Student Name

Mathematics Stage 5 Paths

INDICES B

Book 1 Apply index laws to algebraic expressions involving negative-integer indices

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-IND-P-01 applies the index laws to operate with algebraic expressions involving negative-integer indices

Apply index laws to algebraic expressions involving negative-integer indices

- Apply index notation, patterns and index laws to establish $a^{-1} = \frac{1}{a}$, $a^{-2} = \frac{1}{a^2}$, $a^{-3} = \frac{1}{a^3}$ and $a^{-n} = \frac{1}{a^n}$
- Represent expressions involving negative-integer indices as expressions involving positive-integer indices and vice versa
- Apply the index laws to simplify algebraic products and quotients involving negative-integer indices
- Describe and use x^{-1} as the reciprocal of x and generalise this relationship to expressions of the form $\left(\frac{a}{b}\right)^{-1}$
- Use knowledge of the reciprocal to simplify expressions of the form $\left(\frac{a}{b}\right)^{-n}$

REVIEW OF PRIOR KNOWLEDGE

1 Rewrite the following with a positive index.

a	3^{-2} $= \frac{1}{3^2} = \frac{1}{9}$	**b**	2^{-5}	**c**	4^{-3}	**d**	5^{-4}
e	6^{-3}	**f**	9^{-2}	**g**	3^{-4}	**h**	10^{-4}
i	2^{-6}	**j**	5^{-3}	**k**	6^{-2}	**l**	10^{-3}
m	0.5^{-3}	**n**	$(-2)^{-1}$	**o**	$(-2)^{-2}$	**p**	$(-2)^{-3}$

2 Rewrite the following with a positive index.

a	$\frac{1}{3^{-4}}$	**b**	$\frac{1}{2^{-7}}$	**c**	$\frac{1}{7^{-2}}$	**d**	$\frac{1}{6^{-1}}$
e	$\frac{1}{4^{-5}}$	**f**	$\frac{2}{2^{-7}}$	**g**	$\frac{3}{7^{-2}}$	**h**	$\frac{4}{6^{-1}}$
i	$\frac{6}{4^{-3}}$	**j**	$\frac{5}{8^{-2}}$	**k**	$\frac{4}{7^{-5}}$	**l**	$\frac{3}{2^{-5}}$

NEGATIVE INDEX WITH VARIABLES

- The negative index means to take the reciprocal of the base.

Index Law of Negative Powers	
$a^{-n} = \frac{1}{a^n}$	$ba^{-n} = \frac{b}{a^n}$

- For negative powers:

1. Move the base to the denominator.
2. Power becomes positive.

Example Rewrite expressions involving negative indices with positive indices.

x^{-2} $= \frac{1}{x^2}$	Why is $x^{-2} = \frac{1}{x^2}$ (use the index law of powers) $x^{-2} = (x^{\cdots})^{-1} = \ldots\ldots\ldots\ldots$
$3a^{-2}$ $= \frac{3}{a^2}$	Why is 3 not in the denominator? *3 is not raised to a …….. power,* *so we do not take the reciprocal.*

Express the following with positive indices.

a x^{-1} $\frac{1}{x}$	**b** a^{-4} $\frac{1}{a^4}$	**c** c^{-2} $\frac{1}{c^2}$
d $3m^{-5}$ $\frac{3}{m^5}$	**e** p^7q^{-2} $\frac{p^7}{q^2}$	**f** $x^{-4}y^{-3}$ $\frac{1}{x^4y^3}$

Key ideas

1. A negative index means to take the …… of the base.
2. If a base has a negative index, to make the index positive move it to the ……

1 Express each of the following with positive indices and no brackets.

a	x^{-1}	**b**	a^{-4}	**c**	b^{-6}
	$\frac{1}{x}$		$\frac{1}{a^4}$		$\frac{1}{b^6}$
d	x^{-2}	**e**	$2x^{-2}$	**f**	$(2x)^{-2}$
	$\frac{1}{x^2}$		$\frac{2}{x^2}$		$\frac{1}{4x^2}$
g	$3m^{-4}$	**h**	$(3m)^{-4}$	**i**	mn^{-4}
	$\frac{3}{m^4}$		$\frac{1}{81m^4}$		$\frac{m}{n^4}$
j	p^7q^{-2}	**k**	$(pq)^{-2}$	**l**	$x(yz)^{-3}$
	$\frac{p^7}{q^2}$		$\frac{1}{p^2q^2}$		$\frac{x}{y^3z^3}$
m	x^4y^{-4}	**n**	$2a^{-3}b^{-1}$	**o**	$7r^{-2}s^3$
	$\frac{x^4}{y^4}$		$\frac{2}{a^3b}$		$\frac{7s^3}{r^2}$

2 Simplify these expressions.

a	$y^{13} \times y^4$	**i**	$2x^7 \times 5x^4$
	y^{17}		$10x^{11}$
b	$6y^{13} \times 5y^4$	**j**	$12y^5 \times 5x^4$
	$30y^{17}$		$60x^4y^5$
c	$y^{13} \div y^4$	**k**	$12y^5 \div 6y^4$
	y^9		$2y$
d	$40y^{13} \div 8y^4$	**l**	$12y^5 \div 12y^{-4}$
	$5y^9$		y^9
e	$(y^{13})^4$	**m**	$(12y^5)^2$
	y^{52}		$144y^{10}$
f	$(3y^{13})^4$	**n**	$(12y^{-3})^2$
	$81y^{52}$		$\frac{144}{y^6}$
g	$7y^4 \div y^2$	**o**	$12y^{-3} \div 4y^2$
	$7y^2$		$\frac{3}{y^5}$
h	$7y^4 \times y^2$	**p**	$12y^{-3} \div 4y^{-2}$
	$7y^6$		$\frac{3}{y}$
i	$(7y^4)^2$	**q**	$12y^{-3} \times 4y^{-2}$
	$49y^8$		$\frac{48}{y^5}$

3 Write these expressions without negative indices.

a 2^{-1} $\frac{1}{2}$

b 3^{-1} $\frac{1}{3}$

c 3^{-2} $\frac{1}{9}$

d 3^{-3} $\frac{1}{27}$

e 10^{-3} $\frac{1}{1000}$

f x^{-3} $\frac{1}{x^3}$

g $-x^{-3}$ $-\frac{1}{x^3}$

h $-x^{-4}$ $-\frac{1}{x^4}$

i $(-x)^{-4}$ $\frac{1}{x^4}$

j $(-x)^{-3}$ $-\frac{1}{x^3}$

k $7x^{-3}$ $\frac{7}{x^3}$

l $(7x)^{-3}$ $\frac{1}{343x^3}$

m $(7xy)^{-3}$ $\frac{1}{343x^3y^3}$

n $-(7xy)^{-3}$ $-\frac{1}{343x^3y^3}$

o $(-7xy)^{-3}$ $-\frac{1}{343x^3y^3}$

p $(-7xy)^{-4}$ $\frac{1}{2401x^4y^4}$

q $(-7x^2y)^{-4}$ $\frac{1}{2401x^8y^4}$

r $-(-7x^2y^n)^{-4}$ $\frac{-1}{2401x^4y^{4n}}$

4 Express the following using positive indices.

a x^{-5} $\frac{1}{x^5}$

b a^{-4} $\frac{1}{a^4}$

c $2a^{-4}$ $\frac{2}{a^4}$

d $(2a)^{-4}$ $\frac{1}{16a^4}$

e $3y^{-7}$ $\frac{3}{y^7}$

f $3a^2b^{-3}$ $\frac{3a^2}{b^3}$

g $4m^3n^{-3}$ $\frac{4m^3}{n^3}$

h $10x^{-2}y^5$ $\frac{10y^5}{x^2}$

i $3x^{-5}$ $\frac{3}{x^5}$

j $\frac{x^{-5}}{3}$ $\frac{1}{3x^5}$

k $\frac{2a^{-3}}{5}$ $\frac{2}{5a^3}$

l $\frac{(3a)^{-1}}{b}$ $\frac{1}{3ab}$

m $3x^{-4}y^{-2}z^3$ $\frac{3z^3}{x^4y^2}$

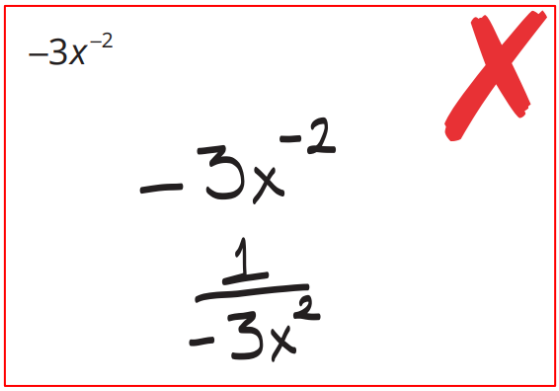
n $\frac{1}{3}p^{-2}q^3r$ $\frac{q^3r}{3p^2}$

o $\frac{1}{5}d^2e^{-4}f^5$ $\frac{d^2f^5}{5e^4}$

p $\frac{3}{8}u^2v^{-6}w^7$ $\frac{3u^2w^7}{8v^6}$

5 Read these examples carefully and answer the questions.

Kelly's work



The –3 should have stayed in the numerator for Kelly's final answer to be correct. Why does the –3 belong in the numerator?

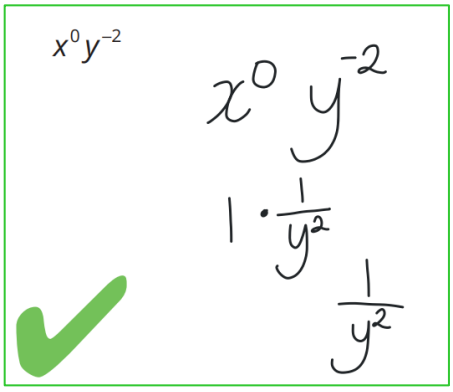
Only the x is being raised to the power of -2

Simplify:

$$-2x^{-4}$$

$-\frac{2}{x^4}$ or $\frac{-2}{x^4}$

Danice's work



Why was x not included in Danice's answer?

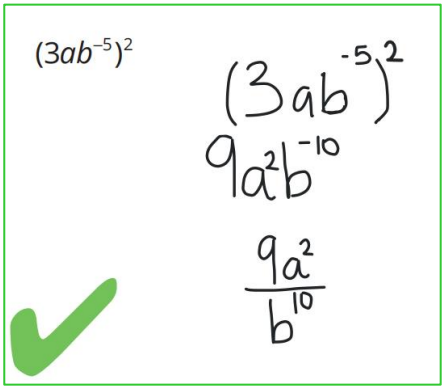
x was raised to the power of 0, meaning it becomes 1.

Simplify:

$$a^{-4}b^0$$

$\frac{1}{a^4}$

Maya's work



Where did the 9 come from in Maya's expression?

The 3 is inside brackets, so is raised to the power of 2.

The original expression did not contain a fraction. Why did Maya's answer contain a fraction?

The negative power means take the reciprocal of b; it becomes $\frac{1}{b}$

Simplify:

$$(5a^{-3}b)^2$$

$\frac{25b^2}{a^3}$

NEGATIVE INDICES IN THE DENOMINATOR

Index Law of Negative Powers	
$\frac{1}{a^{-n}} = a^n$	$\frac{b}{a^{-n}} = ba^n$

- For negative powers in the denominator:
  1. Move the base to the numerator
  2. Power becomes positive.

Example Rewrite expressions involving negative indices with positive indices.

$\frac{1}{c^{-2}}$ $= c^2$	Show this using fraction division. $\frac{1}{c^{-2}} = \frac{1}{\left(\frac{1}{c^2}\right)} =$
$\frac{5}{x^3y^{-4}}$ $= \frac{5y^4}{x^3}$	Why does 5 stay in the numerator and x^3 stay in the denominator? How come only the y term moves? *Only the y term has a ..... power.*

Express the following with positive indices.

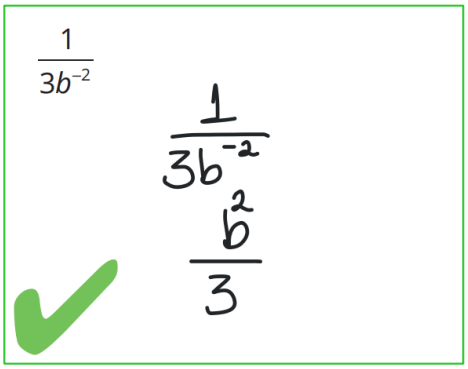
a $\frac{1}{y^{-1}}$ y	**b** $\frac{3}{b^{-2}}$ $3b^2$	**c** $\frac{7}{x^{-4}y^3}$ $\frac{7x^4}{y^3}$
d $\frac{a^2}{b^{-3}}$ a^2b^3	**e** $\frac{1}{u^{-3}b^2}$ $\frac{u^3}{b^2}$	**f** $\frac{3a}{y^{-5}}$ $3ay^5$

Key ideas

1. A negative index means to take the ..... of the base.
2. If a denominator has a negative index, to make the index positive move it to the .....

1 Read the example carefully and answer the questions.

Andy's work



Why did the negative sign disappear when Andy wrote his solution?

Negative powers in the denominator indicate to take the reciprocal of the base (move the base to the numerator) and make power positive.

Why did the 3 stay in the denominator?

The 3 was not raised to a negative power.

Simplify:

$\frac{2}{x^{-3}}$

$2x^3$

2 Express each of the following with positive indices only.

a	$\frac{1}{x^{-2}}$ x^2	**b**	$\frac{2}{y^{-3}}$ $2y^3$	**c**	$\frac{4}{m^{-7}}$ $4m^7$
d	$\frac{4}{p^{-4}}$ $4p^4$	**e**	$\frac{a}{b^{-2}}$ ab^2	**f**	$\frac{e}{d^{-1}}$ ed
g	$\frac{2b^4}{d^{-3}}$ $2b^4d^3$	**h**	$\frac{3m^2}{n^{-4}}$ $3m^2n^4$	**i**	$\frac{4b^4}{3a^{-3}}$ $\frac{4b^4a^3}{3}$
j	$\frac{a^{-3}}{b^{-3}}$ b^3/a^3	**k**	$\frac{x^{-2}}{y^{-5}}$ $\frac{y^5}{x^2}$	**l**	$\frac{g^{-2}}{h^{-3}}$ $\frac{h^3}{g^2}$
m	$\frac{5^{-1}}{7^{-3}}$ $\frac{343}{5}$	**n**	$\frac{3^{-2}}{4^{-3}}$ $\frac{64}{9}$	**o**	$\frac{5^{-2}}{6^{-1}}$ $\frac{6}{25}$
p	$\frac{2n^2}{m^{-3}}$ $2n^2m^3$	**q**	$\frac{y^5}{3x^{-2}}$ $\frac{x^2y^5}{3}$	**r**	$\frac{-3}{7y^{-4}}$ $-\frac{3y^4}{7}$
s	$\frac{-2}{b^{-8}}$ $-2b^8$	**t**	$\frac{-3g}{4h^{-3}}$ $-\frac{3gh^3}{4}$	**u**	$\frac{(-3u)^2}{5t^{-2}}$ $\frac{9u^2t^2}{5}$
v	$\frac{7}{x^{-4}y^3}$ $\frac{7x^4}{y^3}$	**w**	$\frac{1}{u^{-3}v^2}$ $\frac{u^3}{v^2}$	**x**	$\frac{a^{-3}5^{-1}}{y^{-3}}$ $\frac{y^3}{5a^3}$
y	$\frac{5a^2c^{-4}}{6b^{-2}d}$ $\frac{5a^2b^2}{6c^4d}$	**z**	$\frac{2a^{-4}}{b^{-5}c^2}$ $\frac{2b^5}{a^4c^2}$	**aa**	$\frac{5^{-1}h^3k^{-2}}{4^{-1}m^{-2}p}$ $\frac{4h^3m^2}{5k^2p}$

NEGATIVE INDEX AS THE RECIPROCAL

- The negative index means to take the reciprocal of the base, i.e., invert the fraction.
- The rules with negative indices are:

Negative Index is the Reciprocal	
$\left(\frac{a}{b}\right)^{-1} = \frac{a^{-1}}{b^{-1}} = \frac{b}{a}$	$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n}$

Example Simplify expressions using the reciprocal.

$\left(\frac{x}{y}\right)^{-1}$ $= \frac{y}{x}$	What does it mean to take a reciprocal of a number? *"……" the fraction.* *The numerator becomes the ……., and the denominator becomes the ……*
$\left(\frac{x}{y}\right)^{-3}$ $= \left(\frac{y}{x}\right)^{3}$ $= \frac{y^3}{x^3}$	What happened to change $\left(\frac{x}{y}\right)^{-3} = \left(\frac{y}{x}\right)^{3}$? *The negative power means to …… the base inside the brackets.*

Express the following with positive indices.

a $\left(\frac{4}{1}\right)^{-1}$ $\frac{1}{4}$	**b** $\left(\frac{1}{4}\right)^{-1}$ 4	**c** $\left(\frac{3}{4}\right)^{-1}$ $\frac{4}{3}$
d $\left(\frac{2}{3}\right)^{-2}$ $\frac{9}{4}$	**e** $\left(\frac{x}{y}\right)^{-2}$ $\frac{y^2}{x^2}$	**f** $\left(\frac{2x^2}{5y}\right)^{-3}$ $\frac{125y^3}{8x^6}$

Key ideas

1. A negative index means to take the …… of the base.
2. Take the reciprocal by swapping the …… and ……

1 Simplify these expressions by writing them without negative indices.

a $\left(\frac{5}{4}\right)^{-1}$ $\frac{4}{5}$	**b** $\left(\frac{2}{7}\right)^{-1}$ $\frac{7}{2}$	**c** $\left(\frac{x}{3}\right)^{-1}$ $\frac{3}{x}$	**d** $\left(\frac{a}{b}\right)^{-1}$ $\frac{b}{a}$
e $\left(\frac{3}{4}\right)^{-1}$ $\frac{4}{3}$	**f** $\left(\frac{x}{y}\right)^{-1}$ $\frac{y}{x}$	**g** $\left(\frac{bx}{y}\right)^{-1}$ $\frac{y}{bx}$	**h** $\left(\frac{2x^2}{3y}\right)^{-1}$ $\frac{3y}{2x^2}$
i $\left(\frac{2}{3}\right)^{-2}$ $\frac{9}{4}$	**j** $\left(\frac{3}{2}\right)^{-2}$ $\frac{4}{9}$	**k** $\left(\frac{1}{2}\right)^{-5}$ 32	**l** $\left(\frac{2}{5}\right)^{-3}$ $\frac{125}{8}$
m $\left(\frac{xy}{2}\right)^{-4}$ $\frac{16}{x^4y^4}$	**n** $\left(\frac{x}{5}\right)^{-2}$ $\frac{25}{x^2}$	**o** $\left(\frac{5x}{ab}\right)^{-3}$ $\frac{a^3b^3}{125x^3}$	**p** $\left(\frac{7}{3y}\right)^{-3}$ $\frac{27y^3}{343}$
q $\left(\frac{x^3}{ab}\right)^{-5}$ $\frac{a^5b^5}{x^{15}}$	**r** $\left(\frac{xy}{3n^4}\right)^{-2}$ $\frac{9n^8}{x^2y^2}$	**s** $\left(\frac{2x^4}{ab^2}\right)^{-5}$ $\frac{a^5b^{10}}{32x^{20}}$	**t** $\left(\frac{7a^2}{9x^7}\right)^{-1}$ $\frac{9x^7}{7a^2}$

2 Consider $\left(\frac{2}{3}\right)^{-1}$

Complete this working out.

$$\left(\frac{2}{3}\right)^{-1} = \frac{1}{\left(\frac{2}{3}\right)}$$

$$= 1 \div \qquad 1 \div \frac{2}{3}$$

$$= 1 \times \qquad 1 \times \frac{3}{2}$$

$$= \frac{3}{2}$$

MIXED PRACTICE

Summary of Negative Indices		
A negative index means to interchange the numerator and denominator		
$a^{-m} = \frac{1}{a^m}$	$\frac{1}{a^{-n}} = a^n$	$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$

FOUNDATION

1 Write these expressions using positive indices.

a a^{-3} $\frac{1}{a^3}$	**b** $\frac{1}{a^{-3}}$ a^3	**c** $2a^{-3}$ $\frac{2}{a^3}$	**d** $\frac{2}{a^{-3}}$ $2a^3$
e $\frac{a^{-3}}{2}$ $\frac{1}{2a^3}$	**f** $\frac{5a^{-3}}{2}$ $\frac{5}{2a^3}$	**g** $\frac{5}{2a^{-3}}$ $\frac{5a^3}{2}$	**h** $\frac{4a^{-3}}{7b^{-5}}$ $\frac{4b^5}{7a^3}$

2 Simplify these expressions. Write your final answer using positive indices.

a $x^2y \times x^4y^3$ x^6y^4	**b** $x^{-5}y^3 \times x^3y^{-2}$ $\frac{y}{x^2}$	**c** $0.2s^{-2}t^3 \times 5st^{-5}$ $\frac{1}{st^2}$
d $56x^2y^6 \div 8xy^8$ $\frac{7x}{y^2}$	**e** $(s^2y^{-3})^3$ $\frac{s^6}{y^9}$	**f** $(5c^{-2}d^3)^{-1}$ $\frac{c^2}{5d^3}$
g $3a^{-4} \times 2a^2$ $\frac{6}{a^2}$	**h** $5m^{-4} \times (-2m^{-2})$ $-\frac{10}{m^6}$	**i** $\frac{2x^{-2}}{3x^{-3}}$ $\frac{2x}{3}$
j $\frac{7d^{-3}}{10d^{-5}}$ $\frac{7d^2}{10}$	**k** $\frac{6c^{-5}}{12c^{-5}}$ $\frac{1}{2}$	**l** $\frac{5s^{-2}}{3s}$ $\frac{5}{3s^3}$
m $\frac{4f^{-5}}{3f^{-3}}$ $\frac{4}{3f^2}$	**n** $\frac{3d^{-3}}{6d^{-1}}$ $\frac{1}{2d^2}$	**o** $\frac{15t^{-4}}{18t^{-2}}$ $\frac{5}{6t^2}$
p $(2a^{-2}y^3)^{-2} \times (2ay^{-3})^3$ $\frac{2a^7}{y^{15}}$	**q** $(5st^2)^3 \div (5s^{-1}t^3)^2$ $5s^5$	**r** $(2a^{-2}y^3)^{-2} \div (2ay^{-3})^3$ $\frac{ay^3}{32}$

3 Write these expressions with negative indices.

a $\frac{1}{x}$ x^{-1}

b $-\frac{1}{x}$ $-x^{-1}$

c $-\frac{12}{x}$ $-12x^{-1}$

d $-\frac{12}{x^2}$ $-12x^{-2}$

e $\frac{9}{x^2}$ $9x^{-2}$

f $\frac{9}{x^3}$ $9x^{-3}$

g $\frac{9}{x^5}$ $9x^{-5}$

h $\frac{1}{6x}$ $\frac{x^{-1}}{6}$

i $\frac{1}{6x^2}$ $\frac{x^{-2}}{6}$

j $-\frac{1}{6x^2}$ $-\frac{x^{-2}}{6}$

k $-\frac{5}{6x^2}$ $-\frac{5x^{-2}}{6}$

l $-\frac{5}{10x^2}$ $-\frac{x^{-2}}{2}$

4 Simplify these expressions. Write using positive indices.

a $x^{-2} \times x^{-3}$ $\frac{1}{x^5}$

b $k^5 \div k^{-6}$ k^{11}

c $(y^{-3})^2$ $\frac{1}{y^6}$

d $a^5 \div a^7 \times a^{-4}$ $\frac{1}{a^6}$

e $xy^3 \times x^2y^{-2}$ x^3y

f $\frac{h^{10} \times (h^{-3})^5}{h^2}$ $\frac{1}{h^7}$

g $5y^{-2} \times 4xy^6 \div 2xy^{-3}$ $10y^7$

h $(6x^{-4})^2 \div 4x^{-3}$ $\frac{9}{x^5}$

i $\left(\frac{2x^2}{x^3}\right)^4$ $\frac{16}{x^4}$

j $\left(\frac{m^3}{4m^5}\right)^3$ $\frac{1}{64m^6}$

k $2(x^{-7})^3$ $\frac{2}{x^{21}}$

l $4(d^{-2})^3$ $\frac{4}{d^6}$

m $(3t^{-4})^2$ $\frac{9}{t^8}$

n $5(x^2)^{-2}$ $\frac{5}{x^4}$

o $7(h^{-2})^{-4}$ $7h^8$

p $-8(x^5)^{-3}$ $-\frac{8}{x^{15}}$

q $\left(\frac{4}{y^2}\right)^{-2}$ $\frac{y^4}{16}$

r $\left(\frac{3}{h^3}\right)^{-4}$ $\frac{h^{12}}{81}$

5 HSC Advanced Sample Question

Simplify $\left(\frac{p}{q}\right)^3 \div (pq^{-2})$

$\frac{p^2}{q}$

6 NAPLAN B+

Simplify $\frac{36t^4}{9t^{-2}}$

$4t^6$

7 NAPLAN A

Simplify $\frac{24p^{-3}q^4}{8pq^2}$ and express as a fraction involving non-negative indices only.

$\frac{3q^2}{p^4}$

8 NAPLAN A

Simplify $\left(\frac{3y^3}{2}\right)^{-2}$ and express as a fraction involving non-negative indices only.

$\frac{4}{9y^6}$

9 NAPLAN A

Simplify $\left(\frac{4p^2}{8}\right)^{-3}$ and express as a fraction involving non-negative indices only.

$\frac{8}{p^6}$

10 If $x^4 = 3$, find the value of:

a $x^4 - 1$

2

b x^8

9

c $2x^{16}$

162

d $3x^4 - 3x^8$

-18

11 Simplify.

a $(a^3b^2)^3 \times (a^2b^4)^{-1}$

a^7b^2

b $\frac{2a^3b^2}{a^{-3}} \times \frac{2a^2b^5}{b^4}$

$4a^8b^3$

c $\frac{(3rs^2)^4}{r^{-3}s^4} \times \frac{(2r^2s)^2}{s^7}$

$\frac{324r^{11}}{s}$

d $\frac{4(x^{-2}y^4)^2}{x^2y^{-3}} \times \frac{xy^4}{2x^{-2}y}$

$\frac{2y^{14}}{x^3}$

e $\left(\frac{a^2b^3}{b^{-2}}\right)^2 \div \left(\frac{ab^4}{a^2}\right)^{-2}$

a^2b^{18}

f $\left(\frac{m^4n^{-2}}{r^3}\right)^2 \div \left(\frac{m^{-3}n^2}{r^3}\right)^2$

$\frac{m^{14}}{n^8}$

12 Simplify, expressing the answers with positive indices. Evaluate powers where possible.

a $(3a^2b^{-2})^3 \times (2a^4)^{-2}$

$\frac{27}{4a^2b^6}$

b $(5x^4y^6)^{-3} \times (5^2xy^{-1})^3$

$\frac{1}{x^9y^{21}}$

c $(5m^2n^{-3})^{-2} \times 2(m^{-2}n^3)^2$

$\frac{2n^{12}}{25m^8}$

d $\frac{(2a^4b^{-2})^3}{c^2} \times \frac{(2^2a^{-3}b^2)^{-1}}{c}$

$\frac{2a^{15}}{c^3b^8}$

e $\frac{(m^2n^3)^2}{p^{-3}} \times (mnp^{-2})^{-3}$

mn^3p^9

f $\frac{(a^2)^3}{b^3} \div \left(\frac{a}{b^2}\right)^{-2}$

$\frac{a^8}{b^7}$

g $\frac{(2a^4)^2}{b^7} \div \frac{(a^2)^{-3}}{2b}$

$\frac{8a^{14}}{b^6}$

h $\frac{(4c^4d^{-3})^2}{9} \div \frac{3c^{-2}}{d}$

$\frac{16c^{10}}{27d^5}$

i $\frac{(3m^2n^3)^{-2}}{p^4} \div \frac{p^{-3}}{m}$

$\frac{1}{9m^3n^6p}$