

Student Name

Mathematics Stage 5 Core

INDICES A

Book 3 Apply index las to numerical expressions
with negative-integer indices

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CONTENTS

Overview2

The Negative Index3

Writing Fractions with a Negative Index10

Negative Indices in the Denominator12

Check Your Understanding15

OVERVIEW

SYLLABUS CONTENT

MA5-IND-C-01 simplifies algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases

Apply index laws to numerical expressions with negative-integer indices

- Apply index notation, patterns and index laws to establish the meaning of negative indices for numerical bases
- Evaluate numerical expressions involving a negative index by first representing them with a positive index
- Represent given numbers in index form (integer indices and bases only) and vice versa

REVIEW OF PRIOR KNOWLEDGE

1 Find the reciprocal of these numbers.

a 5

b 3

c 113

d $\frac{1}{3}$

e $\frac{1}{5}$

f $\frac{1}{7}$

g $\frac{1}{2753}$

h $\frac{5}{2}$

THE NEGATIVE INDEX

Investigation Establishing the negative index.

Using a calculator, evaluate the following.

$2^{-1} =$

$2^{-2} =$

$3^{-1} =$

$3^{-2} =$

$4^{-1} =$

$4^{-2} =$

$5^{-1} =$

$5^{-2} =$

$6^{-1} =$

$6^{-2} =$

$10^{-1} =$

$10^{-2} =$

$18^{-1} =$

$28^{-2} =$

$40^{-1} =$

$40^{-2} =$

English

expression:

Algebraic

expression:

Explain why this is true using the expressions $x^2 \div x^3$ and $x^2 \div x^4$

$x^2 \div x^3 = \dots\dots\dots \text{ (index form)}$

$x^2 \div x^4 = \dots\dots\dots \text{ (index form)}$

$= \dots\dots\dots \text{ (fraction)}$

$= \dots\dots\dots \text{ (fraction)}$

$= \dots\dots\dots \text{ (simplified fraction)}$

$= \dots\dots\dots \text{ (simplified fraction)}$

NEGATIVE INDICES

- A **negative index** means to take the reciprocal of the base.
- We can rewrite expressions with negative indices with positive indices.

Index Law of Negative Powers

$$a^{-n} = \frac{1}{a^n}$$

Example Rewrite numbers using positive indices.

6^{-1} $= \frac{1}{6}$	What does a power of -1 mean? <i>Take the of the base.</i>
5^{-9} $= \frac{1}{5^9}$	Why is $5^{-9} = \frac{1}{5^9}$ (use the index law of powers) $5^{-9} = (5^{\dots})^{-1} = \dots$
2×5^{-9} $= 2 \times \frac{1}{5^9}$ $= \frac{2}{5^9}$	Why is the 2 is not in the denominator? <i>The 2 was not raised to a power, so we do not take the of 2.</i>

Express the following with positive indices.

a	3^{-1}	b	5^{-1}	c	$(-7)^{-1}$
	$\frac{1}{3}$		$\frac{1}{5}$		$-\frac{1}{7}$
d	3^{-2}	e	4^{-3}	f	2^{-5}
	$\frac{1}{3^2}$		$\frac{1}{4^3}$		$\frac{1}{2^5}$
g	2×8^{-1}	h	3×5^{-2}	i	5×3^{-2}
	$\frac{2}{8}$		$\frac{3}{5^2}$		$\frac{5}{3^2}$

Key ideas

1. A negative index means to take the of the base.

1 Complete these tables.

a

Expression	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Number	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8} = \frac{1}{2^3}$	$\frac{1}{16}$

b

Expression	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}
Number	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$

c

Expression	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}	5^{-4}
Number	625	125	25	5	1	$\frac{1}{5}$	$\frac{1}{25}$	$\frac{1}{125}$	$\frac{1}{625}$

d

Expression	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Number	10000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$

2 Match each card on the top row with a card from the bottom row.

2^{-1}

3^{-1}

4^{-1}

5^{-1}

8^{-1}

10^{-1}

$\frac{1}{3}$

$\frac{1}{5}$

0.5

0.1

$\frac{1}{4}$

$\frac{1}{8}$

$2^{-1} \quad 0.5$
 $3^{-1} \quad \frac{1}{3}$
 $4^{-1} \quad \frac{1}{4}$
 $5^{-1} \quad \frac{1}{5}$
 $8^{-1} \quad \frac{1}{8}$
 $10^{-1} \quad 0.1$

3 Complete the table.

Power	Fraction
2^{-1}	$\frac{1}{2}$
2^{-2}	$\frac{1}{2^2} = \frac{1}{4}$
2^{-3}	
2^{-4}	
2^{-5}	

$\frac{1}{8}$
 $\frac{1}{16}$
 $\frac{1}{32}$

4 Complete the following:

a Simplify using index laws: $2^2 \div 2^5 = 2^{\square} - \square = 2^{\square}$

2^{-3}

b Write in expanded form: $2^2 \div 2^5 = \frac{2 \times 2}{\square} = \frac{1}{2^{\square}}$

$\frac{1}{2^3}$

c Therefore, $2^{-3} = \frac{1}{2^{\square}}$

$2^{-3} = \frac{1}{2^3}$

5 Complete the following:

a $10^{-4} = \frac{1}{10^{\square}}$

4

b $3^{-2} = \frac{1}{3^{\square}}$

2

c $7^{-3} = \frac{1}{7^{\square}}$

3

d $8^{-6} = \frac{1}{8^{\square}}$

6

6 Read the example carefully and answer the questions.

Madeline's work

Write as an expression without indices:

4^{-2}
 4^{-2}
 -16

How does a negative exponent affect its base?

Negative index means to take the reciprocal of the base

What is the correct answer to this problem?

$\frac{1}{16}$

Write as an expression without indices:

5^{-3}

$\frac{1}{125}$

7 Rewrite the following as fractions with positive indices.

a 3^{-2}
 $= \frac{1}{3^2}$

b 4^{-3}

$\frac{1}{4^3}$

c 2^{-5}

$\frac{1}{2^5}$

d 8^{-2}

$\frac{1}{8^2}$

e 12^{-1}

f 9^{-2}

g 6^{-1}

h 7^{-3}

$\frac{1}{12}$

$\frac{1}{9^2}$

$\frac{1}{6}$

$\frac{1}{7^3}$

i 3^{-6}

j 2^{-8}

k 5^{-1}

l 10^{-5}

$\frac{1}{3^6}$

$\frac{1}{2^8}$

$\frac{1}{5}$

$\frac{1}{10^5}$

8 Complete the following to write these expressions as simplified fractions.

a $2^{-3} = \frac{1}{2^{\square}} = \frac{1}{\square}$ $\frac{1}{2^3} = \frac{1}{8}$	b $5^{-1} = \frac{1}{5^{\square}} = \frac{1}{\square}$ $\frac{1}{5^1} = \frac{1}{5}$	c $2^{-4} = \frac{1}{2^{\square}} = \frac{1}{\square}$ $\frac{1}{2^4} = \frac{1}{16}$
d $3^{-2} = \frac{1}{3^2} = \frac{1}{\square}$ $\frac{1}{9}$	e $10^{-2} = \frac{1}{10^2} = \frac{1}{\square}$ $\frac{1}{100}$	f $6^{-2} = \frac{1}{\square^{\square}} = \frac{1}{\square}$ $\frac{1}{6^2} = \frac{1}{36}$

9 Write the following as simplified fractions without powers.

a $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$	b 2^{-5} $\frac{1}{32}$	c 4^{-3} $\frac{1}{64}$	d 5^{-4} $\frac{1}{625}$
e 6^{-3} $\frac{1}{216}$	f 9^{-2} $\frac{1}{81}$	g 3^{-4} $\frac{1}{81}$	h 10^{-4} $\frac{1}{10000}$
i 2^{-6} $\frac{1}{64}$	j 5^{-3} $\frac{1}{125}$	k 6^{-2} $\frac{1}{36}$	l 10^{-3} $\frac{1}{1000}$
m $(-2)^{-1}$ $-\frac{1}{2}$	n $(-2)^{-2}$ $\frac{1}{4}$	o $-(-2)^{-2}$ $-\frac{1}{4}$	p $(-2)^{-3}$ $-\frac{1}{8}$

10 Explain why these two expressions do not have the same value.

$$(-5)^{-2}$$

$$-5^{-2}$$

One has the negative raised to the power and the other doesn't.

$$(-5)^{-2} = \frac{1}{(-5)^2} = \frac{1}{25}$$

$$-5^{-2} = -\frac{1}{5^2} = -\frac{1}{25}$$

11 Write the following as decimals.

a 2^{-3} 0.125	b 4^{-1} 0.25	c 10^{-2} 0.01
d 25^{-1} 0.04	e 5^{-2} 0.04	f $(-10)^{-2}$ 0.01

12 Write the following as simplified fractions.

a 3×2^{-3} $= 3 \times \frac{1}{8}$ $= \dots\dots\dots$	b 5×4^{-3}	c 7×5^{-2}	d 2×3^{-4}
	$\frac{5}{64}$	$\frac{7}{25}$	$\frac{2}{81}$
e 4×3^{-5}	f 9×5^{-2}	g 8×7^{-3}	h 6×5^{-2}
$\frac{4}{243}$	$\frac{9}{25}$	$\frac{8}{243}$	$\frac{6}{25}$

13 Evaluate these expressions.

a 5^{-2} $= \frac{1}{5^2}$ $= \dots\dots$	b 3^{-3}	c 2^{-5}
	$\frac{1}{27}$	$\frac{1}{32}$
d 2^{-4}	e 6^{-2}	f 10^{-3}
$\frac{1}{16}$	$\frac{1}{36}$	$\frac{1}{1000}$
g 5^{-3}	h 2^{-3}	i 3^{-5}
$\frac{1}{125}$	$\frac{1}{8}$	$\frac{1}{243}$
j 4^{-4}	k 10^{-3}	l 10^{-5}
$\frac{1}{256}$	$\frac{1}{1000}$	$\frac{1}{100000}$
m 5×10^{-4}	n 1.5×10^{-3}	o 3.42×10^{-6}
$\frac{5}{10000} = \frac{1}{2000}$	$\frac{1.5}{1000} = \frac{3}{2000}$	$\frac{342}{1\,000\,000} = \frac{171}{500\,000}$

14 Write each pair of numbers as powers of the given base number.

a 8 and $\frac{1}{8}$ as powers of 2

$8^3, 8^{-3}$

b 16 and $\frac{1}{16}$ as powers of 4

$4^2, 4^{-2}$

c 16 and $\frac{1}{16}$ as powers of 2

$2^4, 2^{-4}$

d 25 and $\frac{1}{25}$ as powers of 5

$5^2, 5^{-2}$

e 125 and $\frac{1}{125}$ as powers of 5

$5^3, 5^{-3}$

f 625 and $\frac{1}{625}$ as powers of 5

$5^4, 5^{-4}$

g 100 and $\frac{1}{100}$ as powers of 10

$10^2, 10^{-2}$

h 1 000 000 and $\frac{1}{1\,000\,000}$ as powers of 10

$10^6, 10^{-6}$

15 Mr Ding asks his class to write down the value of 2^{-3} . Here are some of his students' answers.

A -8

B $\frac{1}{6}$

C 0.125

D $\frac{1}{8}$

a Which of the answers are correct?

C and D

b What mistakes have the other students made?

A: a negative index doesn't mean to make a positive number negative. B: They multiplied the power and the index

16

a If $2^{10} = 1024$, then 2^{-10} is equal to

$\frac{1}{1024}$

b If $3^{-5} = \frac{1}{243}$, then 3^5 is equal to

243

17 First change each mixed numeral or decimal to a fraction, then simplify the expression.

a $\left(1\frac{1}{2}\right)^{-1}$

b $\left(2\frac{2}{3}\right)^{-1}$

c $\left(2\frac{1}{2}\right)^{-2}$

d $\left(3\frac{1}{3}\right)^{-3}$

$\frac{4}{9}$

$\frac{3}{8}$

$\frac{4}{25}$

$\frac{27}{64}$

e 0.2^{-1}

f 2.25^{-1}

g 2.5^{-2}

h 0.05^{-2}

5

$\frac{4}{9}$

$\frac{4}{25}$

400

WRITING FRACTIONS WITH A NEGATIVE INDEX

Index Law of Negative Powers	
$\frac{1}{a^n} = a^{-n}$	$\frac{b}{a^n} = ba^{-n}$

Example Rewrite numbers with negative indices.

$\frac{1}{3}$ $= 3^{-1}$	A fraction can be rewritten with a negative index by moving the denominator to the numerator and making the index of the denominator
$\frac{1}{3^2}$ $= (3^2)^{-1}$ $= 3^{-2}$	Why is $(3^2)^{-1} = 3^{-2}$? The index law of of a says to multiply the powers.
$\frac{5}{3^2}$ $= 5 \times 3^{-2}$	Why is 5 not raised to a negative power? $\frac{5}{3^2}$ is the same as $5 \times \frac{1}{3^2}$, the 5 is not raised to a power

Express the following with negative indices.

a $\frac{1}{2}$	b $\frac{1}{52}$	c $-\frac{1}{8}$
2^{-1}	52^{-1}	8^{-1}
d $\frac{1}{4^2}$	e $\frac{1}{5^2}$	f $\frac{1}{7^8}$
4^{-2}	5^{-2}	7^{-8}
g $\frac{3}{5}$	h $\frac{2}{7^3}$	i $\frac{4}{10^2}$
3×5^{-1}	2×7^{-3}	4×10^{-2}

Key ideas

1. We can rewrite fractions as non-fractional bases with a power.
2. This involves moving the denominator to the numerator and making the index

1 Find the missing numbers.

a $10^{-4} = \frac{1}{10^{\square}}$

4

b $3^{-2} = \frac{1}{3^{\square}}$

2

c $7^{-3} = \frac{1}{7^{\square}}$

3

d $8^{-6} = \frac{1}{8^{\square}}$

6

e $9^{-4} = \frac{1}{9^{\square}}$

4

f $5^{-4} = \frac{1}{5^{\square}}$

4

2 Rewrite these fractions as expressions with negative indices.

a $\frac{1}{2}$

2^{-1}

b $\frac{1}{2^2}$

2^{-2}

c $\frac{1}{2^8}$

2^{-8}

d $\frac{1}{2^5}$

2^{-5}

e $\frac{1}{5}$

5^{-1}

f $\frac{1}{4^3}$

4^{-3}

g $\frac{1}{3^4}$

3^{-4}

h $\frac{1}{5^6}$

5^{-6}

i $\frac{1}{6}$

6^{-1}

j $\frac{1}{7^5}$

7^{-5}

k $\frac{1}{4^9}$

4^{-9}

l $\frac{1}{10^2}$

10^{-2}

3 Complete the following:

a $\frac{1}{8} = \frac{1}{2^{\square}} = 2^{\square}$

2^{-3}

b $\frac{1}{9} = \frac{1}{3^{\square}} = 3^{\square}$

3^{-2}

c $\frac{1}{16} = \frac{1}{2^{\square}} = 2^{\square}$

2^{-4}

d $\frac{1}{25} = \frac{1}{5^{\square}} = 5^{\square}$

5^{-2}

4 Write these fractions using the smallest possible index. For example, $\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

a $\frac{1}{4}$

2^{-2}

b $\frac{1}{9}$

3^{-2}

c $\frac{1}{125}$

5^{-3}

d $\frac{1}{27}$

3^{-3}

NEGATIVE INDICES IN THE DENOMINATOR

Index Law of Negative Powers	
$\frac{1}{a^{-n}} = a^n$	$\frac{b}{a^{-n}} = ba^n$

Example Rewrite numbers with positive indices.

$\frac{1}{5^{-3}}$ $= 5^3$	Show this using fraction division. $\frac{1}{5^{-3}} = \frac{1}{\left(\frac{1}{5^3}\right)} =$
$\frac{2}{3^{-4}}$ $= 2 \times 3^4$	Why does 2 not go to the denominator? $\frac{2}{3^{-2}}$ is the same as $2 \times \frac{1}{3^{-2}}$, the 2 is not raised to a power.

Express the following with positive indices.

a $\frac{1}{2^{-7}}$ 2^7	b $\frac{1}{6^{-2}}$ 6^2	c $\frac{1}{4^{-5}}$ 4^5
d $\frac{4}{3^{-2}}$ 4×3^2	e $\frac{4}{5^{-3}}$ 4×5^3	f $\frac{3}{4^{-5}}$ 3×4^5

Key ideas

- When there are negative indices in the denominator, we can rewrite the expression using a power by moving it to the

1 Complete the following:

a $\frac{1}{3^{-4}} = 3^{\square}$

4

b $\frac{1}{7^{-6}} = 7^{\square}$

6

c $\frac{1}{8^{-3}} = 8^{\square}$

3

d $\frac{1}{4^{-6}} = 4^{\square}$

6

2 Write the following with positive indices.

a $\frac{1}{3^{-4}}$

3^4

b $\frac{1}{2^{-7}}$

2^7

c $\frac{1}{7^{-2}}$

7^2

d $\frac{1}{6^{-1}}$

6

e $\frac{1}{4^{-5}}$

4^5

f $\frac{3}{4^{-5}}$

3×4^5

g $\frac{3}{7^{-2}}$

3×7^2

h $\frac{4}{6^{-1}}$

4×6

3 Rewrite the following with positive indices.

a $\frac{6}{4^{-3}}$

6×4^3

b $\frac{5}{8^{-2}}$

5×8^2

c $\frac{4}{7^{-5}}$

4×7^5

d $\frac{3}{2^{-5}}$

3×2^5

e $\frac{12}{5^{-4}}$

12×5^4

f $\frac{6}{10^{-3}}$

6×10^3

g $\frac{7}{9^{-4}}$

7×9^4

h $\frac{7}{10^{-6}}$

7×10^6

4 Evaluate the following without using a calculator. Write your answer without indices.

a 5^{-1}

$\frac{1}{5}$

b 3^{-2}

$\frac{1}{9}$

c $(-4)^{-2}$

$\frac{1}{16}$

d 4×10^{-2}

e -5^{-2}

f -5×10^{-3}

$\frac{1}{25}$

$-\frac{1}{25}$

$-\frac{1}{200}$

g $6^4 \times 6^{-6}$

h $8^{-7} \times (8^2)^3$

i $(5^2)^{-1} \times (2^{-2})^{-1}$

$\frac{1}{6^2}$

$\frac{1}{8}$

$\frac{4}{25}$

j $\frac{1}{8^{-1}}$

8

k $\frac{1}{10^{-2}}$

100

l $\frac{-2}{5^{-3}}$

-250

m $\frac{2}{2^{-3}}$

16

n $\frac{-5}{2^{-1}}$

-10

o $\frac{2^3}{2^{-3}}$

64

p $\left(\frac{3}{8}\right)^{-2}$

$\frac{64}{9}$

q $\left(\frac{-5}{2}\right)^2$

$\frac{25}{4}$

r $\frac{(3^{-2})^3}{3^{-5}}$

$\frac{1}{3}$

5 Complete each collection of equivalent expressions.

a *Example*

5^{-2}	$\frac{1}{25}$	$\frac{1}{5} \times \frac{1}{5}$
$1 \div 5 \div 5$	25^{-1}	$\frac{1}{5^2}$

b

2^{-3}		

Clockwise from top:

$\frac{1}{8}$, $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, $\frac{1}{2^3}$, 8^{-1} , $1 \div 2 \div 2 \div 2$

c

$1 \div 3 \div 3 \div 3 \div 3$		

$\frac{1}{81}$, $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$, $\frac{1}{3^4}$, 81^{-1} , 3^{-4}

d

	$\frac{3}{16}$	

$3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, $\frac{3}{2^4}$, 3×2^{-4} , $3 \div 2 \div 2 \div 2 \div 2$, 3×2^{-4}

6 **NAPLAN A**

Select the expression that is equivalent to $(4^{-3} \div 4^6)^3$

- A. 4^{-27}
- B. 4^{-9}
- C. 4^{-54}
- D. 4^{-6}

A

CHECK YOUR UNDERSTANDING

Negative indices

1	What is 3^{-1}						
a	-3	b	2	c	$\frac{1}{3}$	d	$-\frac{1}{3}$
2	What is 10^{-1}						
a	$\frac{1}{10}$	b	10	c	-10	d	9
3	What is 3^{-2}						
a	$\frac{1}{6}$	b	-9	c	$\frac{1}{9}$	d	1
4	What is 4^{-2}						
a	$\frac{1}{16}$	b	-16	c	$\frac{1}{8}$	d	2
5	What is 10^{-3}						
a	7	b	1000	c	-30	d	$\frac{1}{1000}$
6	Which of these is a power of 5?						
a	55	b	100	c	$\frac{1}{500}$	d	$\frac{1}{25}$