

Student Name

Mathematics Stage 5 Paths

LINEAR RELATIONSHIPS C

Book 1 Apply formulas to find the midpoint and
gradient/slope of an interval on the Cartesian
plane

Version: 260207

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OVERVIEW

SYLLABUS CONTENT

MA5-LIN-P-01 describes and applies transformations, the midpoint, gradient/slope and distance formulas, and equations of lines to solve problems

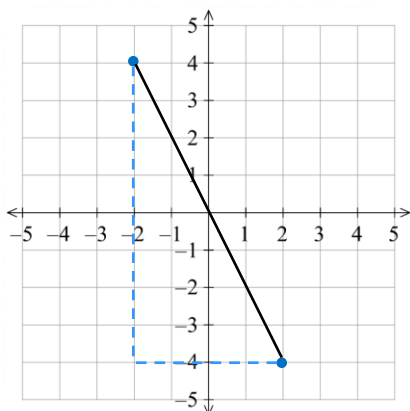
Apply formulas to find the midpoint and gradient/slope of an interval on the Cartesian plane

- Apply the formula to find the midpoint of the interval joining 2 points on the Cartesian plane:
$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$
- Use the relationship $m = \frac{\text{rise}}{\text{run}}$ to establish the formula for the gradient/slope (m) of the interval joining the 2 points (x_1, y_1) and (x_2, y_2) on the Cartesian plane: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Apply the gradient formula to find the gradient of the interval joining 2 points on the Cartesian plane

REVIEW OF PRIOR KNOWLEDGE

1 Find the gradient of these lines.

a $(-2, 4)$ and $(2, -4)$.



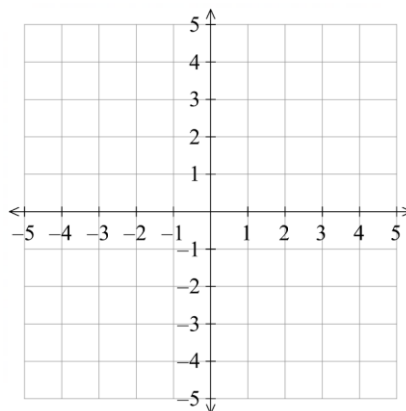
Rise =

Run =

$$m = \frac{\text{Rise}}{\text{Run}}$$

=

b $(0, 5)$ and $(5, 0)$



Rise =

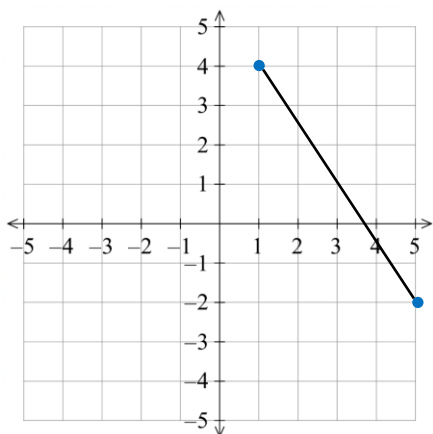
Run =

$$m =$$

=

2 Find the midpoint of these intervals.

a $(5, -2)$ and $(1, 4)$

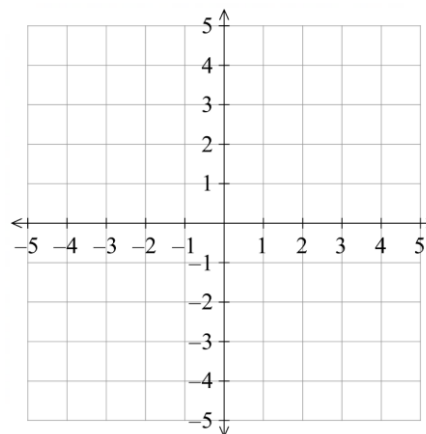


$\bar{x} = \dots\dots\dots$

$\bar{y} = \dots\dots\dots$

$M = (\quad , \quad)$

b $(-2, -5)$ and $(3, -1)$



$\bar{x} = \dots\dots\dots$

$\bar{y} = \dots\dots\dots$

$M = (\quad , \quad)$

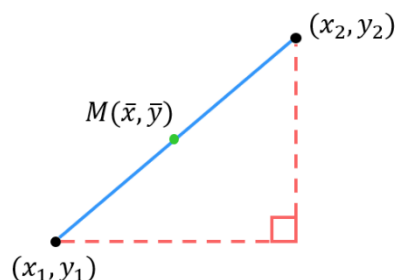
MIDPOINT FORMULA

Midpoint Formula

For two coordinates (x_1, y_1) and (x_2, y_2) :

$M = (\text{mean of } x, \text{Mean of } y)$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



Example Calculate midpoint using the formula.

Calculate the midpoint of the interval through:

$(-4, 3)$ and $(8, -2)$

$x_1 \ y_1 \quad x_2 \ y_2$

$$\bar{x} = \frac{x_1 + x_2}{2}$$

$$= \frac{-4 + 8}{2} = 2$$

$$\bar{y} = \frac{y_1 + y_2}{2}$$

$$= \frac{3 + (-2)}{2} = 0.5$$

$$M = (2, 0.5)$$

$(-3, -5)$ and $(2, 8)$

$x_1 \ y_1 \quad x_2 \ y_2$

$$\bar{x} = \frac{x_1 + x_2}{2}$$

$$= \frac{-3 + 2}{2} = -0.5$$

$$\bar{y} = \frac{y_1 + y_2}{2}$$

$$= \frac{-5 + 8}{2} = 1.5$$

$$M = (-0.5, 1.5)$$

Calculate the midpoint of the interval between these points.

a $(2, 3)$ and $(6, 9)$

$x_1 \ y_1 \quad x_2 \ y_2$

$$\bar{x} = \frac{x_1 + x_2}{2}$$

$$\bar{y} = \frac{y_1 + y_2}{2}$$

$$M = (\quad , \quad)$$

$(4, 6)$

b $(-2, 6)$ and $(8, -1)$

$$\bar{x} = \frac{x_1 + x_2}{2}$$

$$\bar{y} = \frac{y_1 + y_2}{2}$$

$$M = (\quad , \quad)$$

$(3, 2.5)$

c $(-5.5, 4.5)$ and $(7, -1)$

$(0.75, 1.75)$

d $(-5, -6)$ and $(2, 4)$

$(-1.5, -1)$

Key ideas

1. The midpoint is the of the x and y coordinates.

1 Find the midpoint of the x coordinates and the find the midpoint of the y coordinates.

a (2,7) and (14,15)

$$\bar{x} = \frac{\boxed{} + \boxed{}}{2} = \boxed{}$$

$$\bar{y} = \frac{\boxed{} + \boxed{}}{2} = \boxed{}$$

$$M(\bar{x}, \bar{y}) = (\boxed{}, \boxed{})$$

b (5,2) and (23,14)

$$\bar{x} = \frac{\boxed{} + \boxed{}}{2} = \boxed{}$$

$$\bar{y} = \frac{\boxed{} + \boxed{}}{2} = \boxed{}$$

$$M(\bar{x}, \bar{y}) = (\boxed{}, \boxed{})$$

c (1,14) and (19,22)

$$\bar{x} = \frac{\boxed{} + \boxed{}}{2} = \boxed{}$$

$$\bar{y} = \frac{\boxed{} + \boxed{}}{2} = \boxed{}$$

$$M(\bar{x}, \bar{y}) = (\boxed{}, \boxed{})$$

d (14,2) and (8,6)

$$\bar{x} = \dots\dots\dots$$

$$\bar{y} = \dots\dots\dots$$

$$M = (\quad , \quad)$$

e (9,17) and (3,8)

$$\bar{x} = \dots\dots\dots$$

$$\bar{y} = \dots\dots\dots$$

$$M = (\quad , \quad)$$

f (6,0) and (11,13)

$$\bar{x} = \dots\dots\dots$$

$$\bar{y} = \dots\dots\dots$$

$$M = (\quad , \quad)$$

g (-4,-3) and (8,-11)

h (2,-3) and (-6,9)

i (-1,-6) and (-3,10)

j (7,-5) and (-3,9)

k (-9,-2) and (-1,-9)

l (0,-5) and (-10,4)

2 Explain where the midpoint formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ comes from.

.....

3 Calculate the midpoint of these coordinates without drawing the interval.

a $(0,0)$ and $(4,2)$

k $(4,8)$ and $(-1,1)$

b $(0,0)$ and $(2,4)$

l $(4,8)$ and $(-1,-1)$

c $(0,0)$ and $(4,8)$

m $(-4,8)$ and $(-1,-1)$

d $(4,8)$ and $(0,0)$

n $(4,8)$ and $(-2,-2)$

e $(4,8)$ and $(2,2)$

o $(-4,8)$ and $(-2,-2)$

f $(4,8)$ and $(4,4)$

p $(-4,-8)$ and $(-2,-2)$

g $(4,8)$ and $(3,3)$

q $(-2,-4)$ and $(-2,-2)$

h $(4,8)$ and $(9,9)$

r $(-20,-40)$ and $(-20,-20)$

i $(4,8)$ and $(10,10)$

s $(-200,-400)$ and $(-200,-200)$

j $(4,8)$ and $(1,1)$

t $(-200,-200)$ and $(200,200)$

4 Calculate the midpoint of these coordinates without drawing the interval.

a $(-2,-2)$ and $(6,-3)$

b $(0,6)$ and $(6,0)$

c $(2.5,3)$ and $(4,-7)$

d $(0.5,0)$ and $(5,3)$

e $(51,-12)$ and $(-36,11)$

f $(2a,4b)$ and $(4a,6b)$

g $(5p,3q)$ and $(-3p,5q)$

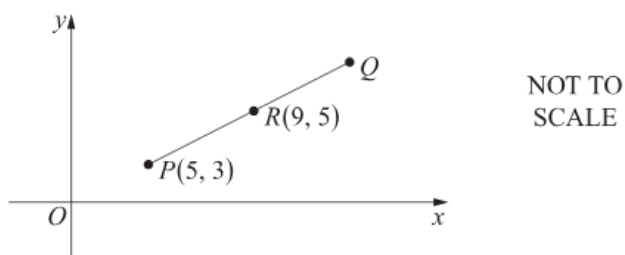
h $(-4r,7s)$ and $(4r,-7s)$

- 5 Find the missing coordinates if M is the midpoint of A and B

A	B	M
$(4,2)$		$(6,1)$
$(7,1)$		$(2,1)$
	$(3,6)$	$(2,1)$
	$(0,-1)$	$(-3,2)$
	$(4,4)$	$(-1,6.5)$

6 **2018 HSC Advanced Band 2**

The point $(9,5)$ is the midpoint of the interval PQ , where P has coordinates $(5,3)$.
What are the coordinates of Q ?



- 7 Find the values of a and b when:

a midpoint of $(a, 3)$ and $(7, b)$ is $(5, 4)$.

$$\bar{x} = \frac{x_1 + x_2}{2} \qquad \bar{y} = \frac{y_1 + y_2}{2}$$

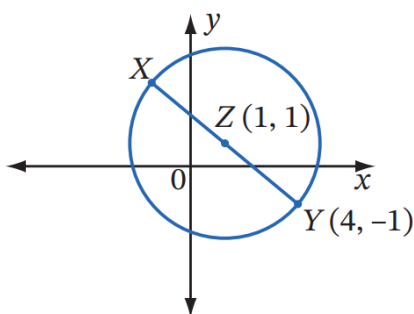
$$5 = \frac{a + 7}{2}$$

b midpoint of $(a, -1)$ and $(2, b)$ is $(-1, 2)$.

c midpoint of $(-3, a)$ and $(b, 2)$ is $(-\frac{1}{2}, 0)$.

d midpoint of $(-5, a)$ and $(b, -4)$ is $(-\frac{3}{2}, \frac{7}{2})$.

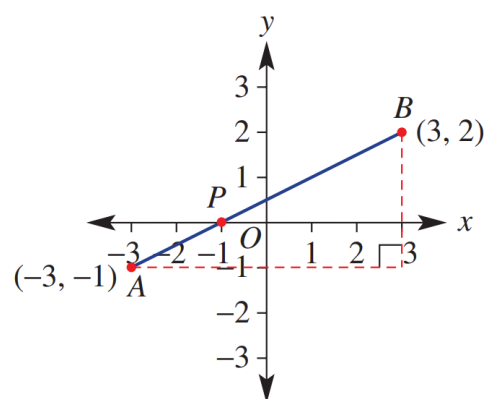
- 8 The circle has diameter XY and centre Z . What are the coordinates of X ?



- 9 From left to right, this line segment shows point $P(-1, 0)$ dividing the segment in the ratio 1 : 2.

a What fraction of the horizontal distance between the end points is P from A?

b What fraction of the vertical distance between the end points is P from A?



- 10 Find the coordinates of point P that divides $A(-4, 9)$ and $B(1, -1)$ in the ratio 2 : 3.

- 11 Find the coordinates of point P that divides $A(-2, -3)$ and $B(4, 0)$ in the ratio 3 : 2.

GRADIENT FORMULA

Gradient Formula	
For two coordinates (x_1, y_1) and (x_2, y_2) :	
$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{y_2 - y_1}{x_2 - x_1}$	

Example Calculate gradient using the formula.

Calculate the gradient of the interval through:

$(4, 2)$ and $(12, 4)$

$x_1 \ y_1 \quad x_2 \ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 2}{12 - 4}$$

$$= \frac{2}{8} = \frac{1}{4}$$

$(-3, 8)$ and $(5, -2)$

$x_1 \ y_1 \quad x_2 \ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - 8}{5 - (-3)}$$

$$= -\frac{10}{8} = -\frac{5}{4}$$

Calculate the gradient of the interval between these points.

a $(2, 3)$ and $(5, 8)$

$x_1 \ y_1 \quad x_2 \ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

=

=

$\frac{5}{3}$

b $(-2, 5)$ and $(0, -6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

=

=

$-\frac{11}{2}$

c $(-1, -2)$ and $(-3, -7)$

$\frac{5}{2}$

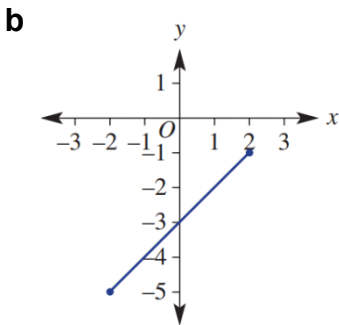
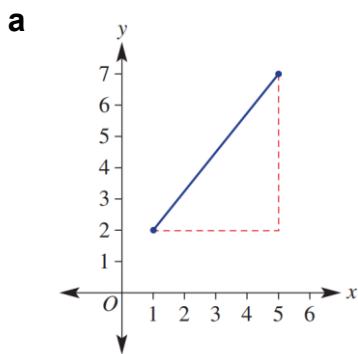
d $(4, -3)$ and $(-4, 1)$

$-\frac{1}{2}$

Key ideas

1. The gradient is the change in divided by the change in

1 Calculate the gradient of the line segment.



2 Find the gradient of each interval.

a A(1, 4), B(5, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

=

=

b A(-2, -7), B(3, 3)

c A(-5, -2), B(3, 2)

d A(3, 6), B(5, 5)

e A(-1, -2), B(1, 4)

f A(-5, 7), B(15, -7)

3 Explain where the gradient formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ comes from.

.....

.....

4 A line joins the two points (-1, 3) and (4, -2).

a Calculate the gradient of the line, using $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (4, -2)$.

.....

b Calculate the gradient of the line, using $(x_1, y_1) = (4, -2)$ and $(x_2, y_2) = (-1, 3)$.

.....

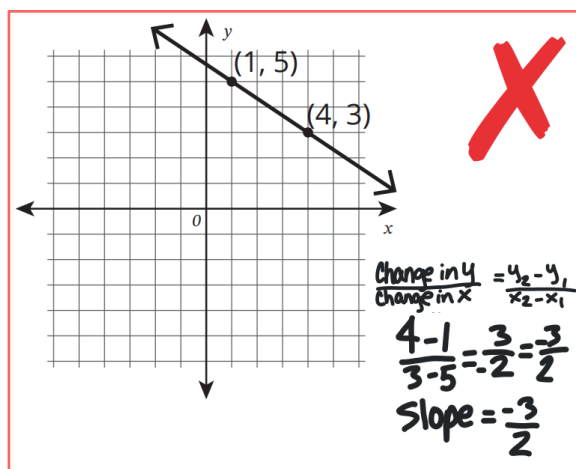
c Do you get the same gradient even if you switch (x_1, y_1) and (x_2, y_2) ?

.....

- 5 Read the example carefully and answer the questions.

Akemi's work:

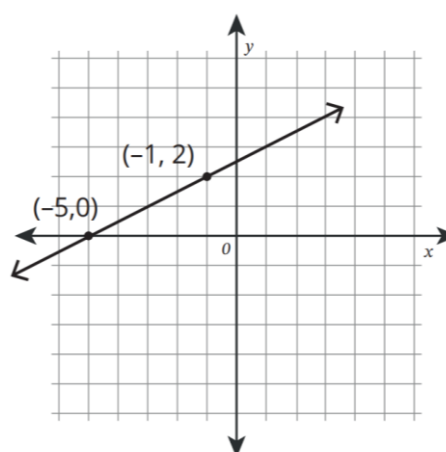
Calculate the gradient.



What did Akemi do wrong when substituting the points into the slope formula?

Using the graph, how could she have checked her work?

Calculate the gradient:



- 6 **Collinear** points lie on the same line. Three points A, B, C are collinear if AB and BC have the same gradient, because then AB and BC are parallel lines passing through a common point B .
- a Determine whether $A(-2, 5)$, $B(1, 3)$ and $C(7, -1)$ are collinear.

$$m_{AB} =$$

$$m_{BC} =$$

collinear

- b $P(-2, 7)$, $Q(1, 1)$, $R(4, -6)$

not collinear

- c $P(-5, -4)$, $Q(-2, -2)$, $R(1, 0)$

collinear