Student Name

Mathematics Stage 4

INDICES

Book 1 Apply index notation to represent whole numbers as products of powers of prime numbers

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CONTENTS

OVERVIEW

MA4-INT-C-01 operates with primes and roots, positive-integer and zero indices involving numerical bases and establishes the relevant index laws

Apply index notation to represent whole numbers as products of powers of prime numbers

- Describe numbers written in index form using terms such as base, power, index and exponent
- Represent numbers in index notation limited to positive powers
- Represent in expanded form and evaluate numbers expressed in index notation, including powers of 10
- Apply the order of operations to evaluate expressions involving indices
- Determine and apply tests for divisibility for 2, 3, 4, 5, 6 and 10
- Represent a whole number greater than one as a product of its prime factors, using index notation where appropriate

SUMMARY

Index Notation and Prime Factorisation

Index Notation

shows repeated multiplication

index

$8^3 = \underbrace{8 \times 8 \times 8}_{\text{3 times}}$

base

$= 512$

basic numeral

Divisibility Tests

Used to find whether a number is a factor of another number.

Divisible by	Test
2	The last digit is even (0, 2, 4 , 6 or 8).
3	The sum of the digits is divisible by 3.
4	The last two digits are divisible by 4.
5	The last digit is 5 or 0.
6	The number is divisible by 2 and 3.
10	The last digit is 0.

Order of Operations

Write a new line for every operation you do

Operations in Brackets $(\square)$

Indices/Roots $\square^2$ $\sqrt{\square}$

Division/Multiplication $\times$ $\div$

Addition/Subtraction $-$ $+$

$7^2 - 6^2$

$= 49 - 36$

$= 13$

Prime Numbers

A prime number has only two factors: 1 and itself.

If a number is divisible by any number other than 1 and itself, then it must be composite.

First 10 prime numbers:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Prime Factorisation

Use a factor tree to represent a number as a product of its prime factors.

30 → 3, 10; 10 → 2, 5

48 → 4, 12; 4 → 2, 2; 12 → 2, 5

$30 = 2 \times 3 \times 5$

$48 = 2^3 \times 5$

REVIEW OF PRIOR KNOWLEDGE

1 Evaluate these expressions, following the order of operations.

a $5 + 2 \times 3$	**b** $4 \times 2 + 3 \times 5$	**c** $8 - 2 \times 3$
d $2 + 4 \times 3 \div 6$	**e** $13 - 3 \times 4 + 2$	**f** $3 + 4 \times (10 - 7)$
g $60 \div (5 \times 2) + 3$	**h** $(5 \times 10) \div (5 \times 5)$	**i** $(3 \times 8 - 6 \times 2) \div 4$
j $\frac{1+3\times4}{2}$	**k** $13 - \big((4 + 6) \div 2\big)$	**l** $\frac{10\times3+2}{4}$

2

a Is 15 a factor of 75?	**b** Does 9 divide evenly into 63?	**c** is 12 a factor of 48?
d Is 22 a factor of 77?	**e** Is 42 divisible by 5?	**f** Is 25 a factor of 100?

3 Simplify these fractions.

a $\frac{6}{8}$	**b** $\frac{24}{36}$	**c** $\frac{56}{72}$
d $\frac{18}{24}$	**e** $\frac{30}{45}$	**f** $\frac{63}{84}$

INDEX NOTATION

- **Powers** show repeated multiplication.
- 8^5 is read: "8 to the power of 5"
- Any number without a power has a power of 1.
- "**index**", "**power**", "**exponent**" all mean the same thing.

Index Notation: https://www.geogebra.org/m/qtvandqt

Index Notation

index

$$8^3 = \underbrace{8 \times 8 \times 8}_{\text{3 times}}$$

base

Example Express repeated multiplication using index form.

$5 \times 5 \times 5 \times 5 \times 5 \times 5$ $= 5^6$	What does a power of 6 mean? *Multiply the ………… number ……… times.*
$(2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3)$ $= 2^3 \times 3^4$	

Write these numbers in index form.

a $3 \times 3 \times 3 \times 3 \times 3$

$=$

3^5

b $5 \times 5 \times 5 \times 5$

$=$

5^4

c $9 \times 9 \times 12$

$=$

$9^2 \times 12$

d 6

$=$

6^1

e $4 \times 4 \times 4 \times 5 \times 5$

$=$

$4^3 \times 5^2$

f $5 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$=$

$2^3 \times 3^2 \times 5^2$

Example Rewrite index form in expanded form.

2^4 $= 2 \times 2 \times 2 \times 2$	$2^3 \times 3^5$ $= (2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3 \times 3)$

Write these numbers in expanded form.

a 3^6 $=$ $3 \times 3 \times 3 \times 3 \times 3 \times 3$	**b** 13^2 $=$ 13×13
c 2×14^2 $=$ $2 \times 14 \times 14$	**d** $2^3 \times 6^2$ $=$ $2 \times 2 \times 2 \times 6 \times 6$

Example Evaluate numbers in index form.

3^2 $= 3 \times 3 = 9$	What does evaluate mean? *Evaluate means to find the ……………… ……………… of an expression.*
2^5 $= 2 \times 2 \times 2 \times 2 \times 2 = 32$	What are two ways to read out 3^2? *"three to the ………… of ……"* *"three ………………"*

Evaluate:

a 6^3 216	**b** 2^6 64	**c** 11^2 121	**d** 10^4 10000

Key ideas

1. 7^5: ………… is the base number and ……… is the index number.
2. We can also call the index number the ………………… or the ……………………
3. We read out 7^5 as "7 to the ……………… of 5"
4. Powers represent repeated ……………………

1 Complete the table:

Index Form	Expanded Form	Basic Numeral
2^3	$2 \times 2 \times 2$	8
5^2	5×5	25
10^3	$10 \times 10 \times 10$	1000
2^6	$2 \times 2 \times 2 \times 2 \times 2 \times 2$	64
3^9	$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$	19683
6^2	6×6	36
4^3	$4 \times 4 \times 4$	64
2^5	$2 \times 2 \times 2 \times 2 \times 2$	32
3^4	$3 \times 3 \times 3 \times 3$	81
1^{500}	$1 \times 1 \times 1 \times 1 \times \ldots (500\text{ times})$	1
0^{500}	$0 \times 0 \times 0 \times 0 \times \ldots (500\text{ times})$	0

2 Evaluate these expressions.

a 4^3

64

b 3^4

81

c 2^4

16

d 1^4

1

e 0^4

0

f 1^{412}

1

g 412^1

412

h $(-412)^1$

-412

i 2^5

32

j 5^2

25

k $(-5)^2$

25

l $(-5)^3$

-125

m $(0.2)^2$

0.04

n $(0.2)^3$

0.008

o $\left(\frac{1}{5}\right)^3$

$\frac{1}{125}$

3 Simplify the following expressions by writing them with powers.

a $3 \times 3 \times 5 \times 5$

$3^2 \times 5^2$

b $8 \times 8 \times 5 \times 5 \times 5$

$8^2 \times 5^3$

c $8 \times 8 \times 8 \times 11 \times 11$

$8^3 \times 11^2$

d $7 \times 7 \times 2 \times 2 \times 7$

$2^2 \times 7^3$

e $12 \times 9 \times 9 \times 12$

$9^2 \times 12^2$

f $6 \times 3 \times 6 \times 3 \times 6 \times 3$

$3^3 \times 6^3$

g $13 \times 7 \times 13 \times 7 \times 7 \times 7$

$7^4 \times 13^2$

h $4 \times 13 \times 4 \times 4 \times 7$

$4^3 \times 7 \times 13$

i $10 \times 9 \times 10 \times 9 \times 9 \times 7$

$7 \times 9^3 \times 10^2$

j $(-2) \times (-2)$

$(-2)^2$

k $(-1) \times (-1) \times (-1)$

$(-1)^3$

l $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}$

$\left(\frac{3}{7}\right)^3$

m $\frac{7}{4} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4}$

$\left(\frac{7}{4}\right)^4$

n $2^5 \times 2$

2^6

o $x \times x \times x \times x \times x$

x^5

p $a \times b \times a \times b \times a \times c$

a^3b^2c

4 Write these expressions in expanded form.

a $3^5 \times 2^3$

$3 \times 3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2$

b $4^5 \times 9^3$

$4 \times 4 \times 4 \times 4 \times 4 \times 9 \times 9 \times 9$

c $7^2 \times 5^3$

$7 \times 7 \times 5 \times 5 \times 5$

d $a^2 \times b^3$

$a \times a \times b \times b \times b$

e $4^3 \times 3^4$

$4 \times 4 \times 4 \times 3 \times 3 \times 3 \times 3$

f $2^2 \times c^3$

$2 \times 2 \times c \times c \times c$

5 There is a disease where the number of people infected doubles each day.
Complete the first row of the table before completing the second row.

	Day 0	Day 1	Day 2	Day 3	Day 4	Day 5
People Infected	1	2	2×2	$2 \times 2 \times 2$	$2 \times 2 \times 2 \times 2$	$2 \times 2 \times 2 \times 2 \times 2$
Index Notation	2^0	2^1	2^2	2^3	2^4	2^5

How many people would be infected on day 40, in index form?

2^{40}

If it takes 32 days for half the population of Earth to be infected, how many days will it take for the entire population to be infected?

33

6 Write each of these as a power of 10.

a $10 \times 10 \times 10$ 10^3	**b** $10 \times 10 \times 10 \times 10 \times 10$ 10^5	**c** 10×10 10^2
d 100 10^2	**e** 1000 10^3	**f** $10\,000$ 10^4
g $10\,000\,000$ 10^7	**h** 1 million (6 zeros) 10^6	**i** $100\,000\,000$ 10^8
j $10\,000\,000\,000$ 10^{10}	**k** 1 billion (9 zeros) 10^9	**l** 100 billion 10^{11}

7 Evaluate these expressions.

a 3×3 9	**b** 7×7 49	**c** 9×9 81	**d** 12×12 144
e -3×-3 9	**f** -7×-7 49	**g** -9×-9 81	**h** -12×-12 144

8 Work out the following:

a $(-1)^1$ -1	**b** $(-1)^2$ 1	**c** $(-1)^3$ -1	**d** $(-1)^4$ 1	**e** $(-1)^5$ -1
f $(-2)^1$ -2	**g** $(-2)^2$ 4	**h** $(-2)^3$ -8	**i** $(-2)^4$ 16	**j** $(-2)^5$ -32
k $(-3)^1$ -3	**l** $(-3)^2$ 9	**m** $(-3)^3$ -27	**n** $(-3)^4$ 81	**o** $(-3)^5$ -243

Using the above, complete these sentences:

A negative number raised to an odd power is

negative

A negative number raised to an even power is

positive

Decide whether the answer to each calculation will be positive or negative. You do not need to work out the answers

a $(-9)^{24}$ positive	**b** $(5)^{19}$ positive	**c** $(-1)^{11}$ negative	**d** $(-200)^{298}$ positive

9 Rewrite each number in expanded form with powers of 10. The first one has been done for you.

a 537 $= 500 + 30 + 7$ $= 5 \times 10^2 + 3 \times 10^1 + 7$	**b** 38 $3 \times 10^1 + 8$	**c** 798 $7 \times 10^2 + 9 \times 10^1 + 8$
d 1235 $1 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 5$	**e** 306 $3 \times 10^2 + 6$	**f** 40007 $4 \times 10^4 + 7$
g $8\,000\,500$ $8 \times 10^6 + 5 \times 10^2$	**h** 94 400 000 $9 \times 10^7 + 4 \times 10^6 + 4 \times 10^5$	**i** 4 350 000 000 $4 \times 10^9 + 3 \times 10^8 + 5 \times 10^7$

10 Rewrite each number as a basic numeral.

a 7×10^1 70	**b** $3 \times 10^2 + 6 \times 10^1 + 4$ 364	**c** $9 \times 10^2 + 8 \times 10^1$ 980
d $7 \times 10^6 + 3 \times 10^4$ 7 030 000	**e** $6 \times 10^5 + 3 \times 10^2 + 5$ 600 305	**f** $8 \times 10^9 + 3 \times 10^6$ 8 003 000 000

11 Find the missing digits.

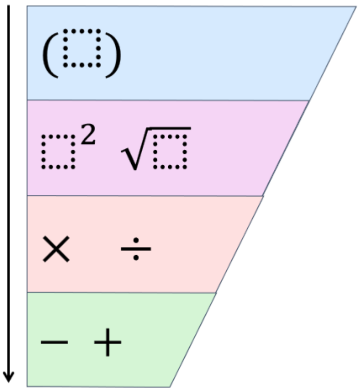
a $36 = 6^{\square}$ 2	**b** $27 = \square^3$ 3	**c** $100\,000 = 10^{\square}$ 5
d $73 = \square^1$ 73	**e** $2401 = 7^{\square}$ 4	**f** $64 = \square^6$ 2
g $16 = 4^{\square}$ 2	**h** $16 = 2^{\square}$ 4	**i** $125 = \square^{\square}$ 5^3

EVALUATE EXPRESSIONS WITH INDICES

ORDER OF OPERATIONS

Write a new line for every operation you do

Operations in **B**rackets		Operations in innermost brackets first
Indices/Roots		Left to right
Division/**M**ultiplication		Left to right
Addition/**S**ubtraction		Left to right



Example Evaluate expressions with indices.

$\mathbf{7^2 - 6^2}$ $= 49 - 36$ $= 13$	Which operations did we do first? *We evaluated the .... before subtraction.*
$\mathbf{(2 + 3^3) \times 10^2}$ $= (2 + 27) \times 10^2$ $= 29 \times 10^2$ $= 29 \times 100$ $= 2900$	Which operation did we do first? *We evaluated the .... inside the ....* Why did we not do $2 + 3$ first? *We do .... before addition.* Which operation did we do second? $2 + 27$ *because they are inside ....*

a $5^3 - 2^4$ — 109

b $3^4 + 7^2$ — 130

c $6^2 - 4^3 \div 2$ — 4

d $2 + 3^2 \times 4$ — 38

e $6 \times (1 + 3^2)$ — 60

f $(5 + 2)^2 \div 9$ — $\frac{49}{9}$

Key ideas

1. We do operations involving .... before multiplication and division.
2. We always do operations inside .... first. If there are lots of operations inside brackets, then we also follow the .... inside brackets.

1 Evaluate these expressions.

a $6 \div 2 + 4$ 7	**b** $6 \div (2 + 4)$ 1	**c** $6^2 \div 2 + 4$ 22
d $6 \div 2^2 + 4$ 5.5	**e** $6 \div (2^2 + 4)$ 0.75	**f** $(6 \div 2)^2 + 4$ 13

2 Evaluate these expressions.

a $4^2 + 5$ 21	**b** $4 + 5^2$ 29	**c** $(4 + 5)^2$ 81	**d** $4^2 + 5^2$ 41

3 Evaluate these expressions.

a $3^2 + 4^2$ $= 9 + 16$ $= \ldots\ldots\ldots\ldots\ldots$ 25	**b** $2 \times 5^2 - 7^2$ $= 2 \times 25 - 49$ $= \ldots\ldots\ldots\ldots\ldots$ 1	**c** $8^2 - 2 \times 3^3$ 10
d $3(9 - 5)^2$ 48	**e** $2^4 \times 2^3$ 128	**f** $1^4 \times 2^3 + 3^2 + 4$ 21
g $10^3 - 10^2$ 900	**h** 2.57×10^4 25 700	**i** $1^{512961} - 1^{595718}$ 0

4 Evaluate these expressions.

a $5 \times 2 + 4$ 14	**b** $5 \times 2^2 + 4$ 24	**c** $5 \times 2 + 4^2$ 26	**d** $5^2 \times 2 + 4$ 54
e $(5 \times 2)^2 + 4$ 104	**f** $(5 \times 2 + 4)^2$ 196	**g** $5^2 - 2 + 4$ 27	**h** $5^2 - (2 + 4)$ 19
i $5 - 2^2 + 4$ 5	**j** $(5 - 2)^2 + 4$ 13	**k** $5 - 2 + 4^2$ 19	**l** $(5 - 2 + 4)^2$ 49

5 Evaluate these expressions.

a $12 \times 4 + 2$ 50	**b** $12 + 4 \times 2$ 20	**c** $12 \div 4 + 2$ 5	**d** $12 + 4 \div 2$ 14
e $12 \times 4 + 2^2$ 52	**f** $12 + 4 \times 2^2$ 28	**g** $12 \div 4 + 2^2$ 7	**h** $12 + 4 \div 2^2$ 13
i $12 \times (4 + 2)$ 72	**j** $(12 + 4)^2 + 2$ 258	**k** $(12 \div 4) + 2^2$ 7	**l** $(12 \times 4) \div 2^2$ 12

6 Read the examples carefully and answer the questions.

Helen's work:

$8-\left(2^2-6\right)$

$8-(2^2-6)$

$8-(4-6)$

$8+2$

10

Which operation did Helen do first, and why?

The square, as it was an index operation inside brackets.

Why did she write +2 in her third line?

Sne skipped a step $8-(-2)$, which simplifies to $8+2$

Evaluate:

$$8+(3^2-12)$$

5

Morgan's work:

$5^2-5\div 2\times 4$

$5^2-5\div 2\cdot 4$

$25-5\div 2\cdot 4$

$20\div 2\cdot 4$

$10\cdot 4$

40

In the step marked with an arrow, Morgan subtracted. What operation should she have done instead of subtraction?

division

Evaluate:

$$5^2+6\div 2\times 4$$

37

John's work:

$\dfrac{-22-2^2}{5-3}$

$-\dfrac{22-2^2}{5-3}$

$-\dfrac{22-4}{5-3}$

$-\dfrac{26}{2}$

-13

Explain how John used the order of operations to do the step marked with an arrow correctly.

He did the square first.

Evaluate:

$$\frac{-22+2^2}{5-3}$$

-9

7 Evaluate the following.

a $4 + 2 \times 3^2$ 22	**b** $(4 + 2 - 3)^2$ 9	**c** $(4 + 2) \times 3^2$ 54	**d** $4 + (2 - 3)^2$ 5
e $4 + (2 \times 3)^2$ 40	**f** $4 \times (2 - 3)^2$ 4	**g** $(4 + 2 \times 3)^2$ 100	**h** $(4 \times 2 - 3)^2$ 25

8 Determine the index number for these basic numerals.

a $16 = 2^{\square}$ 4	**b** $16 = 4^{\square}$ 2	**c** $64 = 4^{\square}$ 3	**d** $64 = 2^{\square}$ 6
e $27 = 3^{\square}$ 3	**f** $100 = 10^{\square}$ 2	**g** $49 = 7^{\square}$ 2	**h** $625 = 5^{\square}$ 4

9 Write 32 as the sum of two indices.

$2^4 + 2^4$, many answers possible

10 Evaluate the expression on each card.

$(-3)^2$ $\qquad$ -3^2

What's different about these questions and answers?

$(-3)^2 = 9$,
$-3^2 = -9$

11 Find possible integers such that:

$\square^{\square} = 64 \qquad \square^{\square} = 64 \qquad \square^{\square} = 64 \qquad \square^{\square} = 64$

$2^6, 4^3, 8^2, 64^1$

12 There are 100 rabbits on Mt Burrow at the start of the year 2000. The rule for the number of rabbits N after t years (from the start of the year 2000) is $N = 100 \times 2^t$.

a Find the number of rabbits at:

i $t = 2$

400

ii $t = 6$

6400

iii The beginning of 2007

12 800

iv The beginning of 2010

102 400

b How many years will it take for the population to first rise to more than 500 000? Give a whole number of years.

13 years

13 Using guess and check, find the missing number.

a $2^{\square} = 128$ 7	**b** $3^{\square} = 27$ 3	**c** $10^{\square} = 10\,000\,000$ 7	**d** $20^{\square} = 8\,000$ 3
e $\square^3 = 64$ 4	**f** $\square^5 = 243$ 3	**g** $\square^7 = 128$ 2	**h** $\square^4 = 10\,000$ 10
i $\square^3 = 1$ 1	**j** $\square^8 = 1$ 1	**k** $2^{\square} = 256 = \square^4$ 8, 4	

12 Given that $12^5 = 248\,832$, what is the value of $(-12)^5$? Explain how you know.

−248832, since a negative number raised to an odd power is negative.

13 $x^2 = 564$. What is the value of $(-x)^2$? Explain how you know.

564, a negative number raised to an even power is positive.

14 Follow the order of operations to evaluate each expression.

a $(7 - 10)^3 + 21 \div -7$

−30

b $(-5)^2 \times 6 \div [10 \times 5 \div (-10)]$

−30

DIVISIBILITY RULES

- Use the **divisibility tests** to test whether a number is a factor of another number.

Divisible by	Test
2	The last digit is even (0, 2, 4 , 6 or 8).
3	The sum of the digits is divisible by 3.
4	The last two digits are divisible by 4.
5	The last digit is 5 or 0.
6	The number is divisible by 2 and 3.
10	The last digit is 0.

Example Apply the divisibility tests.

Number	÷2	÷3	÷4	÷5	÷6	÷10
2022	✓	✓	✗	✗	✓	✗

Explain why 2022 is:

Divisible by 2	Divisible by 3	Not divisible by 4
Not divisible by 5	Divisible by 6	Not divisible by 10

a

Number	÷2	÷3	÷4	÷5	÷6	÷10
196	✓	✗	✓	✗	✗	✗

b

Number	÷2	÷3	÷4	÷5	÷6	÷10
3000	✓	✓	✓	✓	✓	✓

c

Number	÷2	÷3	÷4	÷5	÷6	÷10
135	✗	✓	✗	✓	✗	✗

Key ideas

1. The divisibility tests are used to determine whether a number is a ………………… of another number.

1 Circle the numbers that are divisible by 2.

a 28	**b** 1003	**c** 27
d 66	**e** 59 691	**f** 185
g 1112	**h** 80 000	**i** 77 772

a, d, g, h, i

2 Circle the numbers that are divisible by 4.

a 42	**b** 50	**c** 148
d 86	**e** 532	**f** 102
g 5794	**h** 44 448	**i** 13 212
j 30 006	**k** 91 428	**l** 10 100

c,e, h, i, k, l

3 The final digit of these numbers is missing. These numbers are divisible by 5.
Write two possible final digits.

a 3… 3… 0, 5	**b** 9… 9… 0, 5	**c** 15… 15… 0, 5	**d** 21… 21… 0, 5
e 21… 21… 0, 5	**f** 100… 100… 0, 5	**g** 86… 86… 0, 5	**h** 917… 907… 0, 5

4 Circle the numbers that are divisible by 10.

a 90	**b** 643	**c** 10 101
d 3804	**e** 13 330	**f** 80800

a, e, f

5 Test these for divisibility by 2, 4, 5 and 10. Shade in the circle if it is divisible.

a 30 — 2 4 5 10 — 2, 5, 10	**b** 205 — 2 4 5 10 — 5	**c** 2 080 — 2 4 5 10 — 2, 4, 5, 10
d 60 — 2 4 5 10 — 2, 4, 5, 10	**e** 776 — 2 4 5 10 — 2, 4	**f** 32 070 — 2 4 5 10 — 2, 5, 10
g 110 — 2 4 5 10 — 2, 5, 10	**h** 490 — 2 4 5 10 — 2, 5, 10	**i** 924 — 2 4 5 10 — 2, 4

6 True or false? Every number divisible by 10 is also divisible by 2 and 5.

True

7 Test whether these numbers are divisible by 3. Sum the digits, then decide.

Number		Sum of Digits	Divisible by 3?
a	8022	12	yes
b	78	15	yes
c	93	12	yes
d	112	4	no
e	108	9	yes
f	108 00	9	yes
g	3419	17	no
h	77 322	21	yes

Number		Sum of Digits	Divisible by 3?
i	8301	12	yes
j	10 020	3	yes
k	84 355	25	no
l	99 996	42	yes
m	17 361	18	yes
n	44 004	12	yes
o	14 053	13	no
p	8931	21	yes

8 Test whether these numbers are divisible by 6.
First decide if its divisible by 2, then decide if its divisible by 3.

Number		Divisible by 2?	Sum of Digits	Divisible by 3?	Divisible by 6?
a	38	Yes	11	No	No
b	96	Yes	15	Yes	Yes
c	578	Yes	20	No	No
d	690	Yes	15	Yes	Yes
e	1809	No	18	Yes	No
f	8931	No	21	Yes	No
g	14 053	No	13	No	No
h	35 730	Yes	18	Yes	Yes

9 Which of these numbers have a factor of 2? (circle)

a 43 568 b 23 533 c 16 900 d 303 030

a, c, d

10 Which of these numbers have a factor of 3? (circle)

a 25 961 b 29 505 c 26 320 d 169 026

b, d

11 Which of these numbers have a factor of 4? (circle)

a 28 424 b 63 954 c 32 866 d 147 008

a, d

12 Which of these numbers have a factor of 5? (circle)

a 34 675 b 64 980 c 55 842 d 167 312

a, b

13 Which of these numbers have a factor of 6? (circle)

a 9690 b 14 858 c 195 738 d 39 735

a, c

14 Which of these numbers have a factor of 10? (circle)

a 42 801 b 63 580 c 29 375 d 148 600

b, d

15 Complete these divisibility tables.

	Divisible by					
Number	÷2	÷3	÷4	÷5	÷6	÷10
60	Y	Y	Y	Y	Y	Y
2 016	Y	Y	y	N	Y	N
243 567	N	Y	N	N	N	N
28 080	Y	Y	Y	Y	Y	Y
189 000	Y	Y	Y	Y	Y	Y
1 308 150	Y	Y	N	Y	Y	Y

16 Determine which of the following calculations are possible without leaving a remainder (answer Yes or No).

a	23562 ÷ 3 Y	**b**	39245678 ÷ 4 N
c	3193457 ÷ 6 N	**d**	2000340 ÷ 10 Y
e	51345678 ÷ 5 N	**f**	215364 ÷ 6 Y
g	9543 ÷ 6 N	**h**	25756 ÷ 2 Y
i	324534565 ÷ 5 Y	**j**	329541 ÷ 10 N
k	225329 ÷ 3 N	**l**	2002002002 ÷ 4 N

17 Explain why a number not divisible by 2 would also not be divisible by 4.

The number must be odd, no odd number is divisible by an even number.

18 Explain why a number divisible by 2 and 3 must also be divisible by 6.

The lowest common multiple of 2 and 3 is 6.

19 Emily says that a number divisible by 24 must also be divisible by 4 and 6.
Explain why she is correct.

4 and 6 are factors of 24.

20 Emily says a number divisible by 4 and 6 must also be divisible by 24.

a Give an example to show she is incorrect.

The lowest common multiple of 4 and 6 is not 24. 12 is divisible by 4 and 6, but not 24.

b We need to pick two numbers that are **coprime** (do not have any common factors).
If a number is divisible by …… and ……, it must be divisible by 24

3 and 8

c If we want to test divisibility by 18, we would test for …… and ……

2, 9

d If we want to test divisibility by 45, we would test for …… and ……

5, 9

e If we want to test divisibility by 60, we would test for …… and ……

5, 12
4, 15
3, 20

21 How many of the whole numbers between 1 and 250 inclusive are not divisible by 5?
Explain, in written form, how you arrived at your answer.

There are 250 ÷ 5 = 50 multiples of 5 between 1 and 250. Therefore, there are 200 not divisible by 5.

22 Explain why Julie cannot share $41.75 equally among her three children.

The sum of the digits is 17, which is not divisible by 3.

What is the smallest amount of money Julie needs to add to $41.75 to be able to share it amongst her three children?

1 cent. That would make sum of digits 18, which is divisible by 3.

23 Is the number 968 362 396 392 139 963 359 divisible by 3?

yes

Which digits in the number could you ignore, and why?
(Hint: you only need to consider five digits)

You can ignore any multiple of 3, since the sum of multiples of 3 are divisible by 3.

24 How many two-digit numbers are divisible by 2 and 3? Explain how you arrived at your answer.

How many 2 digit numbers are divisible by 6? 15

25 Find the largest three-digit number that is divisible by both 4 and 5.
Explain how you arrived at your answer.

4 and 5 have a LCM of 20. The largest multiple of 20 less than 1000 is 980.

26 Blake's age is a two-digit number. It is divisible by 2, 3, 6 and 9.
How old is Blake if you know that he is older than 20 but younger than 50?

LCM is 2, 3, 6, 9 is 18.

Blake's age must be a multiple of 18 between 20 and 50

Blake is 36

27 [UKMT IMO]:
Find the smallest positive integer which consists only of 0s and 1s and which is divisible by 12.

Divisible by 12 means divisible by 3 and 4.

Last two digits must be divisible by 4.

Digits must add to a multiple of 3.

11100

28 [UKMT JMO]:
Every digit of a given positive integer is either a 3 or a 4 with each occurring at least once. The integer is divisible by both 3 and 4. What is the smallest such integer?

The number must end in 44 to be divisible by 4.

The smallest possible integer (by systematically listing out all possibilities) is 3444

29 [UKMT JMO]:
The eight-digit number "ppppqqqq", where p and q are digits, is a multiple of 15.
What are all the possible numbers?

Multiple of 15 must be divisible by 3 and 5.

q must be 0 or 5.

Consider pppp5555, sum of digits 4p + 20 must be a multiple of 3.

Possible numbers are 11115555, 44445555, 77775555

pppp0000, sum of digits 4p must be a multiple of 3.

Possible numbers are 33330000, 66660000, 99990000

30 [UKMT JMO]:
Prove that this is always true: The sum of five consecutive integers is divisible by 15 if and only if the third number is a multiple of 3.

The sum of 5 consecutive integers is $(n-2)+(n-1)+n+(n+1)+(n+2)=5n$

For $5n$ to be divisible by 15, n must be a multiple of 3,

therefore, the third number must be a multiple of 3.

31 [UKMT IMO]:
A positive integer N is written using only the digits 2 and 3, with each appearing at least once.
If N is divisible by 2 and by 3, what is the smallest possible integer N?

Divisible by 2 means the last digit must be 2.

The digits must sum to a multiple of 3.

Smallest possible integer (by systematically listing out all possibilities) is 2232

32 [UKMT IMO]:
A palindromic number is one which reads the same when its digits are reversed, for example 23832. What is the largest six-digit palindromic number which is exactly divisible by 15?

Let the number be $abccba$

To be divisible by 5, a must be 5, since the number can't start with 0.

$5bccb5$

The sum of digits $2b + 2c + 10$ is a multiple of 3.

By systematically listing out all possibilities, starting with the largest values of b and c,

597795

33 [UKMT IMO]:
Find the possible values of the digits p and q, given that the five-digit number '$p543q$' is a multiple of 36.
Hint: a number is divisible by 9 if its digits sum to a multiple of 9.

Multiple of 36 means it is divisible by 4 and 9

Last two digits must be a multiple of 4, so 32 or 36

The digits must sum to a multiple of 9.

For p5432: $p + 14$ is a multiple of 9, so $p = 4$

For p5436, $p + 18$ is a multiple of 9, so $p = 9$

So, $p = 4, q = 2$ or $p = 9, q = 6$

34 For each of these expressions, n is an integer. Decide whether the statements are true or false. Explain your answer.

a $5n$ is divisible by 5.

True, as $5n$ is a multiple of 5.

b $4n + 1$ is divisible by 4.

False, a multiple of 4 plus 1 is not divisible by 4.

c $4(n + 1)$ is divisible by 4.

True, since n + 1 is a whole number, $4(n + 1)$ is a multiple of 4.

d $6n$ is divisible by 2.

True, any multiple of 6 is divisible by 2.

e $n(n + 1)$ is divisible by 2.

True, n and $n + 1$ are consecutive, so one of them must be even.

PRIME NUMBERS

Prime Numbers	Composite Numbers
Only 2 factors. Can only form 1 row arrays	More than 2 factors Can form arrays in many ways
2 — 1 × 2	4 — 1 × 4, 2 × 2
3 — 1 × 3	6 — 1 × 6, 2 × 3
5 — 1 × 5	8 — 1 × 8, 2 × 4
7 — 1 × 7	9 — 1 × 9, 3 × 3
11 — 1 × 11	10 — 1 × 10, 2 × 5
13 — 1 × 13	12 — 1 × 12, 2 × 6, 3 × 4
17 — 1 × 17	
19 — 1 × 19	

It is useful to memorise the first 10 prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Example Identify whether a number is prime.

23 1 × 23 23 is prime.	What does it mean for a number to be "prime" *A prime number has only two factors: …………… and ……………*
6 1 × 6 2 × 3 6 is not prime. It is composite.	What does it mean for a number is composite? *A composite number has more than ………… factors.*

DETERMINING WHETHER A NUMBER IS COMPOSITE

- A number is **composite** if it can be divided evenly by any number other than 1 and itself
- It's easier to prove a number is composite than to prove it's prime.
- 0 and 1 are neither prime nor composite.

Example Identify prime and composite numbers.

2	Prime	Composite
3	Prime	Composite
4	Prime	Composite
14	Prime	Composite
23	Prime	Composite
70	Prime	Composite
51	Prime	Composite
99	Prime	Composite
37	Prime	Composite
172 549 172	Prime	Composite
1 888 081 808 881	Prime	Composite

Check your understanding

1 Which of these numbers is composite?

A 11 **b** 13 **c** 15 **d** None of them

2 Which of these numbers is prime?

a 31 **b** 33 **c** 35 **d** 39

3 Which of these numbers is prime?

a 21 **b** 23 **c** 27 **d** None of them

PRIME FACTORISATION

- **Fundamental Theorem of Arithmetic**
  We can represent any integer greater than 1 as a unique product of prime factors.
- **Prime factorisation** breaks a number down into its prime building blocks.

Prime Factorisation: https://www.geogebra.org/m/jvggnptq

Example Write a number in prime factor form.

26 $= 2 \times 13$	What does it mean to find the "prime factorisation" of a number? *Represent a composite number as a product of …………… factors.*
8 $= 2 \times 4$ $= 2 \times 2 \times 2$ $= 2^3$	Why didn't we leave our answer as 2×4? *…… is not a prime number, so 2×4 is not the prime factorisation.* Explain how $2 \times 2 \times 2 = 2^3$ *Repeated ……………… is shown using ………………*

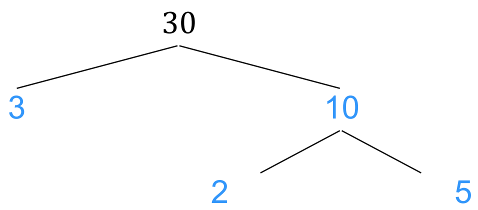
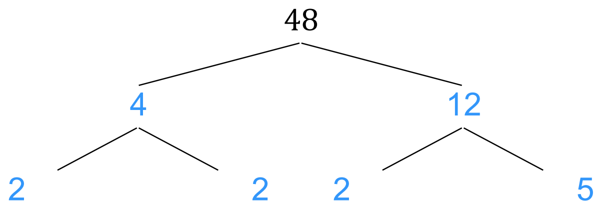
a 10 2×5	**b** 35 5×7
c 8 2^3	**d** 17 17
e 25 5^2	**f** 12 $2^2 \times 3$
g 54 2×3^2	**h** 90 $2^3 \times 3 \times 5^2$

FACTOR TREES

Factor Tree: https://www.geogebra.org/m/qg9uepm9

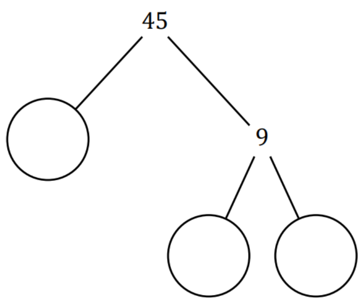
- A factor tree is useful for finding the prime factorisation of large numbers.
  1. Think of a factor pair and draw two branches with the two factors at the ends.
  2. If the factor is a prime number, then end or **terminate** the branch.
     Move on to the other factor and repeat step 1.
  3. Once you have found all the prime factors, write them as a product from smallest to largest.
  4. Use index form to represent multiple prime factors.

Example Write a number in prime factor form using a factor tree.

Find the prime factorisation of 30.  $30 = 2 \times 3 \times 5$	What does it mean when we split 30 into 3 and 10? $30 = 3 \;\ldots\ldots\; 10$ Why did we terminate the branch at 3? *3 is a ………… number.* Why did we not terminate the branch at 10? *10 is not a …………… number.*
Find the prime factorisation of 48.  $48 = 2^3 \times 5$	Do the prime factorisation of 48 another way: 48 What do we mean when we say a number's prime factorisation is ***unique***? *There is only ………… way to write a number as the product of its prime factors.*

Draw the factor tree for these numbers, then write its prime factorisation in index form.

a

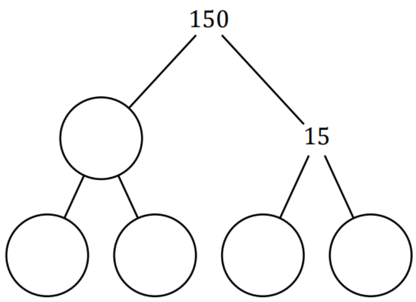


$45 = \ldots\ldots \times \ldots\ldots \times \ldots\ldots$

$= \ldots\ldots \times \ldots\ldots$

5×3^2

b



$150 = \ldots\ldots \times \ldots\ldots \times \ldots\ldots \times \ldots\ldots$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$2 \times 3 \times 5^2$

Key ideas

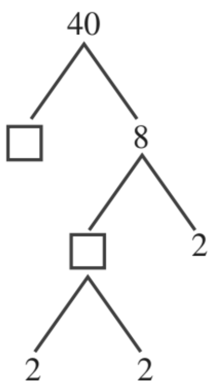
1. A factor tree splits numbers based on ……………………
2. We terminate branches of a factor tree when we reach a ……………… factor.
3. We always write the prime factorisation of a number in ……………… form.

First 10 prime numbers:

2	3	5	7	11	13	17	19	23	29

1 Complete these factor trees and write the number in prime factor form.

a

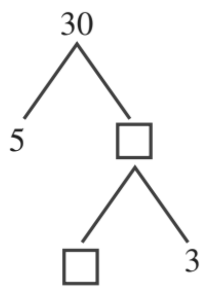


40 = $2 \times 2 \times 2 \times 5$

= ………………

$2^3 \times 5$

b

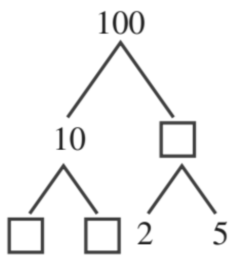


30 = ……………………….

……………………….

$2 \times 3 \times 5$

c



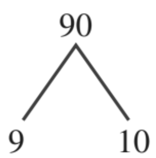
100 = ……………………

……………………

$2^2 \times 5^2$

2 Complete these factor trees and write the number as a product of its prime factors.

a

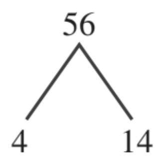


90 = ……………………….

……………………….

$2 \times 3^2 \times 5$

b

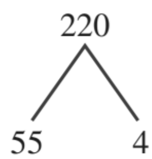


56 = ……………………….

……………………….

$2^3 \times 7$

c

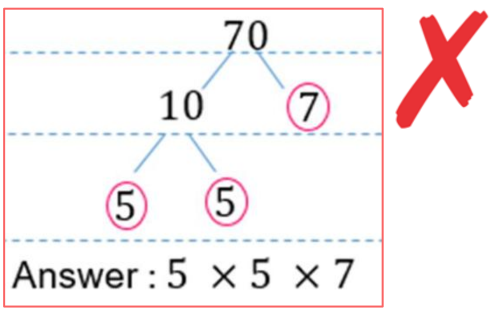


220 = ……………………

……………………

$2^2 \times 5 \times 11$

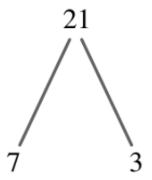
3 A student is asked to express 70 as a product of its prime factors.
Explain why they are incorrect.



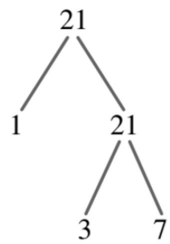
Prime factors of 10 are not 5 and 5, it is 2 and 5.

4 Choose the correct factor tree for 21 and explain why the others are incorrect.

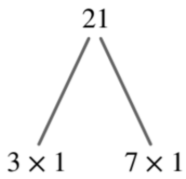
A.



B.



C.



B and C are incorrect as 1 is not a prime number.

5 Express the following numbers as products of its prime factors using index notation.
The divisibility rules will be useful.

a 72

72 = ………………………

$2^3 \times 3^2$

b 24

24 = ………………………

$2^3 \times 3$

c 38

38 = ……………………

2×19

d 44

$2^2 \times 11$

e 124

$2^2 \times 31$

f 80

$2^4 \times 5$

g 96

$2^5 \times 3$

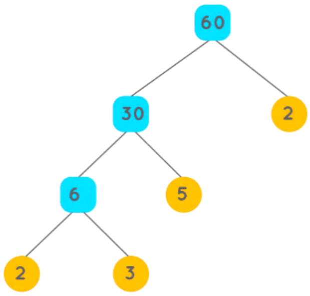
h 240

$2^4 \times 3 \times 5$

i 690

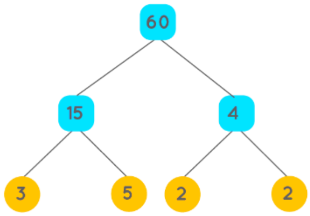
$2 \times 3 \times 5 \times 23$

6 Consider these two factor trees for 60, then complete the sentence.



$2 \times 3 \times 5 \times 2$

$= 2^2 \times 3 \times 5$



$3 \times 5 \times 2 \times 2$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

There are …………… ways to draw a factor tree, but there is only ……… way to write the final prime factorisation. We say that a number's prime factorisation is u……………

Many, one, unique

7 Draw three different factor trees for 24. Show that the prime factorisation is unique. **Note:** simply swapping the order of a pair of factors does not qualify as a different factor tree.

24	24	24

$24 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$2^3 \times 3$

8 Circle the correct prime factorisation for 424:

A $2^2 \times 3^2 \times 5$ **B** $2 \times 3^2 \times 5^2$

C 53×8 **D** $2^3 \times 53$

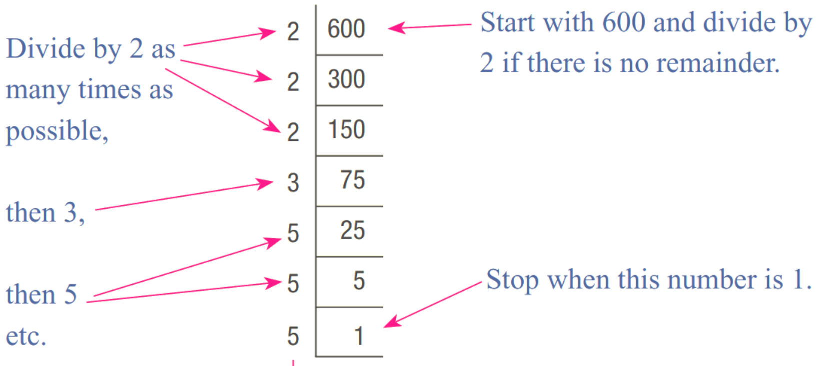
Explain why you can eliminate options A and B immediately.

424 is not divisible by 5.

Explain why C is not the correct answer:

9 is not prime.

10 Instead of a factor tree, we can use repeated division by prime numbers to find the factors:



$600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$

$600 = 2^3 \times 3 \times 5^2$

a

$$\begin{array}{r|l} 2 & 12 \\ 2 & \square \\ \square & 3 \\ & 1 \end{array}$$

$2^2 \times 3$

b

$$\begin{array}{r|l} \square & 30 \\ \square & 15 \\ \square & 5 \\ & 1 \end{array}$$

$2 \times 3 \times 5$

c

$$\begin{array}{r|l} \square & 66 \\ 3 & 33 \\ \square & \square \\ & \square \end{array}$$

$2 \times 3 \times 11$

d 32

2^5

e 40

$2^3 \times 5$

f 81

3^4

g 144

$2^4 \times 3^2$

h 500

$2^2 \times 5^3$

i 1250

2×5^4

11 Match the composite number to its prime factorisation.

a	120	**A**	$2 \times 3 \times 5^2$
b	150	**B**	$2^2 \times 3^2 \times 5$
c	144	**C**	$2^4 \times 3^2$
d	180	**D**	$2 \times 3 \times 2 \times 5 \times 2$

a – D, b – A, c – C, d- B

12 Answer True or False to the following:

a 2 is a factor of $2 \times 3 \times 7 \times 13$ True | False

T

b 3 is a factor of $2 \times 3 \times 7 \times 13$ True | False

T

c 5 is a factor of $2 \times 3 \times 7 \times 13$ True | False

F

d 7 is a factor of $2 \times 3 \times 7 \times 13$ True | False

T

e 4 is a factor of $2 \times 3 \times 7 \times 13$ True | False

F

f 6 is a factor of $2 \times 3 \times 7 \times 13$ True | False

T

g 14 is a factor of $2 \times 3 \times 7 \times 13$ True | False

T

h 21 is a factor of $2 \times 3 \times 7 \times 13$ True | False

T

i 15 is a factor of $2 \times 3 \times 7 \times 13$ True | False

F

j 15 is a factor of $2 \times 3 \times 5 \times 7 \times 13$ True | False

T

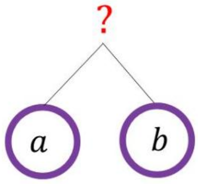
k 30 is a factor of $2 \times 3 \times 5 \times 7 \times 13$ True | False

T

13 a and b are different numbers. The factor tree is completed.

Which of these numbers could the tree be factorising? (circle one)

a 45 | **b** 47 | **c** 49 | **d** 51



D

Explain your answer.

51 is the only number that can be written as 2 different prime factors (3 and 17).

14 Answer these questions using the given facts.

a $6 = 2 \times 3$

What is 30 as the product of its prime factors?

$2 \times 3 \times 5$

b $450 = 2 \times 3^2 \times 5^2$

What is 900 as its product of prime factors?

$2^2 \times 3^2 \times 5^2$

c $12 = 2^2 \times 3$

What is 36 as its product of prime factors?

$2^2 \times 3^2$

15 Answer these questions using the given facts.

a If $21 = 3 \times 7$

What is 84 as the product of its prime factors?

$2^2 \times 3 \times 7$

b if $1575 = 3^2 \times 5^2 \times 7$

What is 15750 as product of prime factors?

$2 \times 3^2 \times 5^3 \times 7$

c if $539 = 7^2 \times 11$

What is 1617 as the product of prime factors?

$3 \times 7^2 \times 11$

16 Fill in the gaps in the prime factorisation, given the information about the factors of the number.

a $2^3 \times 3 \times \ldots\ldots\ldots \times \ldots\ldots\ldots$ has factors of 14 and 15

$5, 7$

b $2^2 \times 3 \times \ldots\ldots\ldots \times \ldots\ldots\ldots$ has factors of 100 and 130

$5^2, 13$

c $2^2 \times 5 \times \ldots\ldots\ldots \times \ldots\ldots\ldots$ has factors of 12, 20, 27, and 28

$3^3, 7$

17 It is possible to factorise a number already in index form:

$$\begin{aligned} 10^3 &= (2 \times 5)^3 \\ &= (2 \times 5) \times (2 \times 5) \times (2 \times 5) \\ &= 2^3 \times 5^3 \end{aligned}$$

Factorise the following:

a 6^5

$2^5 \times 3^5$

b 21^{100}

$3^{100} \times 7^{100}$

c 15^{15}

$3^{15} \times 5^{15}$

18 We will use prime factors to investigate which fractions become recurring decimals.

Reminder: $\frac{1}{2} = 0.5$ (terminating decimal) $\frac{1}{3} = 0.\dot{3}$ (recurring decimal)

a Use a calculator to convert these fractions to decimals, then list the prime factors of each denominator.

i $\frac{3}{20}$

Decimal:

0.15

Prime factors:

2, 5

ii $\frac{79}{80}$

Decimal:

0.9875

Prime factors:

2, 5

iii $\frac{17}{32}$

Decimal:

0.53125

Prime factors:

2

iv $\frac{18}{25}$

Decimal:

0.72

Prime factors:

5

b Use a calculator to convert these fractions to decimals, then list the prime factors of each denominator.

i $\frac{7}{12}$

Decimal:

$0.58\dot{3}$

Prime factors:

2,3

ii $\frac{19}{30}$

Decimal:

$0.6\dot{3}$

Prime factors:

2, 3, 5

iii $\frac{11}{14}$

Decimal:

$0.7\overline{857142}$

Prime factors:

2, 7

iv $\frac{15}{22}$

Decimal:

$0.6\overline{81}$

Prime factors:

2, 11

c What is the link between the denominator's prime factors and whether the fraction will terminate or repeat? (Hint: Decimals are base 10.)

If the prime factors of a denominator are 2 and/or 5, the decimal will terminate, otherwise it will recur.

d Paul says:

"$\frac{15}{30}$ is a recurring decimal,
since the denominator's prime factors are not 2 and 5."

Explain why Paul is not correct.

In fully simplified form, the denominator is 2.

e Would $\frac{16}{30}$ be a recurring decimal? Why?

Recurring, fully simplified the fraction is $\frac{8}{15}$, the denominator has prime factors 3 and 5.

CHECK YOUR UNDERSTANDING

Index notation

1	What is 6^2?							
	a	66	**b**	8	**c**	12	**d**	36
2	What is 2^5?							
	a	32	**b**	64	**c**	10	**d**	16
3	What is 10^2?							
	a	102	**b**	20	**c**	100	**d**	12
4	What is 2^3?							
	a	5	**b**	8	**c**	9	**d**	6
5	Which of these is a power of 5?							
	a	55	**b**	10	**c**	50	**d**	25
6	What is $(-4)^2$?							
	a	-4×-4	**b**	-4×2	**c**	-4×4	**d**	-4^2

Evaluating expressions with indices

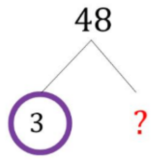
1	$2 \times 3^2 + 4$							
	a	40	**b**	16	**c**	26	**d**	22
2	$1 + 2 \times 3^2$							
	a	37	**b**	19	**c**	81	**d**	37
3	$1 + 3 \times 2^2$							
	a	13	**b**	64	**c**	37	**d**	16
4	$2 + 3^2 \times 4 - 2$							
	a	98	**b**	36	**c**	42	**d**	24
5	$(1 + 2 \times 3)^2$							
	a	45	**b**	49	**c**	81	**d**	37
6	$(5 - 3 \times 2)^2$							
	a	-7	**b**	16	**c**	-1	**d**	1
7	$(15 - 2^2)^2 + 4$							
	a	125	**b**	3	**c**	213	**d**	28 565

Prime factorisation

1 What is 40 as a product of prime factors?

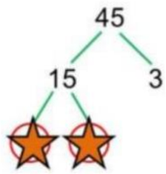
a 4×10 **b** $2^2 \times 5^2$ **c** $2^3 \times 5$ **d** $1^2 \times 2 \times 5$

2 This the first row of a factor tree.
What is the missing number?



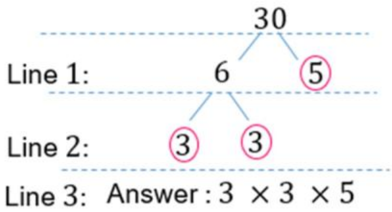
a 45 **b** 144 **c** 24 **d** 16

3 What are the missing numbers?



a 3 and 5 **b** 10 and 5 **c** 7.5 and 7.5 **d** 7.5 and 2

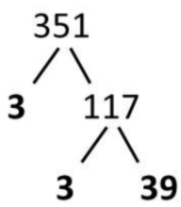
4 On which line of working out does the first error occur?



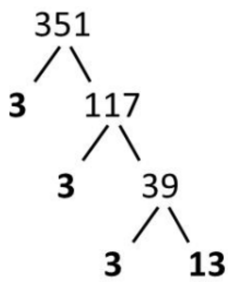
a There's no error **b** Line 1 **c** Line 2 **d** Line 3

5 Which is the correct factor tree for 351?

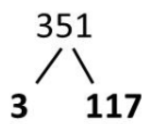
a



b



c



d