Student Name

Mathematics Stage 5 Paths

VARIATION & RATES OF CHANGE B

Book 2 Analyse the relationship between graphs and variable rates of change

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

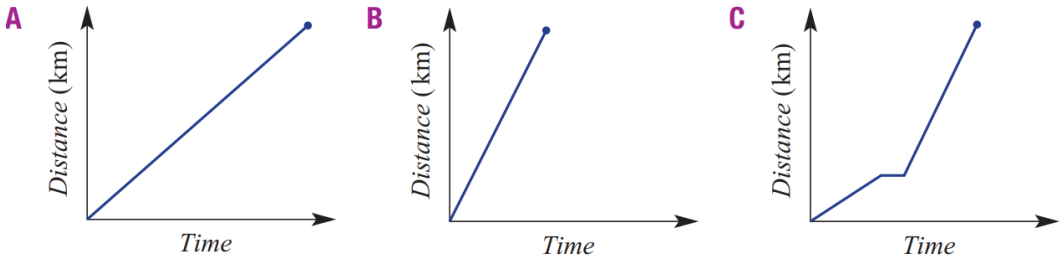
MA5-RAT-P-02 analyses and constructs graphs relating to rates of change

Analyse the relationship between graphs and variable rates of change

- Interpret distance–time graphs when the speed is variable, describing the rate of increase or decrease, the initial and final points, constant relationships represented by straight lines and variable relationships represented by curved lines
- Describe the rate of change between variables in a variety of contexts including direct and inverse variation
- Describe qualitatively the rate of change of a graph, using terms such as *increasing at a decreasing rate*

REVIEW OF PRIOR KNOWLEDGE

1 Three friends, Breanna, Billy and Cianne, travel 5 km from school to the library.
Their journeys are displayed in the three graphs below.
All three graphs are drawn to the same scale.



a Which graph represents the fastest journey?

b If Cianne arrived at the library last, which graph best represents her journey?

c If Breanna walked a short distance before getting picked up by her mum and driven to the library, which graph represents her trip?

2 A 300-litre tank takes 3 hours to empty from a tap.

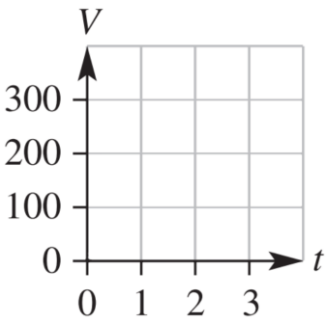
a What is the rate at which water is being emptied from the tank?

b Draw a graph of volume (V litres) vs time (t hours).

c Find the:

i gradient of your graph

ii rule for V



d Use your rule to find the:

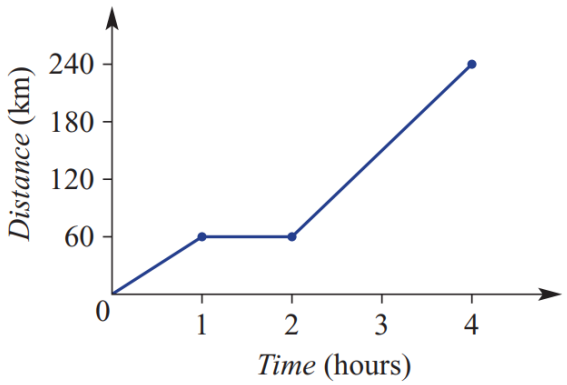
i volume after 1.5 hours

ii time to fill 2000 litres

RATES OF LINEAR GRAPHS WITH VARIABLE RATE OF CHANGE

FOUNDATION

1 A car completes a journey, described by this graph.



a For the first hour, find:

i the total distance travelled

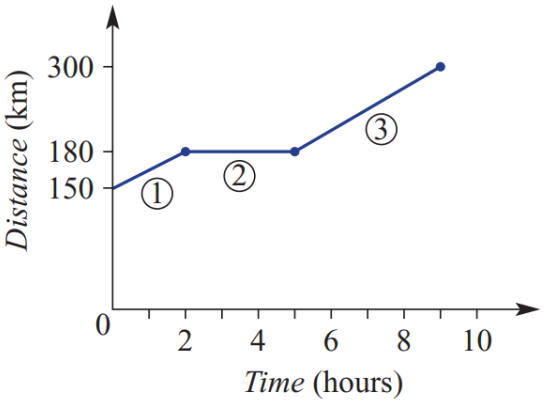
ii the speed

b How fast was the car travelling during:

i the second section?

ii the third section?

2 A cyclist includes a rest stop between two travelling sections.



a For the first hour, find:

i the total distance travelled

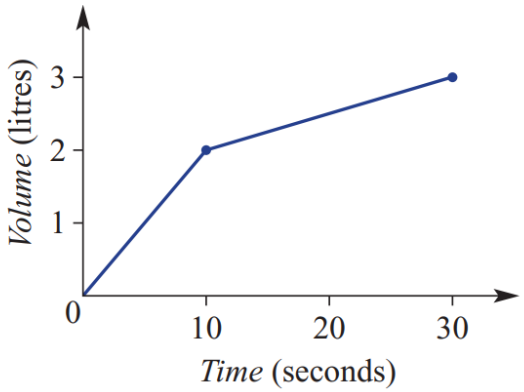
ii the speed

b How fast was the cyclist travelling during:

i the second section?

ii the third section?

3 A container is being filled. How fast (in L/s) is the container being filled:

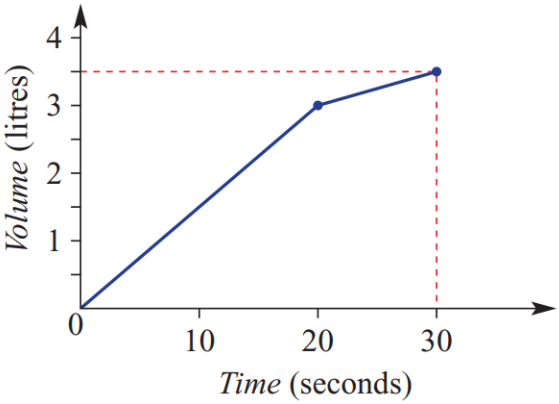


a during the first 10 seconds?

b during the final 10 seconds?

c between the 10- and 20-second marks?

4 A large bottle is being filled. How fast (in L/s) is the bottle being filled:



a during the first 10 seconds?

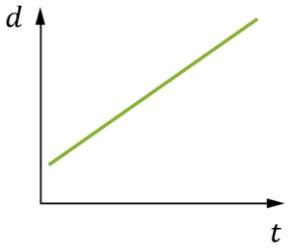
b during the final 10 seconds?

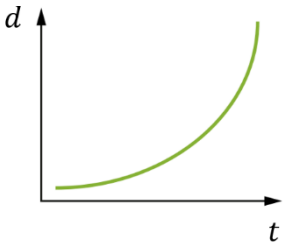
c between the 10- and 20-second marks?

RATES FROM NON-LINEAR GRAPHS

Investigation Rates of change in non-linear graphs

Consider the graphs below.





Both graphs are distance-time graphs.

The left graph shows a car travelling at a constant speed on a highway.

The right graph shows a car speeding up from being stationary at a traffic light.

What do you notice about the two graphs?

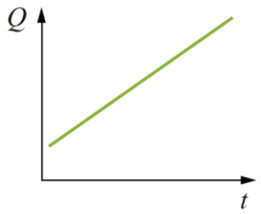
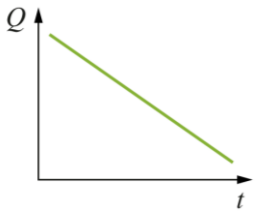
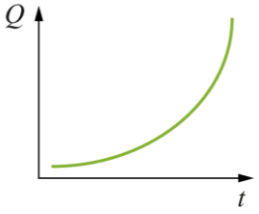
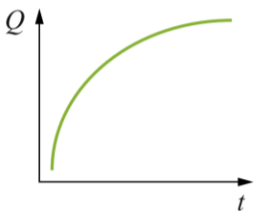
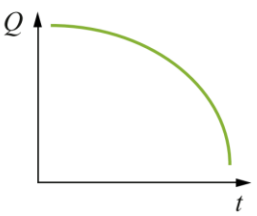
.....

.....

.....

RATES OF NON-LINEAR GRAPHS

- A **linear** graph has a **constant gradient** - the rate stays the same.
- In a **non-linear** graph, the **gradient changes**.
  We can't calculate it (yet), but we can describe how it changes using these words:

Increasing at a constant rate		Decreasing at a constant rate	
			
Increasing at an Increasing rate	**Increasing at a Decreasing rate**	**Decreasing at a Decreasing rate**	**Decreasing at an Increasing rate**
			

Example Matching descriptions to distance-time graphs.

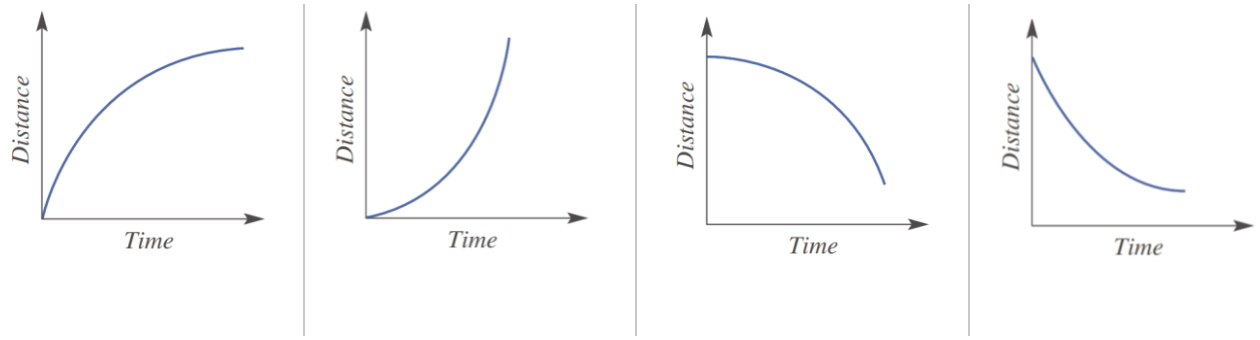
Match the following descriptions to the graph.

A Accelerating away from home. (Distance from home is increasing at a decreasing rate.)

B Accelerating towards home. (Distance from home is increasing at an increasing rate.)

C Decelerating towards home. (Distance from home is decreasing at an increasing rate.)

D Decelerating away from home.(Distance from home is decreasing at a decreasing rate.)



What does accelerating mean in terms of the steepness of the curve?

.....

What does decelerating mean in terms of the steepness of the curve?

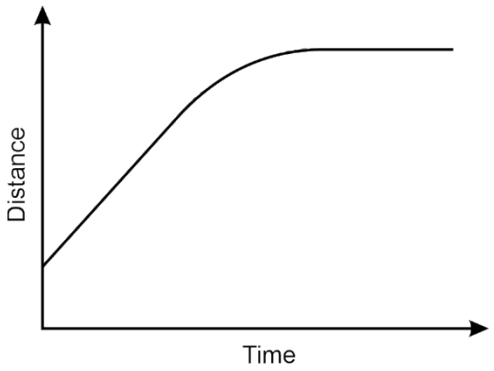
.....

The graph to the right shows a distance-time graph, label the parts of the curve where the object is:

A. Slowing down (decelerating)

B. Stopped

C. Travelling with constant speed



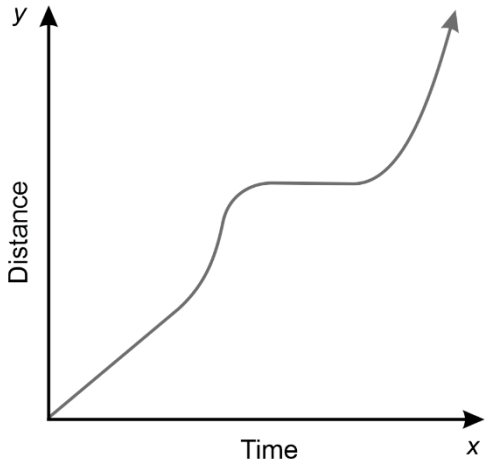
The graph to the right shows a distance-time graph, label the parts of the curve where the object is:

A. Slowing down (decelerating)

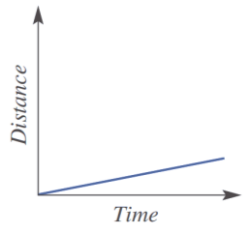
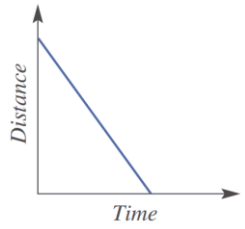
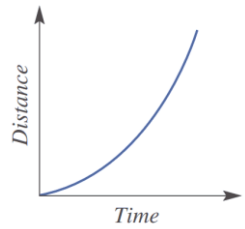
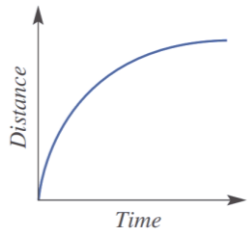
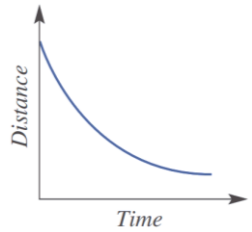
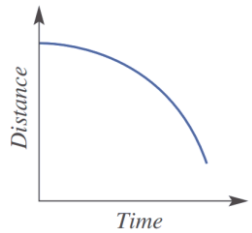
B. Speeding up (accelerating)

C. Stopped

D. Travelling with constant speed



These distance–time graphs show various journeys, each with distance measured from 'home' (i.e. distance = 0). For each graph, select an appropriate description from the list given.

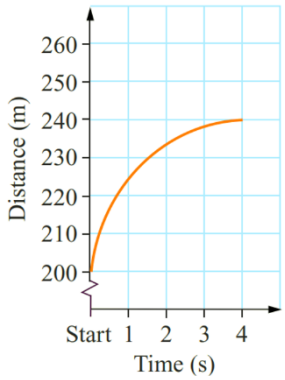
Graph	Distance	Gradient	Speed
a 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
b 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
c 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
d 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
e 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
f 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed

Key ideas

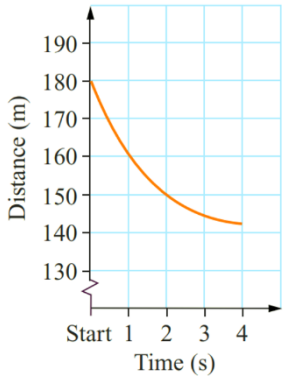
1. A graph that shows change at a constant rate is a …… ……
2. Varying rates of change is shown by curves.
3. An object that is accelerating has a gradient that becomes ……
4. An object that is …… has a gradient that becomes less steep.

1 Each graph below is part of a distance–time graph.

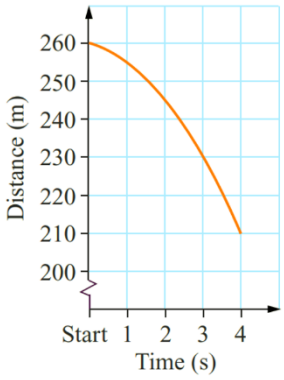
a



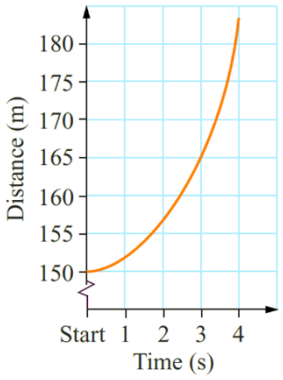
b



c



d



i In the first second of observation, how far did each car travel?

ii In the second second of observation, how far did each car travel?

iii In the third second of observation, how far did each car travel?

iv In the fourth second of observation, how far did each car travel?

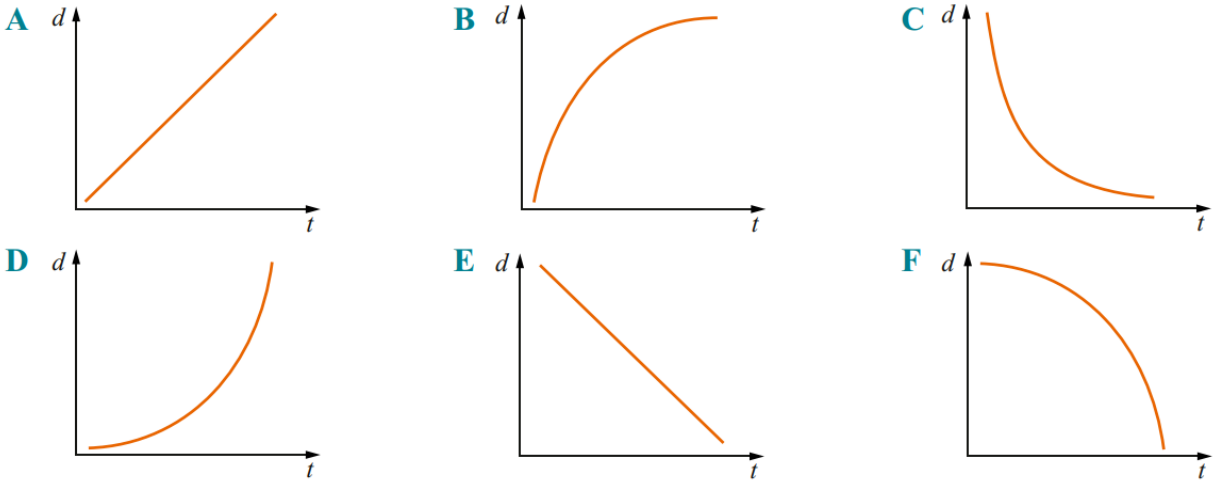
v Describe how distance is changing over this four second period, using the phrase "increasing/decreasing at an increasing/decreasing rate"

vi How does the steepness of a line relate to how quickly distance is changing?

……

……

2 Match each of the statements below to one of the distance–time graphs.



a Distance increases slowly at first and then more quickly. .....

b Distance increases quickly at first and then more slowly. .....

c Distance increases at a constant rate. .....

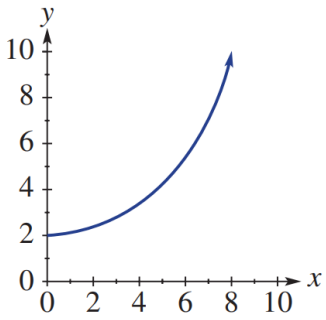
d Distance decreases slowly at first and then more quickly. .....

e Distance decreases quickly at first and then more slowly. .....

f Distance decreases at a constant rate. .....

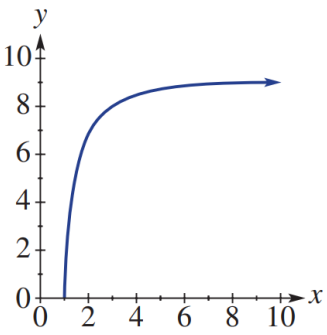
3 For each graph, complete the sentence by supplying the words "*increasing*" or "*decreasing*".

a



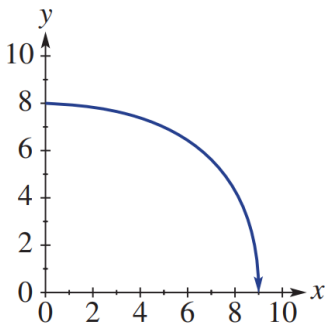
y is ..... at a ..... rate.

b



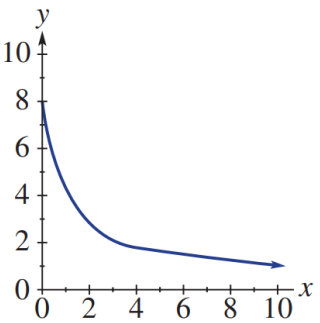
y is ..... at a ..... rate.

c



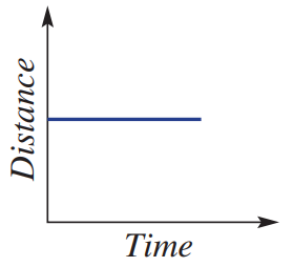
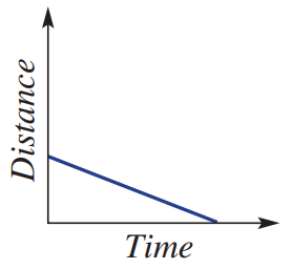
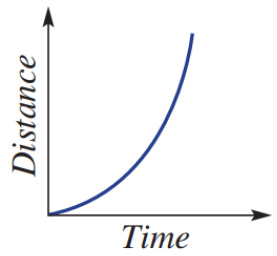
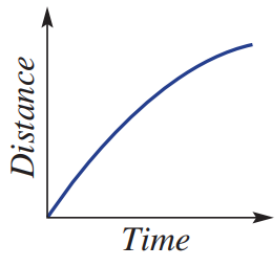
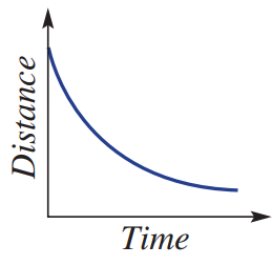
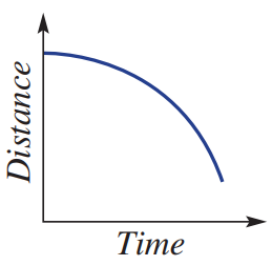
y is ..... at a ..... rate.

d

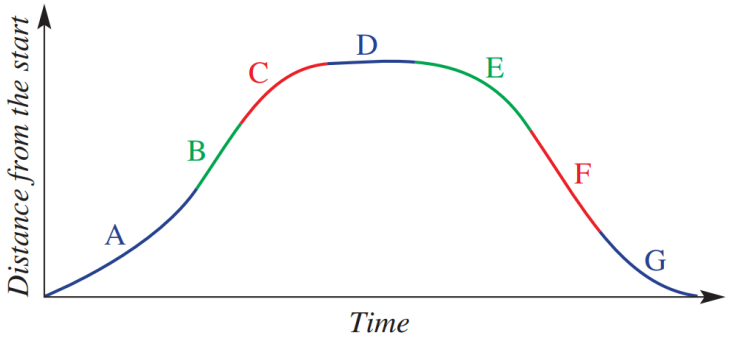


y is ..... at a ..... rate.

4 For each graph, select an appropriate description from the list given.

Graph	Distance	Gradient	Speed
a 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
b 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
c 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
d 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
e 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
f 	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed

5 Select the correct description for the gradient and speed of each segment of this graph.
The rate of change of distance with respect to time is the gradient.



a	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
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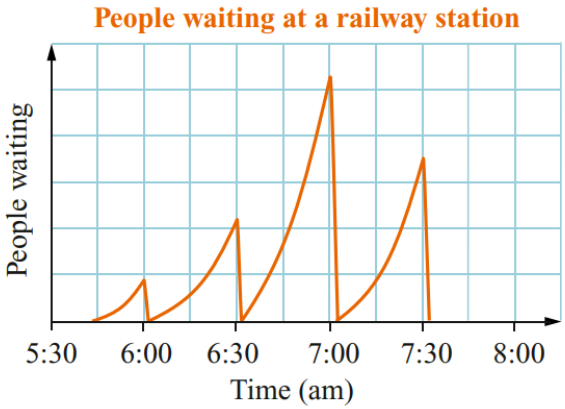
d	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
e	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
f	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed
g	❒ Increasing ❒ Decreasing ❒ Not changing	❒ Positive, constant ❒ Positive, varying ❒ Negative, constant ❒ Negative, varying ❒ Zero gradient	❒ Increasing, accelerating ❒ Decreasing, decelerating ❒ Constant speed ❒ Zero speed

6 The graph shows the number of people waiting on a platform at a railway station one morning.

a How often do trains arrive?

b Describe in words how the number of people varies for this time interval.



7 The graph shows the number of cans of soft drink in the vending machine of a school.

a When do you think the recess and lunch breaks occur in this school?

b Describe in words how the number of cans varies during the day.