Student Name

Mathematics Stage 5 Paths

VARIATION & RATES OF CHANGE B

Book 1 Analyse graphs that are decreasing or increasing at a constant rate

Version: 260207

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CONTENTS

OVERVIEW

SYLLABUS CONTENT

MA5-RAT-P-02 analyses and constructs graphs relating to rates of change

Analyse graphs that are decreasing or increasing at a constant rate

- Describe direct variation graphs as increasing at a constant rate and interpret the gradient of the line as the rate of change
- Interpret and analyse further graphs that exhibit a constant rate of change, including those that are decreasing at a constant rate

RATE FROM A LINEAR MODEL

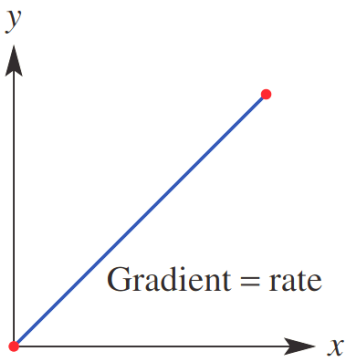
THE GRADIENT IS THE RATE

- A rate compares two quantities.
- When two variables are in a straight-line relationship, the **gradient** is the **rate** y per x

Direct Variation Equation and Graph

$$y = kx$$

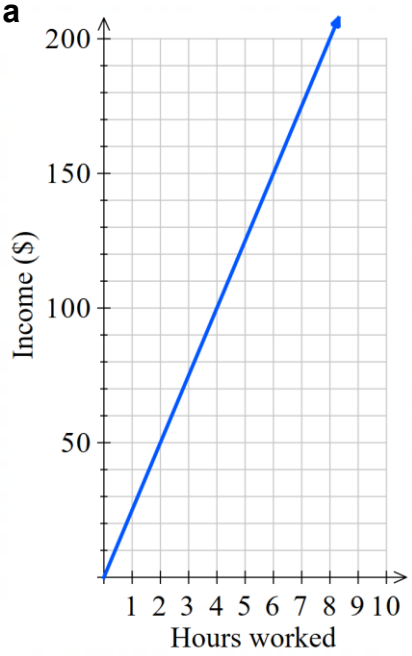
constant of variation (gradient)



Example Interpret the gradient of a direct variation model.

Calculate the gradient, then describe the meaning of the gradient in each of these direct variation graphs.

a



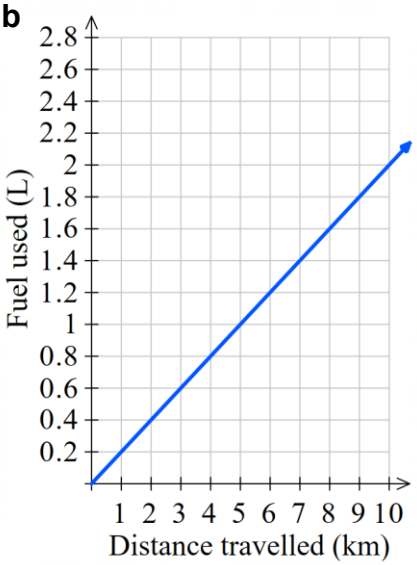
Gradient:

$m = \frac{\text{rise}}{\text{run}} =$

Meaning:

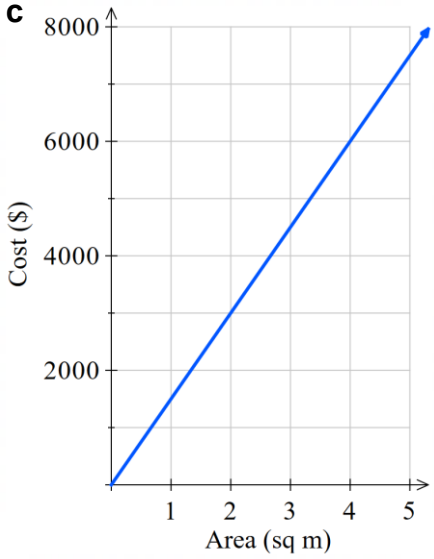
..... per .....

b



0.2 L per km

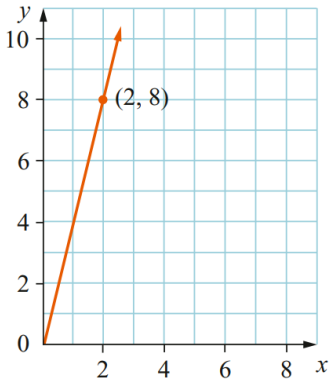
c



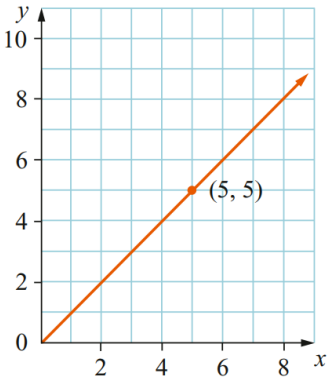
$1500 per sq m

1 For each of these direct variation graphs, find the gradient.

a



b



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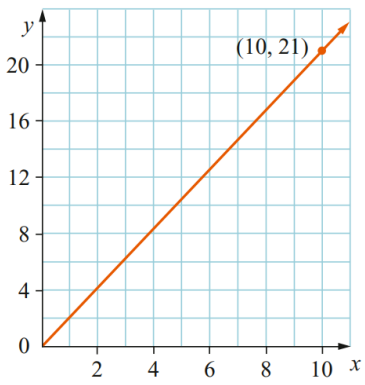
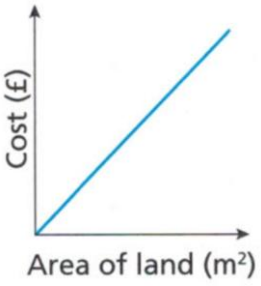


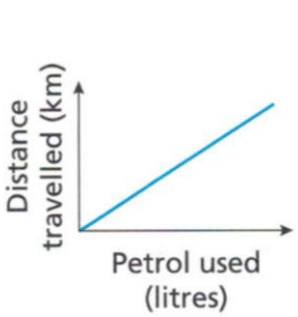
2 Here are some direct variation graphs. Describe what the gradient of each line represents.

a

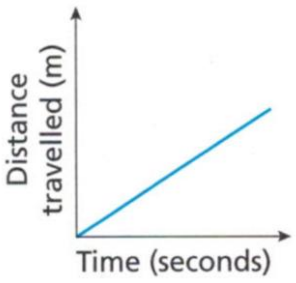


..... per .....

b

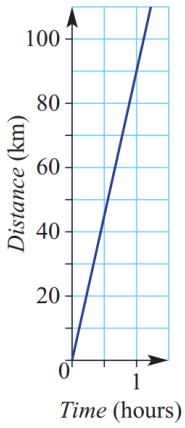


c

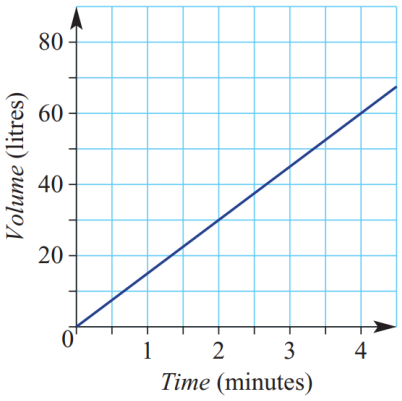


3 Calculate the gradient of each line graph then describe what it represents.

a

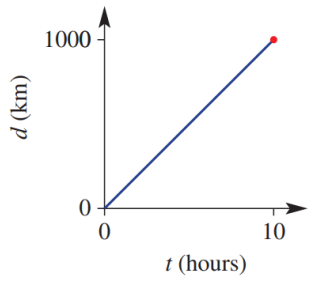


b



4 Use the gradient to find the rate of change of distance over time (speed) for these graphs. Use the units given on each graph.

a



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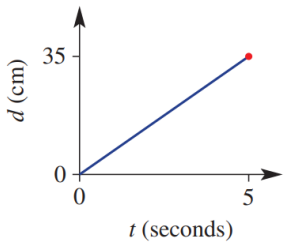

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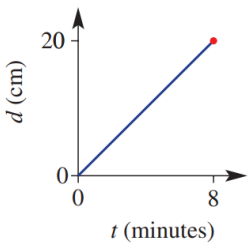
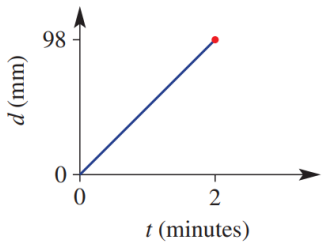


d

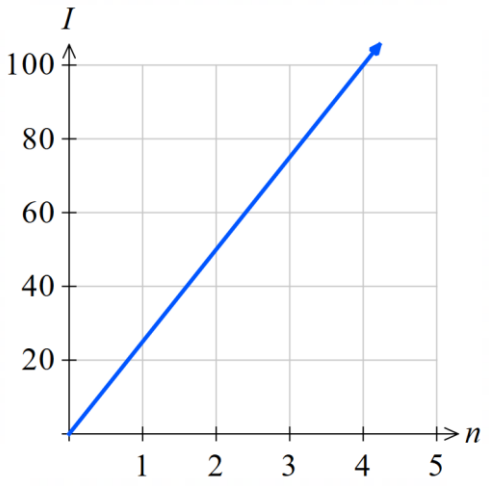


ANALYSING DIRECT VARIATION WITH GRAPHS

Example Analyse a direct variation relationship.

Mr Ding used to work at McDonalds and was paid $20 an hour.
The number of hours he works and his pay are **directly proportional**.
Complete the table of values and draw the graph.

Number of hours n	0	1	2	3
Income I	0	20	40	60



Find the gradient, which is the constant of variation k.

$\frac{\text{rise}}{\text{run}} = 20$

What does k represent in this context?

Pay per hour.

Write an equation relating (I) and (n).

$y = kx$

$I = 20n$

Find his income for 15 hours of work.

$I = 20(15)$

$= 300$

Mr Ding wants to buy a PS5 costing $650. Find how long he must work to earn it.

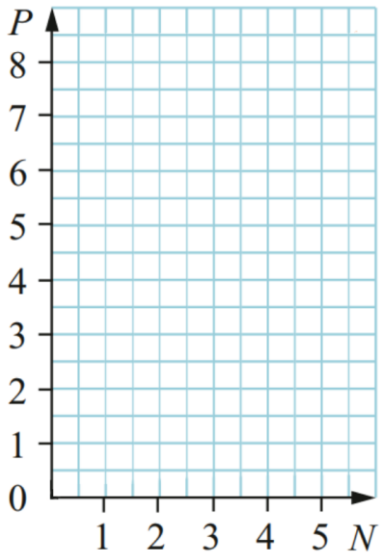
$650 = 20n$

$\frac{650}{20} = n$

$= 32.5$ hours

a Pens cost \$1.50 per pack. Complete the table of values for number of packs N and price P Complete the table and sketch the graph.

N	0	1	2	3
P				



0, 1.5, 3, 4.5

Find the gradient, which is the constant of variation k.

1.5

What does k represent in this context?

Cost per pack

Write the equation relating P and N.

$P = 1.5N$

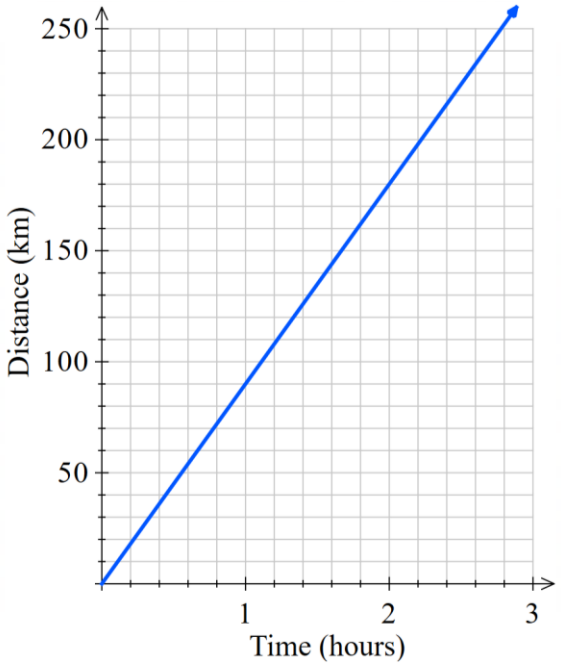
Find the cost of 9 packs of pencils.

\$13.50

How many packs can be bought for \$27?

18 packs

b The graph shows distance over time for an express interstate train.



Find the gradient, which is the constant of variation k.

90

What does k represent in this context?

Speed in km/h

Write an equation relating d and t.

$d = 90t$

How far would the train travel in 12 hours?

1080 km

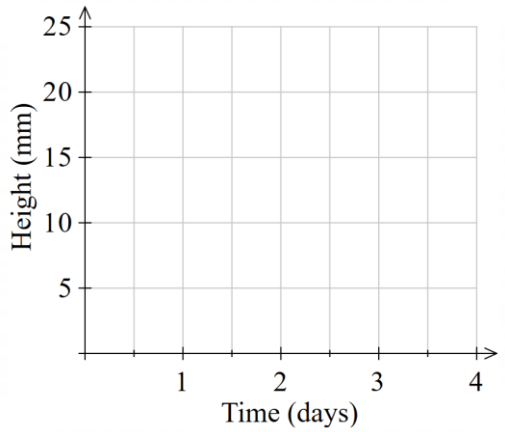
How long would it take to travel between Sydney and Adelaide, a distance of 1260 km?

14 h

1 The rule linking the height of a plant over time is given by $h = 5t$, where h is in millimetres and t is in days.

a Complete the table and graph the relationship.

t	0	1	2	3	4
h					



b What is the gradient of the graph and how does it relate to the equation?

5. It is the coefficient of the independent variable t.

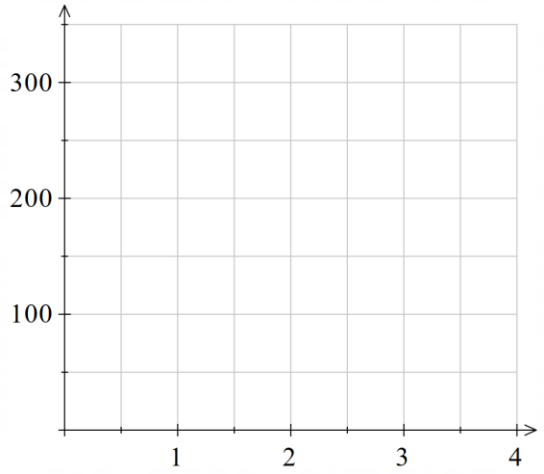
c What rate does the gradient represent?

The plant grows 5 mm per day

2 The distance, d kilometres, travelled by a train in a time of t hours is $d = 80t$

a Complete this table of values using the equation $d = 80t$ and draw the graph.

t	0	1	2	3	4
d					



b What is the gradient and what does it represent?

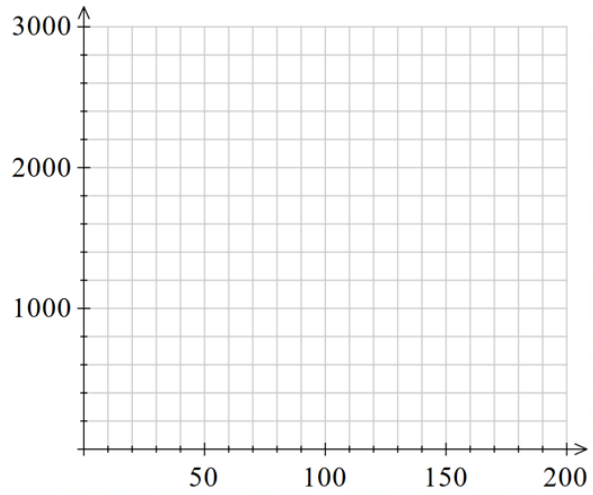
c Use the graph to find the distance of the train after 3.5 hours.

d Use the graph to find when the train is 200 km away.

3 The Supertight Tie Company sells ties for \$15 each.
The sales revenue may be modelled using the equation $R = 15n$.

a Draw the graph of $R = 15n$.
Remember to label the axes.

n	0	50	100	150	200
R					

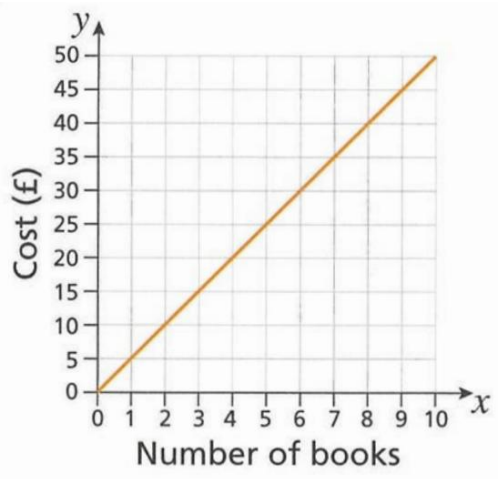


b Find the gradient, which is the constant of variation k.

c What does k represent in this scenario?

d Use the graph to find many ties need to be sold to make \$1800 in revenue.

4 The graph shows the cost of buying a given number of books,



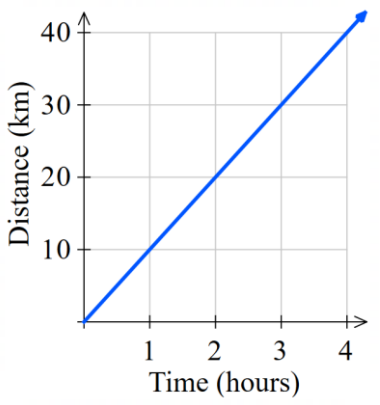
a Use the graph to find the cost of
i 7 books

ii 1 book

b Find the equation of the line $y = kx$.

c Interpret the gradient of the line in the context of the question.

5 This graph shows how far a bike travels over 4 hours.



a Find the gradient of the graph.

10

b What rate does the gradient represent?

speed in km/h

c Write an equation relating distance d and time t.

$d = 10t$

d Use the equation to find:

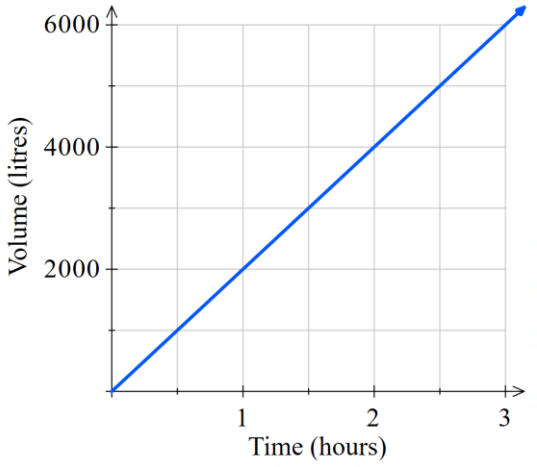
i The distance travelled in 3.5 hours

35 km

ii time required to travel 55 km.

5.5 hrs or 5 hrs 30 min

6 Water is poured into an empty tank at a constant rate.
It takes 3 hours to fill the tank with 6000 litres. The graph is shown.



a Find the gradient.

b What does the gradient represent in this scenario?

c Write the equation relating volume V and time t.

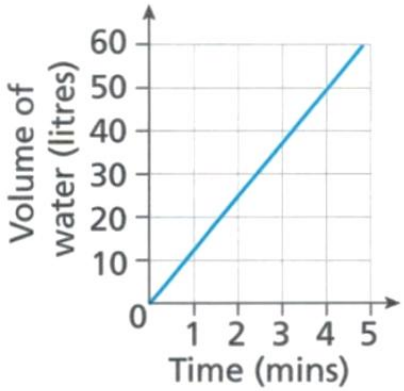
d Use the equation to find:

i volume after 8.5 hours

ii time to fill 7500 litres

7 Here are four graphs showing water flow. For each graph, work out the rate of flow in litres per minute, the volume of water delivered in 30 minutes, and the time it takes to deliver 1000 litres.

a

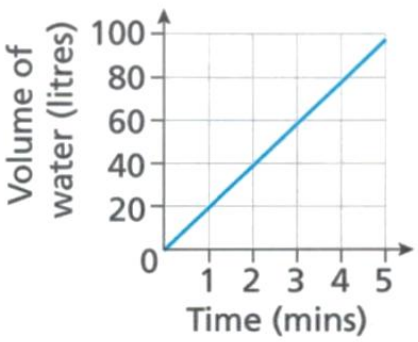


Flow rate:

Volume delivered in 30 minutes:

Time to deliver 1000 litres:

b

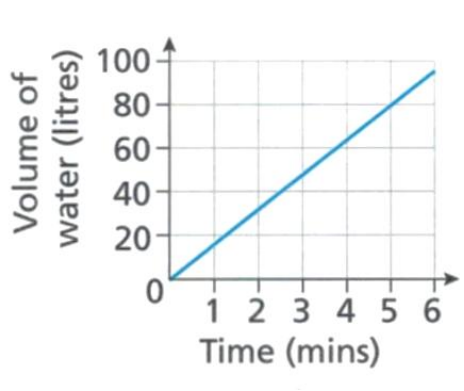


Flow rate:

Volume delivered in 30 minutes:

Time to deliver 1000 litres:

c

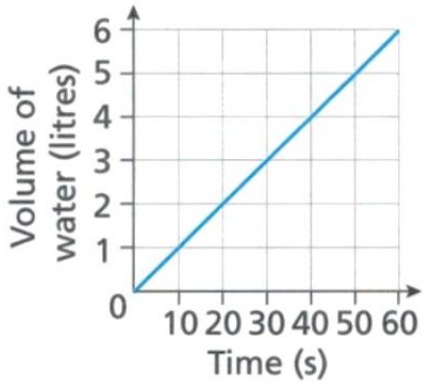


Flow rate:

Volume delivered in 30 minutes:

Time to deliver 1000 litres:

d



Flow rate:

Volume delivered in 30 minutes:

Time to deliver 1000 litres:

8 A solar-powered car travels 100 km in 4 hours.

a What is the rate of change of distance over time? .....

b Draw a graph of distance (d km) vs time (t hours).

c Find the:

i gradient of your graph .....

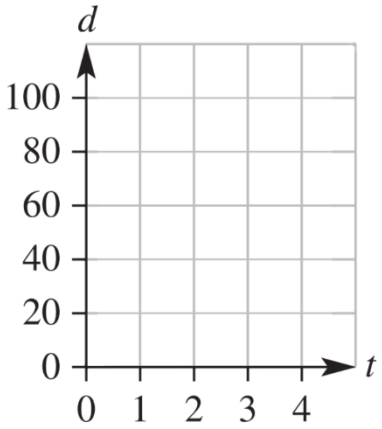
ii rule for d .....

d Use your rule to find the:

i distance after 2.5 hours .....

ii time to travel 40 km

.....



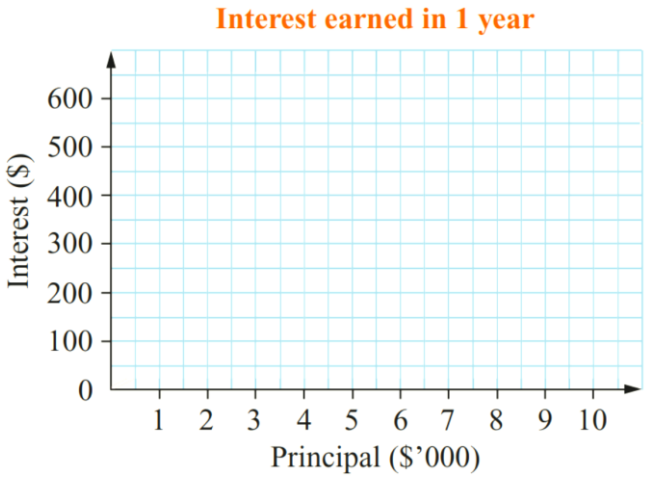
5 The interest, $\$I$, earned on a principal of $\$P$ for 1 year is given by the formula $I = kP$, where k is a constant.

a If an investment of \$6000 earns interest of \$300, find the value of k

.....

b Complete the table and draw the graph.

P	2000	4000	6000	8000	10000
I					



c Calculate the gradient of the graph.

What is its meaning in this context?

(Hint: think of the simple interest formula)

.....

.....

.....

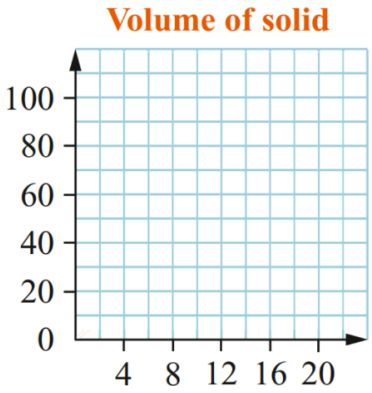
9 The volume V m^3 of a prism is related to its height by $V = kh$, where k is a constant.

a Use the information in the table to find the value of k

h	4	8	12	16	20
V		34			

.....

.....



b Complete the table using your value of k and graph the relationship.

c What is the gradient and what is its meaning in this context?
(Hint: think of the volume of a prism formula)

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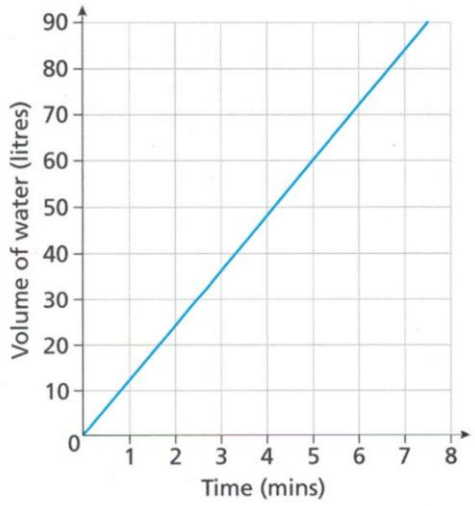
.....

10 Water flows into a tank at a constant rate.
The graph shows how the volume of water in the tank changes with time.



a Find the rate of flow in litres/min.
Choose two points that you can read clearly.

b How much water will flow into the tank in 1 hour?

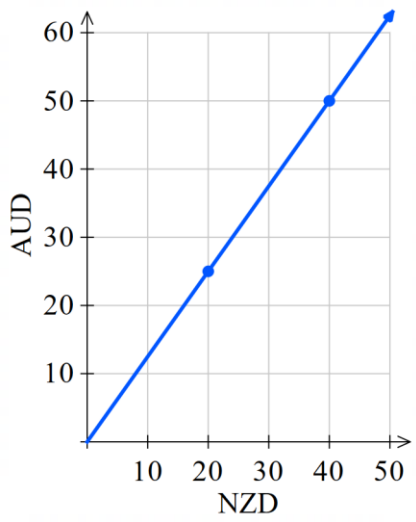
c The capacity of the tank is 1800 litres.
How long will it take to completely fill the tank at the rate you found in part **a**?

CONVERSION GRAPHS

- Many unit conversions are an example of direct variation $y = kx$.
- The gradient represents the conversion factor.

Example Apply direct variation in conversion graphs.

Consider the linear model:



Convert 40 AUD to NZD.

50 NZD.

Convert 25 NZD to AUD.

20 AUD.

Find the gradient of the line, what is its meaning?

Using (0,0) and (20,25):

$m = \frac{25}{20} = 1.25,$

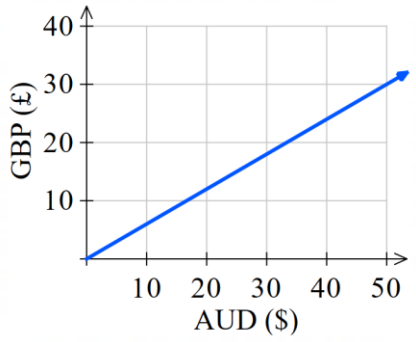
1.25 NZD per AUD

Write an equation relating NZD and AUD.

$y = kx$

$\text{NZD} = 1.25\ \text{AUD}$

The conversion graph is used to convert Australian dollars to British pounds.



a Convert 50 AUD to GBP

£30

b Find the gradient. What is its meaning in this context?

0.6 GBP per AUD

c Write an equation relating AUD and GBP

GBP = 0.6 AUD

d Use your equation to find how much AUD you can buy with 20 GBP.

$33.33

1 Centimetres and inches are directly proportional.
Use the graph to do these conversions:

a 6 inches to cm.

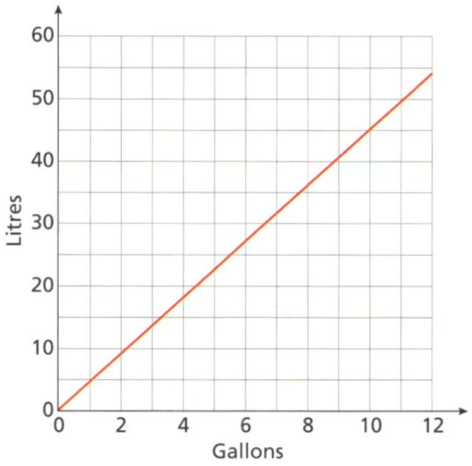
b 20 cm to inches.

c Find the gradient, use the point (10, 25).

d What does the gradient represent?

e Using your equation, how many inches is 200 cm?

2 You can use this graph to convert between gallons and litres.



a Convert 10 gallons to litres.

b Convert 30 litres to gallons.

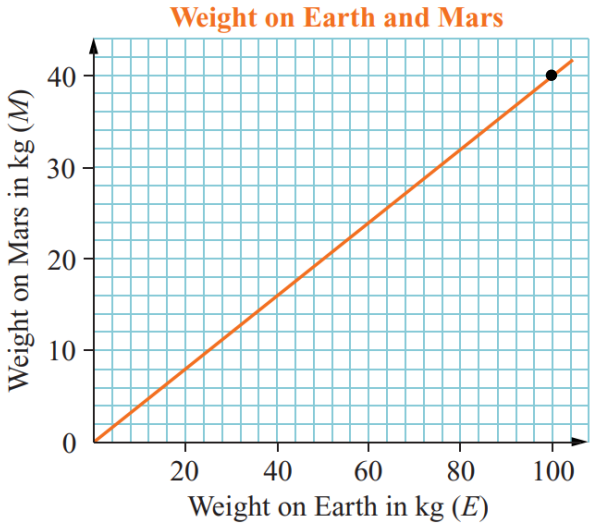
c Why might your answer to part **b** be inaccurate?

d Find the value of k by using the point (10, 45).

e Hence use your equation to find the exact number of gallons for 30 litres.

3 This conversion graph relates weight on Earth E and weight on Mars M.

a Use the point $(100,40)$ to find the value of k and write the equation $M = kE$

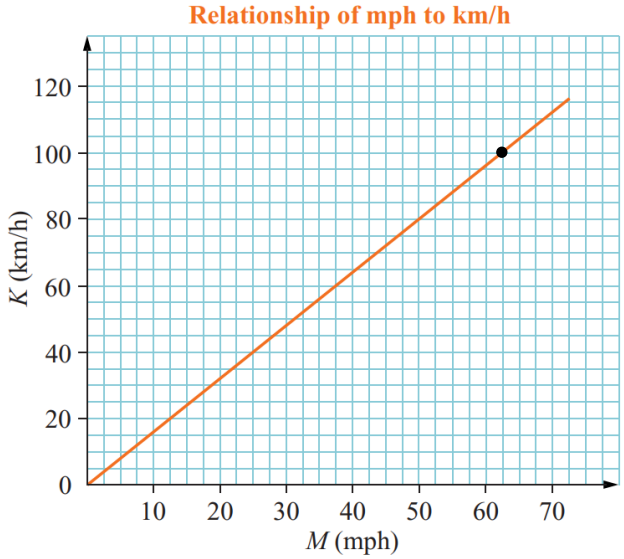


b Use the equation to fill in the table below.

E	30	40		
M			24	40

4 This conversion graph relates miles per hour M and kilometres per hour K.

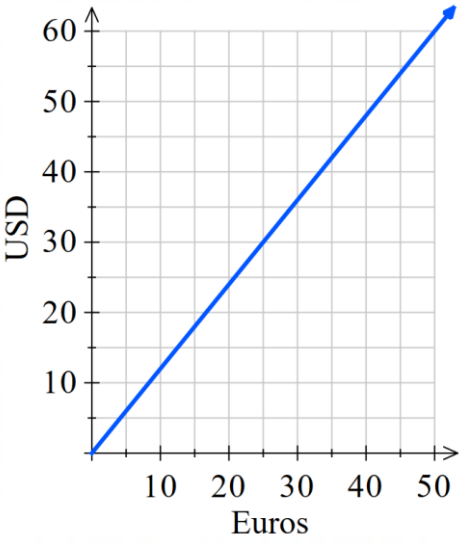
a Use the point $(62,100)$ to find the value of k and write the equation $K = km$



b Use the equation to fill in the table below.

M (mph)	20	50		
K (km/h)			96	112

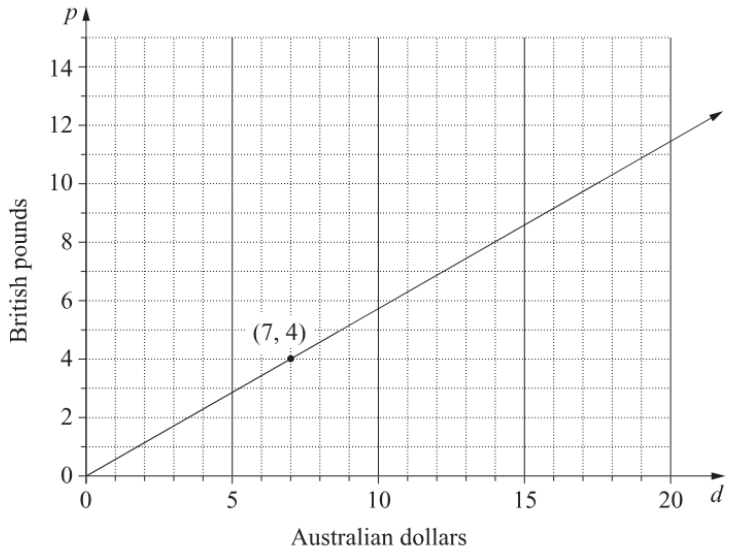
5 The graph shows the conversion between US dollars \$, and euros, €.
How many euros is \$300 US?



6 **2019 HSC Standard 2 Band 5**

The relationship between British pounds (p) and Australian dollars (d) on a particular day is shown in the graph.

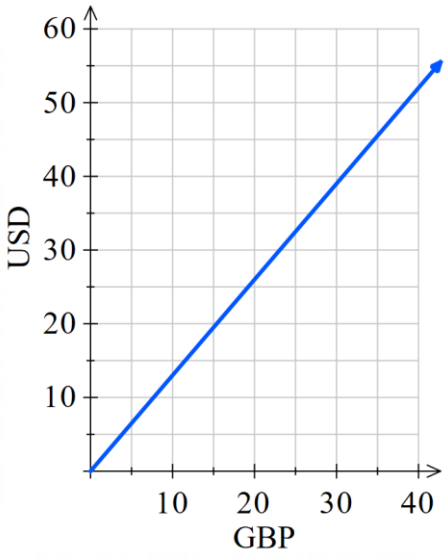
Write the direct variation equation relating pounds to dollars in the form $p = md$.
Leave m as a fraction.

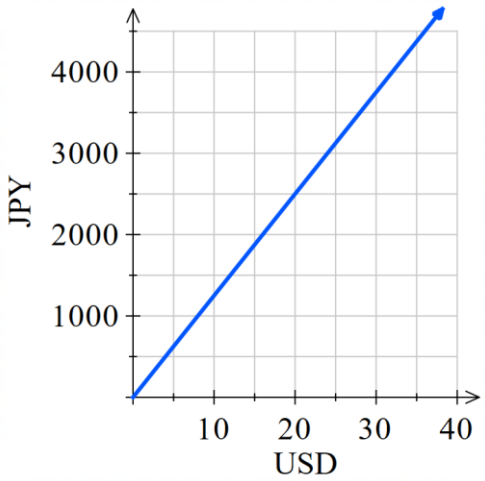


The relationship between Japanese yen (y) and Australian dollars (d) on the same day is given by the equation $y = 76d$. Convert 93 100 Japanese yen to British pounds

7 Use the two graphs below to find the exchange rate between British pounds and Japanese Yen.

Answer as an equation $\text{JPY} = k\ \text{GBP}$.





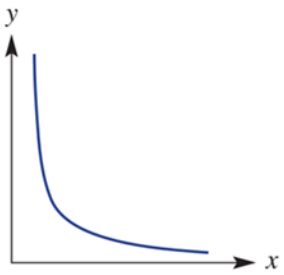
JPY = 150 GBP

ANALYSING INVERSE VARIATION USING GRAPHS

- Two quantities are **inversely proportional** if:
  - An **increase** in one quantity causes a proportional **decrease** in the other.
  - A **decrease** in one quantity causes a proportional **increase** in the other.

Inverse Variation Equation and Graph

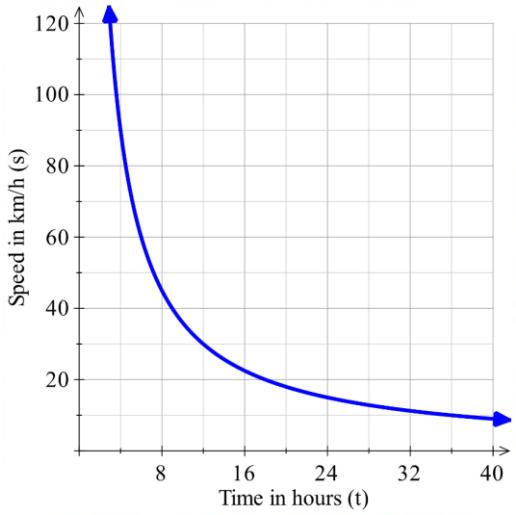
$y = \frac{k}{x}$ ← constant of proportionality



- The graph of inverse variation is a **hyperbola**, also called a **reciprocal relationship**.

Example Interpret a reciprocal model.

The equation $s = \frac{400}{t}$ can be used to model the average speed needed to travel 400 km in a particular time, where s is speed in km/h and t is time in hours.



What is the average speed required to complete a 400 km trip in 5 hours?

Using graph: ≈ 80 km/h

Using equation: $s = \frac{400}{5} = 80$ km/h

How long would it take to complete the trip at 95 km/h?

Using graph: ≈ 4 hours

Using equation: $95 = \frac{400}{t}$

$t = \frac{400}{95} \approx 4.21$ hours

$= 4$ hours 13 minutes.

Why is it impossible for $t = 0$? Explain by referring to the context of the model.

We would have $s = -$, which is u…d…… In the context of the model, we cannot travel …… km in …… hours, this would require …… speed.

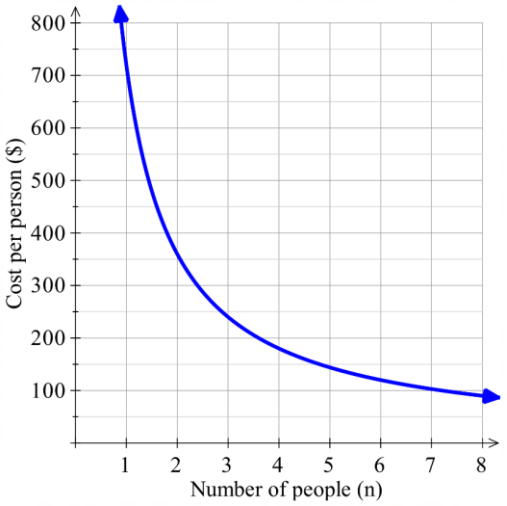
Why is it impossible for $s = 0$? Explain by referring to the context of the model.

We would have $0 = \frac{400}{t}$, which is impossible. In the context of the model, it is not possible to travel …… km at …… speed, it would require …… time.

Why did we not consider the other half of the hyperbola?

…… cannot be negative.

Paul rents a guesthouse from the landlord for $720 per week.
To save money, he has other people stay and divides the expense equally.
The equation that models this situation is $C = \frac{720}{n}$, where C is cost in dollars per person and n is the number of people staying per week.



a What is the cost per person when 3 people are staying?

$240

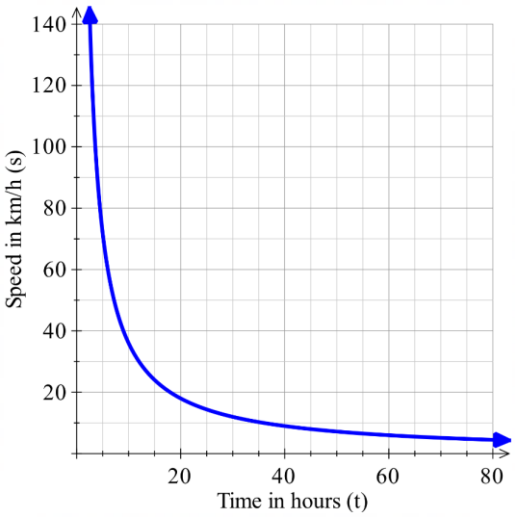
b How many people are needed so cost per person is $144?

5 people

c What are the limitations of this model? (hint: can n be a fraction?)

$n \geq 1$ and must be a whole number.

The equation $s = \frac{360}{t}$ can be used to model the average speed necessary to travel 360 km in a particular time, where t is the time in hours and s is the speed in kilometres per hour.



a What does the 360 represent?

Total distance

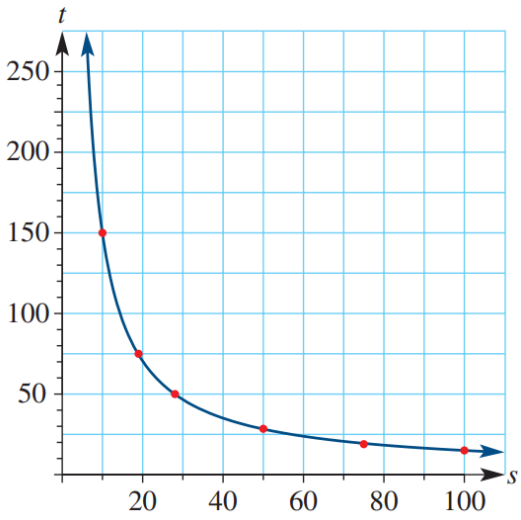
b What average speed is required to complete the 360 km trip in 12 hours?

30 km/h

c How long would it take to complete the trip at 75 km/h?

4.8 h

1 The time taken in hours (t) for a road trip, at an average speed (s), in km/h, is given by $t = \frac{1500}{s}$



a How long would the road trip take at 50 km/h?

30 hours

b How long would the road trip take at 75 km/h?

20 hours

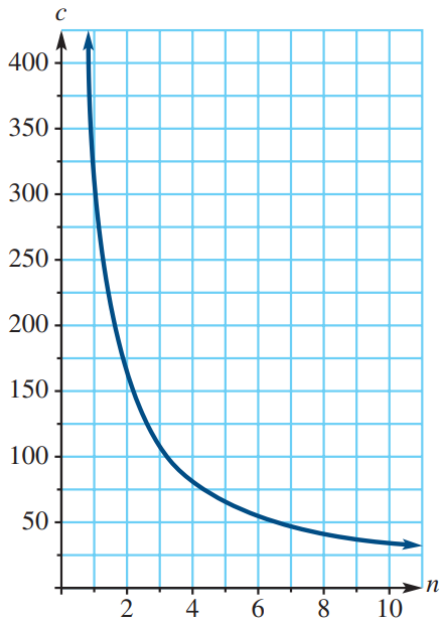
c What is the speed required to complete the road trip in 25 hours?

60 km/h

d Why would it be impossible to complete the road trip in 5 hours?

You would need to travel 300 km/h, which is too fast for a road trip.

2 The total cost (C) of hiring a yacht depends on the number of people (n) sharing the total cost. The equation $C = \frac{320}{n}$ is used to model this situation.



a What is the cost of hiring the yacht for one person?

$320

b What is the cost per person of hiring the yacht if 8 people share the total cost?

$40

c How many people are required to share the cost of hiring a yacht for $80?

4 people

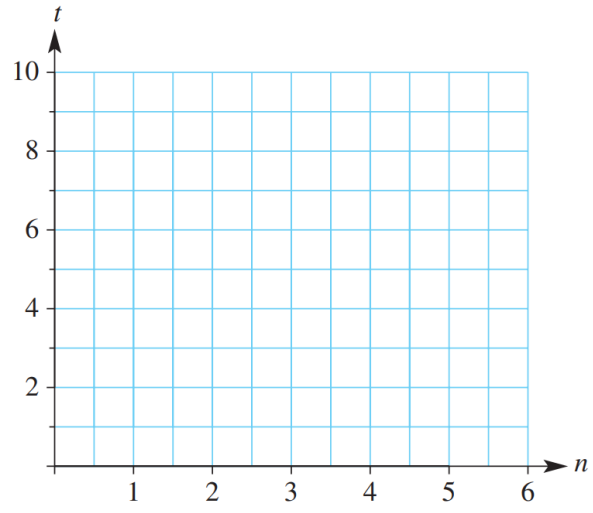
d Is it possible for the cost per person to be $1?

Yes, if 320 people are sharing the cost (it's unlikely).

3 The time taken (t in hours) to dig a hole is dependent on the number of people (n) digging the hole. This relationship is modelled using the formula $t = \frac{6}{n}$

a Complete the table of values and sketch the graph.

n	1	2	3	4	5	6
t						



b How long does it take one person to dig a hole?

6 hours

c How long does it take 3 people to dig a hole?

2 hours

d How long does it take 6 people to dig a hole?

1 hour

e How many people could dig the hole in two hours?

3 people

f How many people could dig the hole in 30 minutes?

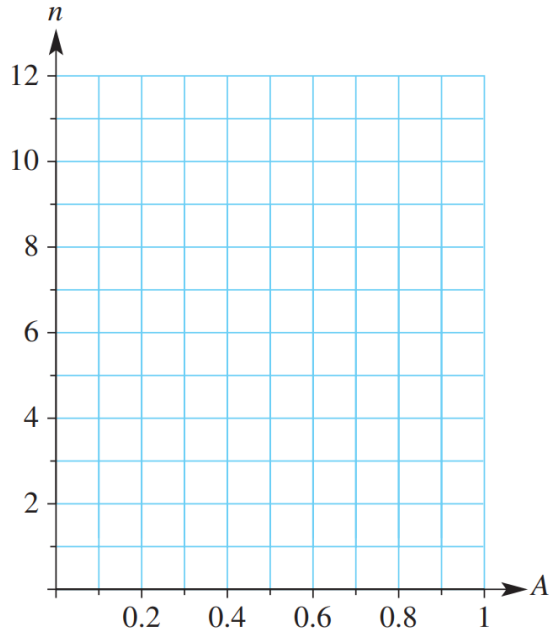
12 people

4 The maximum number of people (n in 1000's) attending an outdoor concert is dependent on the area (A in m^2) allowed per person. The reciprocal equation $n = \frac{1.2}{A}$ models this situation.

a Complete the table of values and sketch the graph.

A	0.1	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1.0
n									

b How many people can attend if the area allowed per person is 0.5 m^2?



2400

c How many people can attend if the area allowed per person is 0.25 m^2?

4800

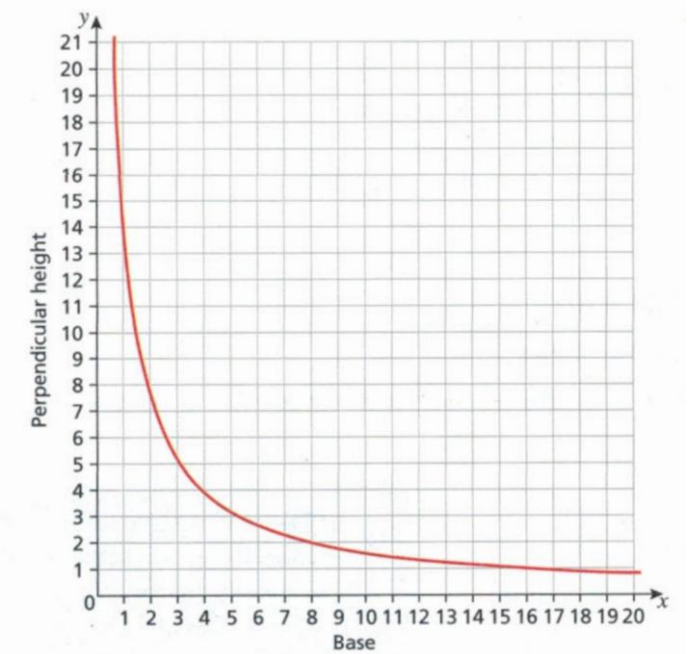
d What is the area allowed per person if the maximum number of people attending the concert is 2000?

0.6 m^2

e Explain why it's impossible for 12 000 people to attend the concert.

Each person would have 0.1 m^2, which is not enough to fit a person

5 The graph shows the relationship between the base length and perpendicular height of a parallelogram with area 16 cm^2.



a Use the graph to find

i the height of a parallelogram with base length 8 cm

2 cm

ii the height of a parallelogram with base length 10 cm

1.6 cm

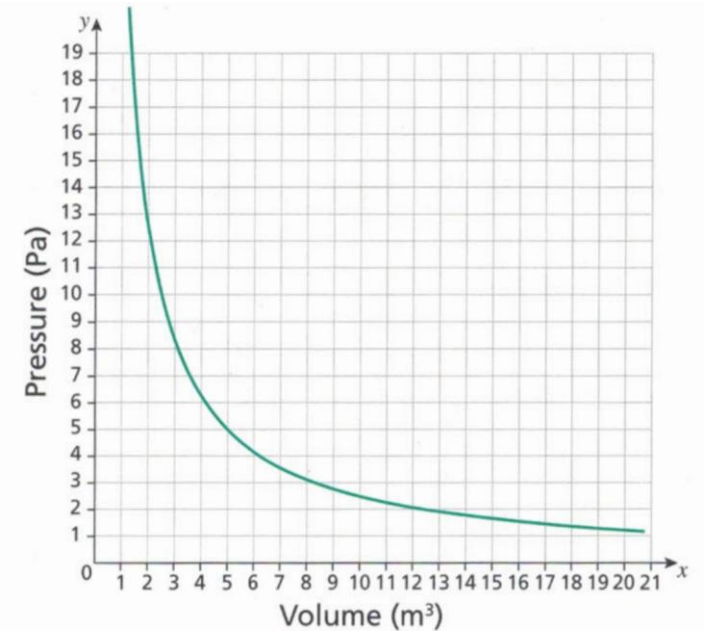
iii the base length of a parallelogram with height 6 cm

$\approx 2.\dot{6}$ cm

b Explain why the equation of the graph is given by $xy = 16$

Area of a parallelogram is $A = bh = 16$, so $xy = 16$

6 The graph below shows Boyle's law.
This is an inversely proportional relationship between the pressure and volume of a gas.



a Use the graph to find

i the volume when the pressure is 15 Pa

≈ 1.6 m^3

ii the pressure when the volume is 4 m^3.

≈ 6.25 Pa

b Write the equation of the graph in the from $xy = k$, where k is the constant of proportionality.

$xy = 24$

7 The diameter of a balloon D varies inversely with the thickness of the rubber x.
The diameter of the balloon is 80 mm when the rubber is 2 mm thick.

a Using $y = \frac{k}{x}$, find the constant of variation k.

160

b Hence write an equation relating diameter D and thickness x.

$D = \frac{160}{x}$

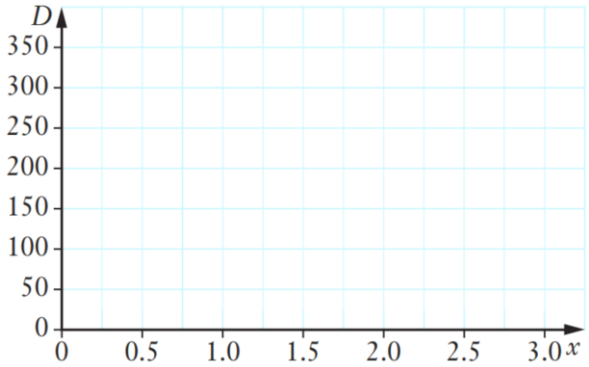
c Find the diameter when the thickness is 0.8 mm.

200 mm

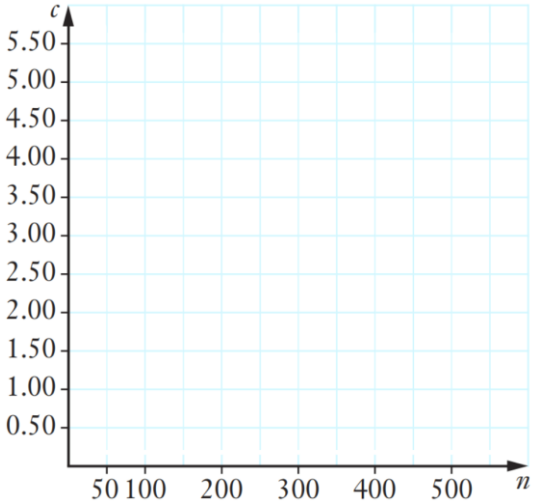
d Find the thickness correct to one decimal place when the diameter of 115.3 mm.

1.4 mm

e Sketch the graph of this information.



8 The number of boxes a factory produces is inversely proportional to the cost to produce each box.
When 128 boxes are produced, it costs \$2 per box.
Find an equation relating cost c and boxes n then sketch the graph.



$c = \frac{256}{n}$

9 2019 HSC Standard 2 Band 4

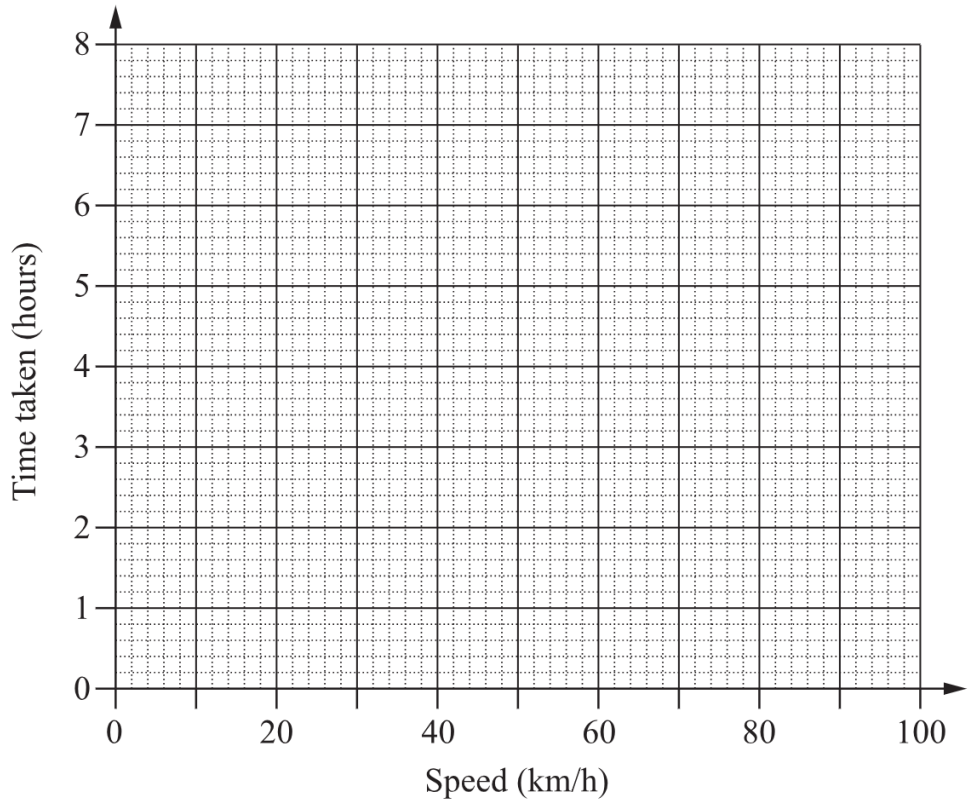
The time taken for a car to travel between two towns at a constant speed varies inversely with its speed.

It takes 1.5 hours for the car to travel between the two towns at a constant speed of 80 km/h

Calculate the distance between the two towns.

120 km

By first plotting four points, draw the curve that shows the time taken to travel between the two towns at different constant speeds.



10 2022 HSC Standard 2 Band 5

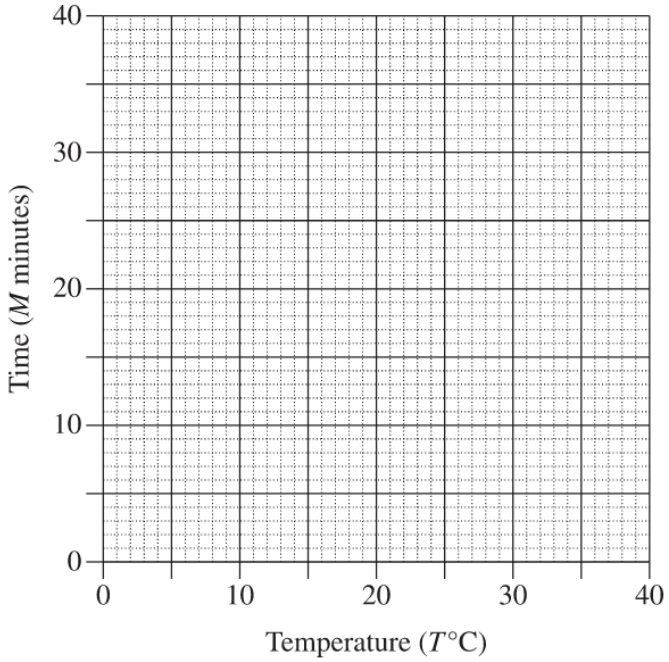
A student believes that the time it takes for an ice cube to melt (M minutes) varies inversely with the room temperature T °C. The student observes that at a room temperature of 15 °C it takes 12 minutes for an ice cube to melt.

Find the equation relating M and T.

$M = \frac{180}{T}$

Using the table, graph the temperature vs time between $T = 5°\text{C}$ and $T = 30°C$

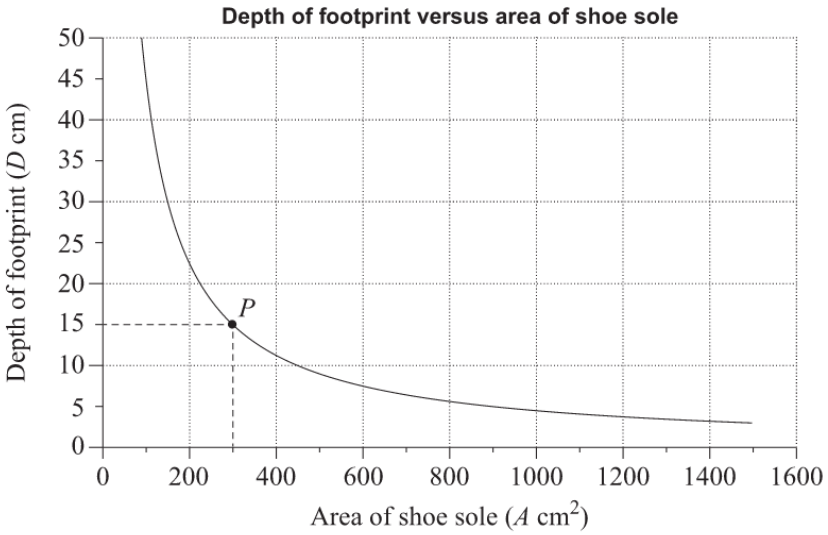
T	5	15	30
M			



36, 12, 6

11 2018 HSC Standard 2 Band 5

When people walk in snow, the depth (D cm) of each footprint depends on both the area (A cm²) of the shoe sole and the weight of the person. The graph shows the relationship between the area of the shoe sole and the depth of the footprint for a group of people of the same weight.



The graph is a hyperbola because D is inversely proportional to A.
Find the equation relating D and A.

$D = \frac{4500}{A}$

A man from this group walks in snow and the depth of his footprint is 4 cm.
Use your equation to calculate the exact area of his shoe sole.

1125 cm^2