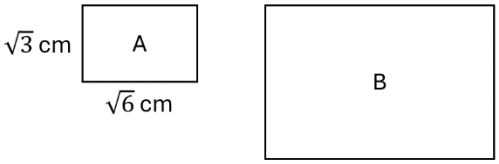
$\frac{1}{2}(7\sqrt{6} - 7\sqrt{2} - \sqrt{3} + 1)$

d



$81 - 30\sqrt{2}$

9 These rectangles are similar. The scale factor is 4.
Find the area of B in its simplest form.



$48\sqrt{2}$ cm²

10 Simplify the following.

a $\left(1+\frac{1}{\sqrt{2}}\right)^2$

$\frac{3}{2}+\frac{2}{\sqrt{2}}$

b $\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)^2$

$\frac{4}{3}$

11 Show that $x=-3+2\sqrt{2}$ satisfies the equation $x^2+6x+1=0$

$LHS=\left(-3+2\sqrt{2}\right)^2+6\left(-3+2\sqrt{2}\right)+1$

$=9-12\sqrt{2}+8-18+12\sqrt{2}+1$

$=0=RHS$

12 Find the value of a and b.

a $\left(\sqrt{3}+1\right)^2=a+b\sqrt{3}$

$a=4,b=2$

b $\left(4-\sqrt{5}\right)^2=a+b\sqrt{5}$

$a=21,b=-8$

c $\left(2\sqrt{3}+\sqrt{2}\right)^2=a+b\sqrt{6}$

$a=14,b=4$

d $\left(2\sqrt{2}+1\right)^2=a+4\sqrt{b}$

$a=9,b=2$

e $\left(2\sqrt{3}-3\sqrt{6}\right)^2=a+b\sqrt{2}$

$a=66,b=-12$

f $\left(1-3\sqrt{2}\right)^2-\left(2+3\sqrt{2}\right)^2=a+b\sqrt{2}$

$a=-3,b=-18$

RATIONALISING THE DENOMINATOR

- Rationalising the denominator means to write a fraction with an integer denominator.
- Multiply the numerator and denominator by the surd in the denominator.

Rationalising the Denominator		
$\frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$	$\frac{a}{b\sqrt{c}} \times \frac{\sqrt{c}}{\sqrt{c}} = \frac{a\sqrt{c}}{bc}$	$\frac{a\sqrt{b}}{c\sqrt{d}} \times \frac{\sqrt{d}}{\sqrt{d}} = \frac{a\sqrt{bd}}{cd}$

Example Rationalise a single-term denominator.

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ $= \frac{\sqrt{2}}{2}$	Why does $\sqrt{2} \times \sqrt{2}$ make the denominator rational? $\sqrt{x} \times \sqrt{x} = (\sqrt{x})^{\cdots} = \ldots\ldots\ldots$ Why do we need to multiply numerator and denominator by the same thing? *To keep the expression* ……
$\frac{2}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{2\sqrt{3}}{15}$	Why did we only need to multiply by $\sqrt{3}$ and not $5\sqrt{3}$? *Our goal is to make* $\sqrt{3}$ *a* …… *number, multiplying by* $\sqrt{3}$ *is sufficient, as* 5 *is already* …… Show why the denominator is 15: $5\sqrt{3} \times \sqrt{3} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Rationalise these expressions.

a $\frac{1}{\sqrt{3}}$ $\frac{\sqrt{3}}{3}$

b $\frac{\sqrt{2}}{\sqrt{3}}$ $\frac{\sqrt{6}}{3}$

c $\frac{1+\sqrt{2}}{\sqrt{3}}$ $\frac{\sqrt{3}+\sqrt{6}}{3}$

d $\frac{\sqrt{6}-5}{\sqrt{2}}$ $\frac{2\sqrt{3}-5\sqrt{2}}{2}$

e $\frac{\sqrt{3}}{2\sqrt{2}}$ $\frac{\sqrt{6}}{4}$

f $\frac{8+3\sqrt{2}}{4\sqrt{5}}$ $\frac{8\sqrt{5}+3\sqrt{10}}{20}$

Key ideas

1. Rationalising the denominator means to write the denominator as an ……
2. For $\frac{a\sqrt{b}}{c\sqrt{d}}$, we rationalise the denominator by multiplying …… and …… by ……

1 Write the missing number.

a	$\sqrt{3} \times \ldots\ldots = 3$	**b**	$\ldots\ldots \times \sqrt{5} = 5$	**c**	$2\sqrt{5} \times \ldots\ldots = 10$
	$\sqrt{3}$		$\sqrt{5}$		$\sqrt{5}$
d	$4\sqrt{3} \times \ldots\ldots = 12$	**e**	$\ldots\ldots \times \sqrt{7} = 21$	**f**	$3\sqrt{6} \times \ldots\ldots = 18$
	$\sqrt{3}$		$3\sqrt{7}$		$\sqrt{6}$
g	$\frac{1}{\sqrt{3}} \times \text{—} = \frac{\sqrt{3}}{3}$	**h**	$\frac{1}{\sqrt{7}} \times \text{—} = \frac{\sqrt{7}}{7}$	**i**	$\frac{2}{\sqrt{13}} \times \text{—} = \frac{2\sqrt{13}}{13}$
	$\frac{\sqrt{3}}{\sqrt{3}}$		$\frac{\sqrt{7}}{\sqrt{7}}$		$\frac{\sqrt{13}}{\sqrt{13}}$

2 Rationalise these denominators.

a	$\frac{1}{\sqrt{2}}$	**b**	$\frac{1}{\sqrt{7}}$	**c**	$\frac{3}{\sqrt{11}}$
	$\frac{\sqrt{2}}{2}$		$\frac{\sqrt{7}}{7}$		$\frac{3\sqrt{11}}{11}$
d	$\frac{4}{\sqrt{5}}$	**e**	$\frac{\sqrt{5}}{\sqrt{3}}$	**f**	$\frac{\sqrt{2}}{\sqrt{7}}$
	$\frac{4\sqrt{5}}{\sqrt{5}}$		$\frac{\sqrt{15}}{3}$		$\frac{\sqrt{14}}{7}$

3 Rewrite the following in the form $\frac{\sqrt{a}}{\sqrt{b}}$ and then rationalise the denominators.

a	$\sqrt{\frac{2}{3}}$	**b**	$\sqrt{\frac{5}{7}}$	**c**	$\sqrt{\frac{2}{5}}$
	$\frac{\sqrt{6}}{3}$		$\frac{\sqrt{35}}{7}$		$\frac{\sqrt{10}}{5}$
d	$\sqrt{\frac{7}{3}}$	**e**	$\sqrt{\frac{6}{7}}$	**f**	$\sqrt{\frac{17}{2}}$
	$\frac{\sqrt{21}}{3}$		$\frac{\sqrt{42}}{7}$		$\frac{\sqrt{34}}{2}$

4 Rationalise these denominators.

a	$\frac{4\sqrt{2}}{\sqrt{7}}$	**b**	$\frac{5\sqrt{2}}{\sqrt{3}}$	**c**	$\frac{3\sqrt{5}}{\sqrt{2}}$
	$\frac{4\sqrt{14}}{7}$		$\frac{5\sqrt{6}}{3}$		$\frac{3\sqrt{10}}{2}$
d	$\frac{3\sqrt{6}}{\sqrt{7}}$	**e**	$\frac{7\sqrt{3}}{\sqrt{10}}$	**f**	$\frac{2\sqrt{7}}{\sqrt{15}}$
	$\frac{3\sqrt{42}}{7}$		$\frac{7\sqrt{30}}{10}$		$\frac{2\sqrt{105}}{15}$

5 By rationalising the denominator, show that $\frac{n}{\sqrt{n}} = \sqrt{n}$.

$\frac{n\sqrt{n}}{n} = \sqrt{n}$

Therefore, simplify:

a	$\frac{2}{\sqrt{2}}$	**b**	$\frac{5}{\sqrt{5}}$	**c**	$\frac{93}{\sqrt{93}}$	**d**	$\frac{ab}{\sqrt{ab}}$
	$\sqrt{2}$		$\sqrt{5}$		$\sqrt{93}$		$\sqrt{ab}$

6 Rationalise these denominators.

a $\frac{5}{\sqrt{2}}$

$\frac{5\sqrt{2}}{2}$

b $\frac{6}{\sqrt{2}}$

$3\sqrt{2}$

c $\frac{6}{\sqrt{3}}$

$2\sqrt{3}$

d $\frac{4}{\sqrt{3}}$

$\frac{4\sqrt{3}}{3}$

e $\frac{4}{\sqrt{6}}$

$\frac{2\sqrt{6}}{3}$

f $\frac{5}{\sqrt{6}}$

$\frac{5\sqrt{6}}{6}$

g $\frac{5}{2\sqrt{6}}$

$\frac{5\sqrt{6}}{12}$

h $\frac{5}{3\sqrt{6}}$

$\frac{5\sqrt{6}}{18}$

i $\frac{5}{10\sqrt{6}}$

$\frac{\sqrt{6}}{12}$

j $\frac{5}{10\sqrt{5}}$

$\frac{\sqrt{5}}{10}$

k $\frac{5}{\sqrt{5}}$

$\sqrt{5}$

l $\frac{8}{\sqrt{8}}$

$\sqrt{8}$

m $\frac{10}{\sqrt{8}}$

$\frac{5\sqrt{8}}{4}$

n $\frac{10}{\sqrt{12}}$

$\frac{5\sqrt{12}}{6}$

o $\frac{10}{2\sqrt{3}}$

$\frac{5\sqrt{3}}{3}$

7 Rationalise these denominators.

a $\frac{4\sqrt{7}}{5\sqrt{3}}$

$\frac{4\sqrt{21}}{15}$

b $\frac{2\sqrt{3}}{3\sqrt{2}}$

$\frac{\sqrt{6}}{3}$

c $\frac{5\sqrt{7}}{3\sqrt{5}}$

$\frac{\sqrt{35}}{3}$

d $\frac{5\sqrt{12}}{3\sqrt{27}}$

$\frac{10}{9}$

e $\frac{9\sqrt{6}}{2\sqrt{3}}$

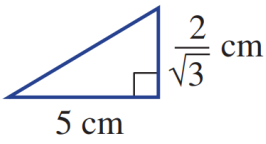
$\frac{9\sqrt{2}}{2}$

f $\frac{7\sqrt{90}}{2\sqrt{70}}$

$\frac{3\sqrt{7}}{2}$

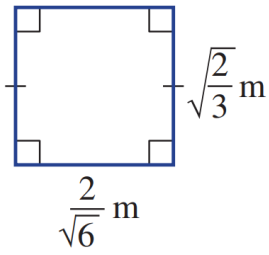
8 Determine the exact value of the area of the following shapes.
Express your answers using a rational denominator.

a



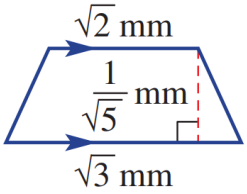
$\frac{5\sqrt{3}}{3}$

b



$\frac{2}{3}$

c



$\frac{\sqrt{10}+\sqrt{15}}{10}$

9 Rationalise these denominators.

a $\frac{1+\sqrt{2}}{\sqrt{3}}$

$\frac{\sqrt{3}+\sqrt{6}}{3}$

b $\frac{2-\sqrt{3}}{\sqrt{5}}$

$\frac{2\sqrt{5}-\sqrt{15}}{5}$

c $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{7}}$

$\frac{\sqrt{35}+\sqrt{14}}{7}$

d $\frac{\sqrt{2}+\sqrt{7}}{\sqrt{6}}$

$\frac{2\sqrt{3}+\sqrt{42}}{6}$

e $\frac{6-\sqrt{10}}{\sqrt{5}}$

$\frac{6\sqrt{5}-5\sqrt{2}}{5}$

f $\frac{3\sqrt{5}+5\sqrt{2}}{\sqrt{10}}$

$\frac{3\sqrt{2}+2\sqrt{5}}{2}$

13 Simplify the following.

$$xy\left(\frac{\sqrt{x}}{\sqrt{y}}-\frac{\sqrt{y}}{\sqrt{x}}\right)^2$$

$(x-y)^2$

RATIONALISING BINOMIAL DENOMINATORS

THE CONJUGATE

- The **conjugate** of $a + b\sqrt{c}$ is $a - b\sqrt{c}$.

Example Identify the conjugate of an expression.

Expression	Conjugate
$7 + \sqrt{3}$	
$8 + \sqrt{5}$	
$\sqrt{5} + 8$	
$\sqrt{5} - 8$	
$4 - \sqrt{7}$	
$5 + 3\sqrt{2}$	
$6 - 2\sqrt{3}$	
$4\sqrt{6} - 3$	

CONJUGATE PRODUCT

- The product of an expression and its conjugate is a difference of two squares.
- A conjugate product has no surds.

Conjugate Product	
$(a + \sqrt{c})(a - \sqrt{c})$ $= a^2 - c$	$(a + b\sqrt{c})(a - b\sqrt{c})$ $= a^2 - b^2c$

Example Determine the conjugate product.

a $(2 + \sqrt{3})(2 - \sqrt{3})$

$= 2^2 - (\sqrt{3})^2$

$= 4 - 3$

$= 1$

b $(5 - \sqrt{6})(5 + \sqrt{6})$

19

c $(7 - \sqrt{3})(7 + \sqrt{3})$

46

d $(5 + 3\sqrt{6})(5 - 3\sqrt{6})$

−29

e $(7 + 2\sqrt{5})(7 - 2\sqrt{5})$

29

f $(12 - 2\sqrt{3})(12 + \sqrt{3})$

132

RATIONALISING A BINOMIAL DENOMINATOR

- For binomial denominators, rationalise by **multiplying by the conjugate**.

Rationalising a Binomial Denominator
$\frac{a}{\sqrt{b}+\sqrt{c}} \times \frac{\sqrt{b}-\sqrt{c}}{\sqrt{b}-\sqrt{c}} = \frac{a\sqrt{b}-a\sqrt{c}}{b^2-c^2}$ $\frac{a}{\sqrt{b}-\sqrt{c}} \times \frac{\sqrt{b}+\sqrt{c}}{\sqrt{b}+\sqrt{c}} = \frac{a\sqrt{b}+a\sqrt{c}}{b^2-c^2}$

Example Rationalise a binomial denominator.

$\frac{3}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$ $= \frac{15-3\sqrt{3}}{25-3}$ $= \frac{15-3\sqrt{3}}{22}$	How did we know to multiply by $5-\sqrt{3}$? *$5-\sqrt{3}$ is the conjugate of ……* How did multiplying by $5-\sqrt{3}$ make the denominator a rational number? *…… of two ……* *$(a+b)(a-b) = a^2-b^2$ means the surd is gone.*
$\frac{\sqrt{5}}{2\sqrt{2}-3} \times \frac{2\sqrt{2}+3}{2\sqrt{2}+3}$ $= \frac{2\sqrt{10}+3\sqrt{5}}{4\times 2-9}$ $= -(2\sqrt{10}+3\sqrt{5})$	How do we know what to multiply by to rationalise a binomial denominator? *Multiply by the …… of the denominator.*

Rationalise these expressions.

a $\frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $\frac{\sqrt{3}+1}{2}$

b $\frac{1}{\sqrt{5}-\sqrt{2}}$ $\frac{\sqrt{5}+\sqrt{2}}{3}$

c $\frac{4}{\sqrt{3}+\sqrt{2}}$ $4\sqrt{3}-4\sqrt{2}$

d $\frac{\sqrt{3}-4}{\sqrt{3}+4}$ $\frac{-19+8\sqrt{3}}{13}$

e $\frac{2\sqrt{7}}{2\sqrt{7}-5}$ $\frac{28+10\sqrt{7}}{3}$

f $\frac{2\sqrt{3}}{\sqrt{5}+2\sqrt{6}}$ $\frac{-2\sqrt{15}+12\sqrt{2}}{19}$

Key ideas

1. To rationalise expressions with binomial denominators, we multiply by the ……
2. The conjugate of $a+b$ is ……, and the conjugate of $a-b$ is ……

1 Expand these products of conjugate pairs.

a $(4-\sqrt{3})(4+\sqrt{3})$

13

b $(5+\sqrt{7})(5-\sqrt{7})$

18

c $(\sqrt{6}-2\sqrt{10})(\sqrt{6}+2\sqrt{10})$

-34

d $(a+\sqrt{b})(a-\sqrt{b})$

a^2-b

Do you get a rational number as a result in each example?

yes

2 Why is $\frac{2}{4+\sqrt{3}}$ the same value as $\frac{2}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}$?

You multiply the numerator and denominator by the same expression, so the resulting expression is equivalent.

Rationalise these denominators.

a $\frac{2}{4+\sqrt{3}}$

$\frac{8-2\sqrt{3}}{13}$

b $\frac{1}{5-\sqrt{7}}$

$\frac{5+\sqrt{7}}{9}$

c $\frac{\sqrt{5}}{8-\sqrt{5}}$

$\frac{8\sqrt{5}+5}{59}$

d $\frac{5}{\sqrt{6}-1}$

$\sqrt{6}+1$

e $\frac{6}{\sqrt{2}+\sqrt{11}}$

$\frac{2\sqrt{11}-2\sqrt{2}}{3}$

f $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

$7+4\sqrt{3}$

3 The expression $\frac{\sqrt{6}+1}{\sqrt{3}+\sqrt{2}}$ can be written as $a\sqrt{3}+b\sqrt{2}$. Find a and b.

$a=-1, b=2$

10 Rationalise these denominators.

a $\frac{2}{\sqrt{2}+3}$

$6-2\sqrt{2}$

b $\frac{2}{3+\sqrt{2}}$

$\frac{6-2\sqrt{2}}{7}$

c $\frac{2}{3-\sqrt{2}}$

$\frac{6+2\sqrt{2}}{7}$

d $\frac{7}{3-\sqrt{2}}$

$3+\sqrt{2}$

e $\frac{7}{\sqrt{2}-3}$

$-\sqrt{2}-3$

f $\frac{7}{\sqrt{7}-3}$

$\frac{-7\sqrt{7}-21}{2}$

g $\frac{\sqrt{7}}{\sqrt{7}-3}$

$\frac{-7-3\sqrt{7}}{2}$

h $\frac{\sqrt{5}}{\sqrt{5}-3}$

$\frac{-5-3\sqrt{5}}{4}$

i $\frac{\sqrt{5}}{5-\sqrt{3}}$

$\frac{5\sqrt{5}+\sqrt{15}}{22}$

j $\frac{2\sqrt{5}}{5-\sqrt{3}}$

$\frac{5\sqrt{5}+\sqrt{15}}{11}$

k $\frac{4\sqrt{5}}{5-\sqrt{3}}$

$\frac{10\sqrt{5}+2\sqrt{15}}{11}$

l $\frac{4\sqrt{5}}{5-2\sqrt{3}}$

$\frac{20\sqrt{5}+8\sqrt{15}}{13}$

m $\frac{4\sqrt{5}}{5-3\sqrt{3}}$

$-10\sqrt{5}-6\sqrt{15}$

n $\frac{4\sqrt{3}}{5-3\sqrt{3}}$

$-10\sqrt{3}-18$

o $\frac{4\sqrt{3}-\sqrt{2}}{5-3\sqrt{3}}$

$\frac{-20\sqrt{3}-36+5\sqrt{2}+3\sqrt{6}}{2}$

4 Simplify by forming the lowest common denominator:

a $\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{3}-1}$

$\sqrt{3}$

b $\frac{3}{2\sqrt{5}-\sqrt{7}}-\frac{3}{2\sqrt{5}+\sqrt{7}}$

$\frac{6\sqrt{7}}{13}$

5 Show that the following expression is rational:

$$\frac{4}{2+\sqrt{5}}-\frac{1}{9-4\sqrt{5}}$$

$= \frac{4}{2+\sqrt{5}} \times \frac{(2-\sqrt{5})}{(2-\sqrt{5})} - \frac{1}{9-4\sqrt{5}} \times \frac{(9+4\sqrt{5})}{(9+4\sqrt{5})}$

$= \frac{8-4\sqrt{5}}{-1} - \frac{9+4\sqrt{5}}{81-80}$

$= -17$

6 Simplify the following by rationalising the denominator.

a $\frac{5+\sqrt{2}}{5-\sqrt{2}}$

$\frac{27+10\sqrt{2}}{23}$

b $\frac{1}{\sqrt{3}+1}+\frac{\sqrt{3}-1}{4}$

$\frac{3\sqrt{3}-3}{4}$

c $\frac{42}{\sqrt{63}}-\frac{6}{\sqrt{7}+2}$

4

7 Simplify the following by rationalising the denominator.

a $\frac{a\sqrt{b}}{c\sqrt{d}}$

$\frac{a\sqrt{bd}}{cd}$

b $\frac{1}{a+\sqrt{b}}$

$\frac{a-\sqrt{b}}{a^2-b}$

c $\frac{1}{\sqrt{a}}-\frac{1}{\sqrt{b}}$

$\frac{\sqrt{a}}{a}-\frac{\sqrt{b}}{b}=\frac{b\sqrt{a}-a\sqrt{b}}{ab}$

d $\frac{1}{a\sqrt{b}-c}$

$\frac{a\sqrt{b}+c}{a^2b-c^2}$

e $\frac{1}{a\sqrt{b}+c\sqrt{d}}$

$\frac{a\sqrt{b}-c\sqrt{d}}{a^2b-c^2d}$

8 Show these expressions are rational by rationalising the denominator.

a $\frac{1}{3+\sqrt{6}}+\frac{2}{\sqrt{6}}$

1

b $\frac{4}{2+\sqrt{2}}+\frac{1}{3-2\sqrt{2}}$

7

c $\frac{8}{3-\sqrt{7}}-\frac{6}{2\sqrt{7}-5}$

2

9

a Rationalise the denominator of

$$\frac{1}{\sqrt{n}+\sqrt{n+1}}$$

$\sqrt{n+1}-\sqrt{n}$

b Hence simplify

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}$$

$= (\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\cdots(\sqrt{100}-\sqrt{99})$

$= -1+\sqrt{100} = 9$

10

a Expand $\left(x+\frac{1}{x}\right)^2$

$x^2+2+\frac{1}{x^2}$

b Suppose that $x=\sqrt{7}+\sqrt{6}$

i Show that $x+\frac{1}{x}=2\sqrt{7}$

$\sqrt{7}+\sqrt{6}+\frac{1}{\sqrt{7}+\sqrt{6}}$

$= \sqrt{7}+\sqrt{6}+\frac{\sqrt{7}-\sqrt{6}}{1}$

$= 2\sqrt{7}$

ii Use the result in part **a** to find the value of $x^2+\frac{1}{x^2}$

$\left(x+\frac{1}{x}\right)^2 = x^2+2+\frac{1}{x^2}$

$\therefore x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2-2$

$= \left(2\sqrt{7}\right)^2-2$

$= 26$

11 Find the value of $\sqrt{7-4\sqrt{3}}\times\sqrt{7+4\sqrt{3}}$

$= \sqrt{(7-4\sqrt{3})(7+4\sqrt{3})}$

$= \sqrt{49-48}$

$= 1$

12

a Write down the expansion of $(a-b)^2$

$a^2-2ab+b^2$

b Use the expansion in part **a** to square $\sqrt{6+\sqrt{11}}-\sqrt{6-\sqrt{11}}$.

Let $a=\sqrt{6+\sqrt{11}}, b=\sqrt{6-\sqrt{11}}$

$(a-b)^2=(6+\sqrt{11})-2\sqrt{6+\sqrt{11}}\sqrt{6-\sqrt{11}}+(6-\sqrt{11})$

$=12-2\sqrt{(6+\sqrt{11})(6-\sqrt{11})}$

$=12-2\sqrt{36-11}$

$=12-2\sqrt{25}$

$=2$

c Hence evaluate $\sqrt{6+\sqrt{11}}-\sqrt{6-\sqrt{11}}$

$\sqrt{2}$

13 If $x=\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}$..., find the value of x.

Notice that since this repeats infinitely, we can write as $x=\sqrt{1+x}$

Square both sides

$x^2=1+x$

Solve using quadratic formula

$x=\frac{1\pm\sqrt{5}}{2}$

x must be positive as it is a root, so

$x=\frac{1+\sqrt{5}}{2}$

(this number $\phi\approx 1.618$ is the golden ratio)

14 If $\sqrt{x+\sqrt{x^2-4}}+\sqrt{x-\sqrt{x^2-4}}=4$, find the value of x.

Let $a=\sqrt{x+\sqrt{x^2-4}}$, $b=\sqrt{x-\sqrt{x^2-4}}$

It is given that $a+b=4$

$$ab=\sqrt{\left(x+\sqrt{x^2-4}\right)\left(x-\sqrt{x^2-4}\right)}$$

$$=\sqrt{x^2-(x^2-4)}$$

$$=\sqrt{4}=2$$

$$a+b=4$$

$$ab=2$$

Solving simultaneously,

$$a(4-a)=2$$

$$a^2-4a+2=0$$

$a=2-\sqrt{2}$ or $2+\sqrt{2}$

You can similarly find values of b:

$b=2+\sqrt{2}$ or $2-\sqrt{2}$

Using values of a:

$$\sqrt{x+\sqrt{x^2-4}}=2+\sqrt{2}$$

$$x+\sqrt{x^2-4}=6+4\sqrt{2}$$

$$x+\sqrt{x^2-4}=6+\sqrt{32}$$

By equating coefficients, $x=6$

$$\sqrt{x+\sqrt{x^2-4}}=2-\sqrt{2}$$

$$x+\sqrt{x^2-4}=6-4\sqrt{2}$$

$$x+\sqrt{x^2-4}=6-\sqrt{32}$$

No solution.

You can repeat for values of b, the only solutions is again $x=6$

Therefore, $x=6$

CHECK YOUR UNDERSTANDING

Surd laws

1 Write in simplest form $\sqrt{8} \times \sqrt{8}$

a 64 **b** 8 **c** $\sqrt{16}$ **d** $\sqrt{64}$

2 Write in simplest form $\sqrt{8} \div \sqrt{4}$

a $\sqrt{2}$ **b** 2 **c** 4 **d** Does not simplify

3 Write in simplest form $\sqrt{14} \div \sqrt{2}$

a $\sqrt{2}$ **b** 2 **c** $\sqrt{7}$ **d** Does not simplify

4 Write in simplest form $\frac{\sqrt{70}}{\sqrt{75}}$

a $\sqrt{35}$ **b** $\sqrt{2}$ **c** $\frac{\sqrt{14}}{\sqrt{7}}$ **d** $\frac{\sqrt{70}}{\sqrt{35}}$

5 Tom says $(\sqrt{5})^2 = \sqrt{25}$

Katie says $(\sqrt{5})^2 = 5$

Who is correct?

a Only Tom **b** Only Katie **c** Both **d** Neither

6 Which option is equivalent to $(\sqrt{5})^3$?

a 5 **b** $5\sqrt{5}$ **c** $\sqrt{15}$ **d** 25

Simplifying surds

1 $\sqrt{20} = 2\sqrt{n}$ What is the value of n?

a 10 **b** 0 **c** 4 **d** 5

2 $\sqrt{75} = n\sqrt{3}$ What is the value of n?

a 5 **b** 25 **c** 9 **d** 72

3 Simplify $\sqrt{18}$

a $2\sqrt{9}$ **b** $9\sqrt{2}$ **c** $3\sqrt{2}$ **d** $2\sqrt{3}$

4 Simplify $\sqrt{48}$

a $16\sqrt{3}$ **b** $4\sqrt{12}$ **c** $4\sqrt{3}$ **d** $2\sqrt{12}$

5 $4\sqrt{3} = \sqrt{n}$ What is the value of n?

a 24 **b** 12 **c** 36 **d** 48

6 $5\sqrt{13} = \sqrt{n}$ What is the value of n?

a 325 **b** 845 **c** 130 **d** 65

7 Simplify $3 + \sqrt{5}$

a $3\sqrt{5}$ **b** $\sqrt{8}$ **c** $\sqrt{15}$ **d** Can't simplify

8 Simplify $\sqrt{3} + \sqrt{5}$

a $3\sqrt{5}$ **b** $\sqrt{8}$ **c** $\sqrt{15}$ **d** Can't simplify

Simplifying surds

1 Tom says $\sqrt{5}+\sqrt{5}=\sqrt{10}$

Katie says $\sqrt{5}+\sqrt{5}=2\sqrt{5}$

Who is correct?

a Only Tom **b** Only Katie **c** Both **d** Neither

2 $2\sqrt{7}+3\sqrt{7}=$

a $5\sqrt{7}$ **b** $5\sqrt{14}$ **c** 35 **d** Can't simplify

3 $2\sqrt{7}-\sqrt{7}=$

a 2 **b** $\sqrt{7}$ **c** 1 **d** Can't simplify

4 Tom says $5\sqrt{2}-3\sqrt{2}=2\sqrt{2}$

Katie says $5\sqrt{2}-3\sqrt{2}=2$

Who is correct?

a Only Tom **b** Only Katie **c** Both **d** Neither

5 $2\sqrt{7}-\sqrt{5}=$

a $\sqrt{9}$ **b** $2\sqrt{2}$ **c** 4 **d** Can't simplify

6 $\sqrt{12}+\sqrt{75}=7\sqrt{b}$ What is the value of b?

a 29 **b** 87 **c** 3 **d** 4

7 $5\sqrt{2}-\sqrt{8}=c\sqrt{2}$ What is the value of c?

a 2 **b** 1 **c** 3 **d** 4

8 $2\sqrt{5}+2\sqrt{45}=d\sqrt{5}$ What is the value of d?

a 7 **b** 20 **c** 8 **d** 4

Multiplying surds

1	$\sqrt{7} \times \sqrt{7}$						
	a $\sqrt{14}$	**b**	49	**c**	7	**d**	Can't simplify
2	$3 \times \sqrt{7}$						
	a $3\sqrt{7}$	**b**	$9\sqrt{7}$	**c**	$\sqrt{21}$	**d**	Can't simplify
3	$2\sqrt{5} \times \sqrt{10}$						
	a $2\sqrt{50}$	**b**	$50\sqrt{2}$	**c**	10	**d**	$10\sqrt{2}$
4	$4\sqrt{3} \times \sqrt{6}$						
	a $4\sqrt{18}$	**b**	$36\sqrt{2}$	**c**	$6\sqrt{2}$	**d**	$12\sqrt{2}$
5	$2\sqrt{3} \times 5\sqrt{2}$						
	a $10\sqrt{6}$	**b**	$7\sqrt{5}$	**c**	$10\sqrt{5}$	**d**	Can't simplify
6	$2\sqrt{2} \times 3\sqrt{8}$						
	a 96	**b**	$\sqrt{96}$	**c**	24	**d**	$6\sqrt{16}$
7	$2\sqrt{3} \times 2\sqrt{3}$						
	a 6	**b**	$3\sqrt{3}$	**c**	$2\sqrt{9}$	**d**	$2\sqrt{6}$

Dividing surds

1 Simplify $\frac{\sqrt{75}}{\sqrt{3}}$

a 25 **b** 5 **c** $\sqrt{25}$ **d** $\sqrt{5}$

2 Simplify $\frac{12\sqrt{6}}{3}$

a $4\sqrt{6}$ **b** $12\sqrt{2}$ **c** $4\sqrt{2}$ **d** Does not simplify

3 Simplify $\frac{15\sqrt{30}}{5}$

a $3\sqrt{6}$ **b** $15\sqrt{6}$ **c** $3\sqrt{30}$ **d** Does not simplify

4 Simplify $6\sqrt{18} \div 2\sqrt{2}$

a $3\sqrt{9}$ **b** $3\sqrt{36}$ **c** 9 **d** 18

5 Tom says $8 \div \sqrt{2} = \frac{8\sqrt{2}}{2}$
Katie says $8 \div \sqrt{2} = 4\sqrt{2}$
Who is correct?

a Only Tom **b** Only Katie **c** Both **d** Neither

Distributing surds

1 $3(2\sqrt{5}-1)=a-3$ What is the value of a?

a $2\sqrt{15}$ **b** $6\sqrt{15}$ **c** $5\sqrt{5}$ **d** $6\sqrt{5}$

2 $\sqrt{5}(2\sqrt{3}-5)=b-5\sqrt{5}$ What is the value of b?

a $2\sqrt{8}$ **b** $2\sqrt{15}$ **c** $10\sqrt{15}$ **d** $\sqrt{30}$

3 Expand and simplify $5(\sqrt{7}+1)$

a $\sqrt{35}+6$ **b** $\sqrt{35}+5$ **c** $5\sqrt{7}+5$ **d** $5\sqrt{7}+6$

4 Expand and simplify $3(\sqrt{2}-5)$

a $\sqrt{6}-8$ **b** $3\sqrt{2}-15$ **c** $\sqrt{6}-15$ **d** $3\sqrt{2}-8$

5 Expand and simplify $\sqrt{5}(\sqrt{5}+2)$

a $\sqrt{5}+5\sqrt{2}$ **b** $5+2\sqrt{5}$ **c** $5+5\sqrt{2}$ **d** $25+2\sqrt{5}$

6 Expand and simplify $\sqrt{5}(3+\sqrt{5})$

a $\sqrt{15}+\sqrt{25}$ **b** $8\sqrt{5}$ **c** $3\sqrt{5}+5$ **d** $\sqrt{15}+5$

Binomial expansion with surds

1 Expand and simplify $(\sqrt{7}-5)(\sqrt{7}+5)$

a $\sqrt{70}-18$ **b** $10\sqrt{7}-18$ **c** 24 **d** -18

2 Expand and simplify $(9+\sqrt{3})^2$

a $84+\sqrt{54}$ **b** $84+18\sqrt{3}$ **c** 12 **d** 84

3 Expand and simplify $(\sqrt{7}-5)^2$

a $-18-10\sqrt{7}$ **b** 4 **c** $32-10\sqrt{7}$ **d** 32

4 Expand and simplify $(9+\sqrt{3})(9-\sqrt{3})$

a $81-\sqrt{6}$ **b** $81-2\sqrt{3}$ **c** 78 **d** 72

Rationalising denominator

1 Which of the following is equivalent to $\frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

a $\frac{5\sqrt{3}}{3}$ **b** $\frac{5\sqrt{3}}{9}$ **c** $\frac{\sqrt{15}}{\sqrt{9}}$ **d** $\frac{5\sqrt{3}}{\sqrt{3}}$

2 Which of the following is equivalent to $\frac{11}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

a $\frac{11}{25}$ **b** $\frac{\sqrt{55}}{25}$ **c** $\frac{11\sqrt{5}}{5}$ **d** $\frac{\sqrt{55}}{5}$

3 Rationalise the denominator. $\frac{2}{\sqrt{3}}$

a $\frac{2\sqrt{3}}{9}$ **b** $\frac{2\sqrt{3}}{\sqrt{3}}$ **c** $\frac{2\sqrt{3}}{3}$ **d** $\frac{\sqrt{6}}{3}$

4 Rationalise the denominator and simplify. $\frac{2}{\sqrt{8}}$

a $\frac{\sqrt{2}}{2}$ **b** $\frac{2\sqrt{8}}{\sqrt{8}}$ **c** $\frac{4}{8}$ **d** Can't rationalise

5 Rationalise the denominator and simplify. $\frac{\sqrt{3}}{\sqrt{15}}$

a $\frac{\sqrt{5}}{5}$ **b** $\frac{1}{\sqrt{5}}$ **c** $\frac{1}{5}$ **d** $\frac{\sqrt{45}}{15}$

6 Rationalise the denominator and simplify. $\frac{\sqrt{7}}{\sqrt{35}}$

a $\frac{1}{25}$ **b** $\frac{1}{\sqrt{5}}$ **c** $\frac{7\sqrt{5}}{35}$ **d** $\frac{\sqrt{5}}{5}$

7 Rationalise the denominator $\frac{9}{2\sqrt{10}}$

a $\frac{9}{20}$ **b** $\frac{9\sqrt{10}}{10}$ **c** $\frac{9\sqrt{10}}{20}$ **d** $\frac{9}{2}$